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Research Paper

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Approximation of Reciprocal-Cubic Functional Equation in Non-Archimedean Normed Space

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Abstract- The aim of this paper is to study the stability of reciprocal-cubic functional equation using direct method in non-Archimedean normed spaces.

Keywords- Generalized Hyers-Ulam stability, Reciprocal-cubic functional equation and non-archimedean normed spaces.
Mathematical subject classification- 39B72, 39B82.

I. Introduction

The stability of different types of functional equation was studied by many mathematicians after Ulam[1] presented various unsolved problems in his famous talk in 1940.Ulam's problem was solved by Hyers [10], T. Aoki [2], Th.M.Rassias[13] and Gavruta[9] under different adaptations.After that several stability results for various functional equations in different spaces have been widely studied.

In 2010,K.Ravi and B.V. Senthil Kunar[17] introduced and proved the generalized Hyers-Ulam stability of the reciprocal functional equation

$$f(x+y)=\frac{f(x)f(y)}{f(x)+f(y)} \qquad (1)$$

in the space of nonzero real numbers.It is easily seen that the reciprocal function $f(x)=\frac{a}{x}$ is a solution of the functional equation (1).

In 2014, kim and Bodaghi[5] introduced and proved the generalized Hyers-Ulam stability of the quadratic reciprocal functional equation

$$f(2x+y)+f(2x-y)=\frac{2f(x)f(y)\big(4f(x)+f(y)\big)}{(4f(y)-f(x))^2} \qquad (2)$$

Recently, K.Ravi et al.[18] and Kim et al.[12] investigated the generalized Hyers-Ulam stability of the cubic reciprocal functional equations

$$f(2x+y)+f(2x-y)=\frac{4f(x)f(y)(4f(y)+3f(x)^{2/3}f(y)^{2/3})}{(4f(y)^{2/3}-f(x)^{2/3})^3} \qquad (3)$$

and

$$f(2x+y)+f(x+2y)=\frac{9f(x)f(y)(f(x)+f(y)+2f(x)^{1/3}f(y)^{1/3}(f(x)^{1/3}+f(y)^{1/3}))}{(2f(x)^{2/3}+2f(y)^{2/3}+5f(x)^{1/3}f(y)^{1/3})^3} \qquad (4)$$

in non-Archimedean fields respectively.Some more results about the stability of various types of reciprocal functional equation can be studied from[6,14 − 16,19,20].

In this paper, we generalize equation (4) and investigate the generalized Hyers-Ulam stability of this reciprocal-cubic functional equation in the framework of non-Archimedean Normed spaces.

$$\frac{f((k+1)x+ky)+f(kx+(k+1)y)}{f(x)f(y)}=$$
$$\frac{(k^3+(k+1)^3)(f(x)+f(y))+3k(2k+1)(k+1)(f(x)f(y))^{\frac{1}{3}}(f(x)^{\frac{1}{3}}+f(y)^{\frac{1}{3}})}{[k(k+1)(f(x)^{\frac{2}{3}}+f(y)^{\frac{2}{3}})+(2k(k+1)+1)f(x)^{\frac{1}{3}}f(y)^{\frac{1}{3}}]^3} \qquad (5)$$

It can be easily seen that the reciprocal-cubic function $f(x)=\frac{c}{x^3}$ is a solution of the reciprocal-cubic functional equation (5).

In section 2, we will discuss terminologies and definitions to be used throughout the paper.In section 3, we will provide the generalized Hyers-Ulam stability of (5) in non-Archimedean Normed spaces. We will provide counter example for the generalized Hyers-Ulam stability of reciprocal-cubic functional equation for some special case.

II. Preliminaries

A *Non-Archimedean field* is a field K equipped with a function $|\diamond|: K\to R$ such that for any $\alpha,\beta\in K$ we have

- $|\alpha|\geq 0$ and equality holds if and only if $\alpha=0$,
- $|\alpha\beta|=|\alpha||\beta|$,
- $|\alpha+\beta|\leq max\{|\alpha|,|\beta|\}$ (strict triangle inequality).

From above conditions, we have $|1|=|-1|=1$ and $|n|\leq 1$ for each integer n.Also if we set $|x|=1$ for all nonzero $x\in K$ and $|0|=0$, we have an non-Achimedean valuation on K, called the *trivial non-Achimedean valuation*.In addition we assume that$|\diamond|$ is non trivial, i.e.

there is an $\alpha_o \in K$ such that $|\alpha_o| \notin \{0,1\}$.

