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Intuitionistic Fuzzy Approach in Solving Travelling Salesman Problem

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Abstract **-** The Travelling Salesman Problem is to find out the shortest route such that the salesman start from a particular place, visit all the places exactly once and return to the starting place with minimum cost. A Fuzzy version of Hungarian algorithm for the solution of Intuitionistic Fuzzy Travelling Salesman Problem using Triangular Intuitionistic Fuzzy numbers was studied earlier[1]. In this paper we introduce a new representation of Triangular Intuitionistic Fuzzy Number (TIFN) called Arithmetic F-Average using which an Intuitionistic Fuzzy Travelling Salesman Problem is solved. This Arithmetic F- Average is applied in Fuzzy Hungarian algorithm which will be more elegant and simpler than given in [1]. Also we defined some of the properties of this average and a ranking method for this average. The Fuzzy Hungarian algorithm of this average is illustrated with an example.

Keywords *– Arithmetic F- Average*

1. INTRODUCTION

Lotfi. A. Zadeh introduced Fuzzy sets as a generalization of classical crisp set in 1965 and later in 1983 Krassimir T. Atanassov introduced the notion of Intuitionistic Fuzzy set. In 19th century, W. R. Hamilton, an Irish mathematician introduced the Travelling Salesman Problem (TSP). In recent years, Fuzzy Travelling Salesman Problem has got great attraction and have been approached using several methods. Mukerjee and Basu [2] proposed a new method to solve Fuzzy TSP. K. Prabhakaran and K. Ganeshan [1] used Fuzzy Hungarian method to solve Intuitionistic Fuzzy TSP in terms of Triangular Intuitionistic Fuzzy Number (TIFN). Here we introduce a new representation of TIFN called Arithmetic F- Average to solve Intuitionistic Fuzzy TSP. Also we introduce a new ranking method of Arithmetic F-Average .

This paper is organized as follows: Section 2, consists of concepts of Intuitionistic Fuzzy Set (IFS), definition of TIFN and Intuitionistic Fuzzy Travelling Salesman Problem (IFTSP). In section 3, we define Arithmetic F-Average denoted by Å which is a new representation of TIFN, some basic operations on Å, ranking of Å and Fuzzy Hungarian Algorithm of Arithmetic F- Average. An illustrative example is given in section 4. Section 5 concludes this paper.

2. PRELIMINARIES

Definition 2.1. Intuitionistic Fuzzy Set [3]

An Intuitionistic Fuzzy Set (IFS) in the universe of discourse X is defined as

$$\hat{A} = \{(x, \mu_{\hat{A}}(x), \nu_{\hat{A}}(x), / x \in X \}$$

where $\mu_{\hat{A}} : X \to [0,1]$ and $\nu_{\hat{A}} : X \to [0,1]$ determine the degree of membership and degree of non - membership of the element x in Â respectively and for every $x \in X$, $0 \leq \mu_{\hat{A}}(x) + \nu_{\hat{A}}(x) \leq 1$.

Definition 2.2. Triangular Intuitionistic Fuzzy Number (TIFN) [1]

A Triangular Intutionistic Fuzzy Number (TIFN) with parameters $\hat{a}_1 \leq a_1 \leq a_2 \leq a_3 \leq \hat{a}_3$ is an Intuitionistic Fuzzy Number having the membership function and non-membership function as follows:

$$\mu_{\hat{A}}(x) = \begin{cases} 0, & \text{for } x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & \text{for } a_1 \le x \le a_2 \\ 1 & \text{for } x = a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{for } x > a_3 \end{cases}$$

$$\nu_{\hat{A}}(x) = \begin{cases} 1 & \text{for } x < \hat{a}_1 \\ \dfrac{a_2-x}{a_2-\hat{a}_1} & \text{for } \hat{a}_1 \le x \le a_2 \\ 0 & \text{for } x = a_2 \\ \dfrac{x-a_2}{\hat{a}_3-a_2} & \text{for } a_2 \le x \le \hat{a}_3 \\ 1 & \text{for } x > \hat{a}_3 \end{cases}$$

It is denoted by ä= (a_1, a_2, a_3; $\hat{a}_1$, a_2, $\hat{a}_3$)

Definition 2.3. Intuitionistic Fuzzy Travelling Salesman Problem [1]

A salesman has to visit n cities. He wishes to start from a particular city, visit each city exactly once and then return to his starting point. The objective is to select the sequence in which the cities are visited in such a way that his total travelling cost is minimized. The mathematical form of the Fuzzy Travelling Salesman Problem is :

$$\text{Minimize} \quad Z = \sum_{i=1}^{n}\sum_{j=1}^{n} \ddot{c}_{ij}\ddot{x}_{ij}$$

Subject to $\sum_{i=1}^{n} \ddot{x}_{ij} \approx \ddot{1}$, $j = 1,2 \dots n$

$\sum_{j=1}^{n} \ddot{x}_{ij} \approx \ddot{1}$, $i = 1,2 \dots n$

Where

$$\ddot{x}_{ij} \approx \begin{cases} \ddot{1}, & \text{if the salesman travel from city i to city j} \\ \ddot{0}, & \text{otherwise} \end{cases}$$

And $\ddot{c}_{ij}$ denote the Intuitionistic fuzzy cost of travelling from city i to city j.