2.1 Definition

Let X be a vector space over a field K with a non-Archimedean valuation $|*|$. A function $||*||: X \to [0,\infty)$ is called a *non-Archimedean norm* if the following conditions hold:

- $||x|| = 0$ if and only if $x = 0$ for all $x \in X$;
- $||rx|| = |r|||x||$ for all $r \in K$ and $x \in X$;
- the strong triangle inequality:
  $||x+y|| \leq max\{||x||,||y||\}$for all $x,y \in X$.

Then $(X,||*||)$is called a non-Archimedean normed space.

2.2 Definition

Let $\{x_n\}$ be a sequence in a non-Archimedean normed space X.

1.
A sequence $\{x_n\}_{n=1}^{\infty}$ in a non-Archimedean space is a *Cauchy sequence* if and only if, the sequence $\{x_{n+1} - x_n\}_{n=1}^{\infty}$ converges to zero.

2.
The sequence $\{x_n\}$ is said to be convergent if, for any $\varepsilon > 0$, there are a positive integer N and $x \in X$ such that $||x_n - x|| \leq \varepsilon$ for all $n \geq N$. Then, the point $x \in X$ is called the limit of the sequence $\{x_n\}$, which is denoted by $lim_{n\to\infty} x_n = x$.

3. If every Cauchy sequence in X converges, then the non-Archimedean normed space X is called a *non-Archimedean Banach space.*

Throughout this paper, we consider that **X** and **Y** are a non-Archimedean field and a complete non-Archimedean field, respectively.Let us define the function $D_f: X \times X \to Y$ by

$$D_f(x,y) = f((k+1)x+ky) + f(kx+(k+1)y) - \frac{(k^3+(k+1)^3)(f(x)+f(y))+3k(2k+1)(k+1)(f(x)f(y))^{\frac{1}{3}}(f(x)^{\frac{1}{3}}+f(y)^{\frac{1}{3}})}{[k(k+1)(f(x)^{\frac{2}{3}}+f(y)^{\frac{2}{3}})+(2k(k+1)+1)f(x)^{\frac{1}{3}}f(y)^{\frac{1}{3}}]^3} \quad (6)$$

for all $x,y \in X$. Also let us assume numerator and denominator of equation (5) is non zero for all $x \in X$ and $x \neq y$.

III. Stability of Functional Equation (5) in Non-Archimedean Space

In this section, using direct method, we will prove the generalized Hyers-Ulam stability of cubic functional equation (5) in non-Archimedean normed spaces.

3.1 Theorem

Let $\Phi: X \times X \to Y$ be a function such that

$$lim_{n\to\infty}|(2k+1)|^{(-3n)}\Phi\left(\frac{x}{(2k+1)^{(n+1)}},\frac{x}{(2k+1)^{(n+1)}}\right) = 0 \quad (7)$$

for all $x,y \in X$ and suppose that $f: X \to Y$ be a mapping satisfying the following inequality

$$|D_f(x,y)| \leq \Phi(x,y) \quad (8)$$

for all $x,y \in X$ .Then,the limit $C(x) = lim_{n\to\infty}(2k+1)^{-3n} f\left(\frac{x}{(2k+1)^n}\right)$ exists for all $x \in X$ and defines a unique cubic reciprocal mapping $C: X \to Y$ such that for all $x \in X$, we have,

$$|f(x) - C(x)| \leq max\left\{\frac{|(2k+1)|^{-3j}}{2}\Phi\left(\frac{x}{(2k+1)^{j+1}},\frac{x}{(2k+1)^{j+1}}\right); j \in N \cup \{0\}\right\} \quad (9)$$

Moreover if,

$$|f(x) - C(x)| \leq lim_{j\to\infty} lim_{n\to\infty} max\left\{\frac{|(2k+1)|^{-3(m+j)}}{2}\Phi\left(\frac{x}{(2k+1)^{m+j+1}},\frac{x}{(2k+1)^{m+j+1}}\right); j \leq m < n+j\right\} = 0 \quad (10)$$

then $C(x)$ is unique cubic reciprocal mapping satisfying (9).

Proof: Existence- Replacing (x,y) by (x,x) and substituting x by $\frac{x}{(2k+1)}$ in (8), we get

$$\left|f(x) - (2k+1)^{-3} f\left(\frac{x}{(2k+1)}\right)\right| \leq \frac{1}{2}\Phi\left(\frac{x}{(2k+1)},\frac{x}{(2k+1)}\right) \quad (11)$$

for all $x \in X$ .In (11) replacing x by $x/(2k+1)^n$ and multiplying by $(2k+1)^{-3n}$,we obtain

$$\left|(2k+1)^{-3n} f\left(\frac{x}{(2k+1)^n}\right) - (2k+1)^{-(3n+3)} f\left(\frac{x}{(2k+1)^{n+1}}\right)\right|$$

$$\leq \frac{1}{2}|(2k+1)|^{-3n}\Phi\left(\frac{x}{(2k+1)^{n+1}},\frac{x}{(2k+1)^{n+1}}\right) \quad (12)$$

From equations (7) and (12) we can say that the sequence $(2k+1)^{-3n} f\left(\frac{x}{(2k+1)^n}\right)$ is a cauchy sequence.As Y is complete,therefore the sequence $\{(2k+1)^{-3n} f\left(\frac{x}{(2k+1)^n}\right)\}_{n\geq 1}$ is convergent also.Consider

$$C(x) = lim_{n\to\infty}\left\{(2k+1)^{-3n} f\left(\frac{x}{(2k+1)^n}\right)\right\}.$$

Next by using induction, we can easily show that

$$\left|(2k+1)^{-3n} f\left(\frac{x}{(2k+1)^n}\right) - f(x)\right|$$
$$\leq \left|\sum_{j=0}^{n-1}\left\{(2k+1)^{-3(j+1)} f\left(\frac{x}{(2k+1)^{j+1}}\right) - (2k+1)^{-3j} f\left(\frac{x}{(2k+1)^j}\right)\right\}\right|$$
$$\leq max\left\{\left|(2k+1)^{-3(j+1)} f\left(\frac{x}{(2k+1)^{j+1}}\right) - (2k+1)^{-3j} f\left(\frac{x}{(2k+1)^j}\right)\right| : 0 \leq j < n\right\}$$
$$\leq max\left\{\frac{|(2k+1)|^{-3j}}{2}\Phi\left(\frac{x}{(2k+1)^{j+1}},\frac{x}{(2k+1)^{j+1}}\right): 0 \leq j < n\right\} \quad (13)$$

for all $x \in X$ and $n \in N$.By taking limit $n \to \infty$ in (13) and using definition of $C(x)$, we can say that the inequality (9) is true.With the help of (7),(8)and again using definition of $C(x)$,for all $x,y \in X$, we can say that

$$|D_C(x,y)| = lim_{n\to\infty}\left|\frac{1}{(2k+1)}\right|^{3n}\left|D_f\left(\frac{x}{(2k+1)^n},\frac{y}{(2k+1)^n}\right)\right|$$
$$\leq lim_{n\to\infty}\left|\frac{1}{(2k+1)}\right|^{3n}\Phi\left(\frac{x}{(2k+1)^n},\frac{y}{(2k+1)^n}\right) = 0.$$

Thus ,the mapping $C(x)$ satisfies(5) and hence it is a cubic-reciprocal functional equation.

Uniqueness-To prove the uniqueness of C ,let $Q: X \to Y$ be another function which satisfies (9).Then,for all $x \in X$

$$|C(x)-Q(x)| = lim_{j\to\infty}\left|(2k+1)\right|^{-3j}\left|C\left(\frac{x}{(2k+1)^j}\right) - Q\left(\frac{x}{(2k+1)^j}\right)\right|$$
$$\leq lim_{j\to\infty}|(2k+1)|^{-3j}max\{|C(\frac{x}{(2k+1)^j}) - f(\frac{x}{(2k+1)^j})|, |f(\frac{x}{(2k+1)^j}) - Q(\frac{x}{(2k+1)^j})|\}$$

$$\leq lim_{j\to\infty}lim_{n\to\infty}max\{\frac{|(2k+1)|^{-3(m+j)}}{2}\Phi(\frac{x}{(2k+1)^{m+j+1}}, \frac{x}{(2k+1)^{m+j+1}}); j\leq m<n+j\} = 0$$

therefore $C=Q$ .Hence the proof.

3.2 Theorem

Let $\Phi: X\times X\to Y$ be a function such that

$lim_{n\to\infty}|(2k+1)|^{3n}\Phi((2k+1)^n x, (2k+1)^n x) = 0$(14)

for all $x,y\in X$ and suppose that $f: X\to Y$ be a mapping satisfying the inequality (8) for all $x,y\in X$. Then,the limit $C(x) = lim_{n\to\infty}(2k+1)^{3n} f(x(2k+1)^n)$ exists for all $x\in X$ and defines a unique cubic reciprocal mapping $C: X\to Y$ such that for all $x\in X$

$$|f(x)-C(x)| \leq max\{\frac{|(2k+1)|^{3(j+1)}}{2}\Phi(x(2k+1)^j, x(2k+1)^j); j\in N\cup\{0\}\}. \quad (15)$$

moreover if, $|f(x)-C(x)| \leq$

$$lim_{j\to\infty}lim_{n\to\infty}max\{\frac{|(2k+1)|^{3(m+j+1)}}{2}\Phi((2k+1)^{m+j}x, (2k+1)^{m+j}x); j\leq m<n+j\} = 0$$

then $C(x)$ is unique cubic reciprocal mapping satisfying (15).