3. ARITHMETIC F- AVERAGE

Definition 3.1.

For an arbitrary TIFN ä= (a_1, a_2, a_3 ; $\hat{a}_1$, a_2, $\hat{a}_3$) the Arithmetic F- Average denoted by Å is defined as:

Å = (a_m, a_n)

where $a_m = \dfrac{a_1 + a_2 + a_3}{3}$ and $a_n = \dfrac{\hat{a}_1 + a_2 + \hat{a}_3}{3}$

3.2. Properties of Arithmetic F- Average

We define the arithmetic operations on Arithmetic F- Average as follows :

For any two Arithmetic F- Average Å = (a_m, a_n) and Ů = (u_m, u_n)

Addition

Å + Ů = (a_m, a_n) + (u_m, u_n) = ($a_m + u_m$, $a_n + u_n$)

Subtraction

Å - Ů = (a_m, a_n) - (u_m, u_n) = ($a_m - u_m$, $a_n - u_n$)

Multiplication

Å × Ů = (a_m, a_n) × (u_m, u_n) = ($a_m \times u_m$, $a_n \times u_n$)

Division

Å ÷ Ů = (a_m, a_n) ÷ (u_m, u_n) = ($a_m \div u_m$, $a_n \div u_n$), $u_m \neq 0$, $u_n \neq 0$

3.3. Ranking of Arithmetic F- Average

Ranking of Arithmetic F- Average is necessary in decision making.

For an arbitrary Arithmetic F- Average Å = (a_m, a_n), the magnitude of Arithmetic F- Average Å is denoted by Mag (Å) and is defined as

$$\text{Mag (Å)} = \frac{a_m + a_n}{6}$$

For any two Arithmetic F- Average Å = (a_m, a_n) and Ů = (u_m, u_n) we define the ranking of Arithmetic F- Average by comparing their magnitude as follows :

1. Å $\geq$ Ů if and only if Mag (Å) ≥ Mag (Ů)

2. Å $\leq$ Ů if and only if Mag (Å) ≤ Mag (Ů)

3. Å ≈ Ů if and only if Mag (Å) = Mag (Ů)

3.4. Fuzzy Hungarian Algorithm of Arithmetic F-Average

The Fuzzy Hungarian Algorithm of Arithmetic F- Average for solving Intuitionistic Fuzzy Travelling Salesman Problem in Intuitionistic Fuzzy context is as follows:

Step 1. Express all the Triangular Intuitionistic Fuzzy Numbers in the given problem in terms of Arithmetic F-Average given in definition 3.1.

Step 2. Identify the smallest element in each row using ranking of Arithmetic F- Average and subtract it from all other elements of that row. Then each row has at least one Intuitionistic Fuzzy zero.

Step 3. Identify the smallest element in each column of the resulting Assignment table of step 2. Subtract it from all other elements of each column. Now each column and row has at least one Intuitionistic Fuzzy zero.

Step 4 Intuitionistic Fuzzy optimal assignment is allotted to the resulting assignment table obtained from step 3 as follows:

a) Examine all the rows continuously until a row with exactly one Intuitionistic Fuzzy zero is identified. Allocate this single Intuitionistic Fuzzy zero and then cancel every other Intuitionistic Fuzzy zeros in that column. Repeat this process for all other rows.
b) Do the same process for all the columns of the Assignment Table of step 4 (a).
c) If a row and or column has two or more Intuitionistic Fuzzy zeros, then allocate any one of them randomly and cancel all other Intuitionistic Fuzzy zeros in that row or column.

Repeat this step until all the Intuitionistic Fuzzy zeros either get allocated or crossed off.

Step 5. If the number of allocations equals order of the matrix or if each row and column gets exactly one assignment, then an optimal solution is attained. Otherwise go to step 6.

Step 6. Draw least number of horizontal and or vertical lines to cover all the Intuitionistic Fuzzy zeros in the assignment table obtained by step 5, by using the following procedure :

a) Note the rows without any assignment.
b) Note the columns having Intuitionistic Fuzzy zero in the noted rows .
c) Note the rows having assigned Intuitionistic Fuzzy zero in the columns considered above.
d) Repeat till no further columns or rows can be marked.
e) Draw lines through each marked columns and unmarked rows.

This will give required number of least lines covering all the Intuitionistic Fuzzy zeros.

Step 7. Develop the new revised Intuitionistic Fuzzy cost matrix as follows: From among the entries not covered by any of the lines in the assignment table obtained by step 6, choose the smallest element. Subtract it from all entries of the table not covered by any line, add it to all the entries lying at the intersection of two lines. Entries covered by just one line are left unchanged.

Step 8. Repeat steps 6 to 8 until an optimal solution is obtained.

4. NUMERICAL EXAMPLE

Consider an Intuitionistic Fuzzy Travelling salesman Problem. The cost matrix whose entries represent the travelling cost from one city to other which are expressed using Arithmetic F- Average is given below. The Fuzzy Hungarian Algorithm is used to find a route for a travelling salesman that starts from his home city A, go through other cities B, C, D, E exactly once and return to A at lowest possible cost

	A	*B*	*C*	*D*	*E*
A	∞	(25,25)	(40,40)	(10,10)	(12,12)
B	(25,25)	∞	(20,20)	(23,23)	(11,11)
C	(40,40)	(20,20)	∞	(23,23)	(33,33)
D	(10,10)	(23,23)	(23,23)	∞	(20,20)
E	(12,12)	(11,11)	(33,33)	(20,20)	∞

The Magnitude matrix of the above matrix is

	A	*B*	*C*	*D*	*E*
A	∞	8.3	13.3	3.3	4
B	8.3	∞	6.6	7.6	3.6
C	13.3	6.6	∞	7.6	11
D	3.3	7.6	7.6	∞	6.6
E	4	3.6	11	6.6	∞

Applying step 2 we get,

	A	*B*	*C*	*D*	*E*
A	∞	(15,15)	(30,30)	(0,0)	(2,2)
B	(14,14)	∞	(9,9)	(12,12)	(0,0)
C	(20,20)	(0,0)	∞	(3,3)	(13,13)
D	(0,0)	(13,13)	(13,13)	∞	(10,10)
E	(1,1)	(0,0)	(22,22)	(9,9)	∞

Calculate Magnitude matrix of the above matrix and apply step 3

	A	*B*	*C*	*D*	*E*
A	∞	(15,15)	(21,21)	(0,0)	(2,2)
B	(14,14)	∞	(0,0)	(12,12)	(0,0)
C	(20,20)	(0,0)	∞	(3,3)	(13,13)
D	(0,0)	(13,13)	(4,4)	∞	(10,10)
E	(1,1)	(0,0)	(13,13)	(9,9)	∞

Applying step 4 we get,

	A	*B*	*C*	*D*	*E*
A	∞	(15,15)	(21,21)	[(0,0)]	(2,2)
B	(14,14)	∞	[(0,0)]	(12,12)	~~(0,0)~~
C	(20,20)	[(0,0)]	∞	(3,3)	(13,13)
D	[(0,0)]	(13,13)	(4,4)	∞	(10,10)
E	(1,1)	~~(0,0)~~	(13,13)	(9,9)	∞

Apply steps 6 and7

	A	*B*	*C*	*D*	*E*
A	∞	(16,16)	(21,21)	[(0,0)]	(2,2)
B	(14,14)	∞	[(0,0)]	(12,12)	~~(0,0)~~
C	(19,19)	[(0,0)]	∞	(2,2)	(12,12)
D	[(0,0)]	(14,14)	(4,4)	∞	(10,10)
E	~~(0,0)~~	~~(0,0)~~	(12,12)	(8,8)	∞

Again apply steps 6 and 7

	A	*B*	*C*	*D*	*E*
A	∞	(18,18)	(21,21)	[(0,0)]	(2,2)
B	(16,16)	∞	[(0,0)]	(12,12)	~~(0,0)~~
C	(19,19)	[(0,0)]	∞	~~(0,0)~~	(10,10)
D	[(0,0)]	(14,14)	(2,2)	∞	(8,8)
E	~~(0,0)~~	~~(0,0)~~	(10,10)	(6,6)	∞

Again apply steps 6 & 7

	A	*B*	*C*	*D*	*E*
A	∞	(18,18)	(19,19)	[(0,0)]	~~(0,0)~~
B	(18,18)	∞	~~(0,0)~~	(14,14)	[(0,0)]
C	(19,19)	[(0,0)]	∞	~~(0,0)~~	(8,8)
D	~~(0,0)~~	(14,14)	[(0,0)]	∞	(6,6)
E	[(0,0)]	~~(0,0)~~	(8,8)	(6,6)	∞

Here each row and column got one assignment. That is Intuitionistic Fuzzy optimal solution is obtained. The Intuitionistic Fuzzy Optimal assignment using Fuzzy Hungarian Algorithm for Arithmetic F- Average for the Fuzzy travelling salesman problem is

A → D, B → E, C→B, D→C, E→ A

The route for the salesman is A→ D→C→B→E→A

5. CONCLUSION

Here we have defined a new representation of TIFN called Arithmetic F- Average. Also we have introduced some of its properties and a new ranking method. A numerical example which was discussed in [1] is solved using Arithmetic F- Average. It is more elegant and simpler than that was discussed in [1].

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