Proof: Existence- Replacing (x,y) by (x,x) in (8) and multiplying by $(2k+1)^3$, then applying same arguments as in previous theorem we can easily get the proof.

3.3 Corollary

Let $\varepsilon\geq 0$ and $a+b=p\neq -3$ be constants.If $f: X\to Y$ satisfies

$$|D_f(x,y)| = \begin{cases}\varepsilon(|x|^p+|y|^p) & p<-3 \quad or \quad p>-3\\ \varepsilon(|x|^a|y|^b) & p<-3 \quad or \quad p>-3,\\ \varepsilon(|x|^{p/2}|y|^{p/2}+(|x|^p+|y|^p)) & p<-3 \quad or \quad p>-3\end{cases}$$

for all $x\in X$,then there exists a unique cubic-reciprocal functional equation $C(x): X\to Y$ satisfying (5) and

$$|C(x)-f(x)| = \begin{cases}\frac{\varepsilon|x|^p}{(2k+1)^p} & p>-3\\ \varepsilon(2k+1)^3|x|^p & p<-3\\ \frac{\varepsilon|x|^p}{2(2k+1)^p} & p>-3\\ \frac{\varepsilon(2k+1)^3|x|^p}{2} & p<-3\\ \frac{3\varepsilon|x|^p}{2(2k+1)^p} & p>-3\\ \frac{3\varepsilon(2k+1)^3|x|^p}{2} & p<-3\end{cases},$$

for all $x\in X$.

Proof: Applying theorem 3.1 and 3.2 with appropriate choice of $\Phi(x,y)$ we can get the desired result.

IV. Counter-examples

In this section we will provide examples to show non-stability of functional equation (5) for $p=-3$ and $\alpha=\frac{-1}{k}$ in **R** with usual metric $|.|$ in corollary (3.3) using well-known counter example provided by Z. Gajda[8].Consider the function $\Phi: R^{å}\to R$ defined as

$$\Phi(x) = \begin{cases}\frac{\vartheta}{x^3} & for \quad x \in (1,\infty)\\ \vartheta & otherwise\end{cases}$$

where $\vartheta>0$ is a constant, and let for all $x\in R^*$ the function $f: R^*\to R$ be defined as

$$f(x) = \sum_{n=0}^{\infty}\frac{\Phi((2k+1)^{-n}x)}{(2k+1)^{3n}} \quad (16)$$

4.1 Theorem

If $f: R^*\to R$ as defined in (16) satisfies the functional inequality

$$|D_f(x,y)| \leq \frac{2\vartheta((2k+1)^3+1)}{(2k+1)^3-1}(|x|^{-3}+|y|^{-3}) \quad (17)$$

for all $x\in R^*$.Then there do not exist a cubic reciprocal mapping $C: R^*\to R$ and a constant $\rho>0$ such that

$$|f(x)-Q(x)| \leq \rho|x|^{-3} \quad (18)$$

for all $x\in R^*$.

Proof: $|f(x)| = |\sum_{n=0}^{\infty}\frac{\Phi((2k+1)^{-n}x)}{(2k+1)^{3n}}| \leq \sum_{n=0}^{\infty}\frac{\vartheta}{(2k+1)^{3n}} = \frac{(2k+1)^3\vartheta}{2k(4k^2+6k+3)}$.

Hence the function is bounded. If $(|x|^{-3}+|y|^{-3})\geq 1$ then L.H.S. of (17) is less than $\frac{2\vartheta((2k+1)^3+1)}{(2k+1)^3-1}$. Suppose that $0<(|x|^{-3}+|y|^{-3})<1$.Then there exists a positive number m such that $\frac{1}{(2k+1)^{3(m+1)}} \leq (|x|^{-3}+|y|^{-3}) \leq \frac{1}{(2k+1)^{3m}}$ (19)

Hence we can say that

$$(2k+1)^{3m}(|x|^{-3}+|y|^{-3}) < 1$$

or $\frac{x^3}{(2k+1)^{3m}} > 1, \frac{y^3}{(2k+1)^{3m}} > 1$

or $\frac{x}{(2k+1)^{(m-1)}} > (2k+1) > 1, \frac{y}{(2k+1)^{(m-1)}} > (2k+1) > 1$

and consequently $\frac{(k+1)x+ky}{(2k+1)^{(m-1)}} > 1, \frac{kx+(k+1)y}{(2k+1)^{(m-1)}} > 1$. Therefore, for each n= 0,1,2,....,m-1,we have

$$\frac{(k+1)x+ky}{(2k+1)^{n}} > 1, \frac{kx+(k+1)y}{(2k+1)^{n}} > 1.$$

and $D_{\Phi}((2k+1)^{-n}x, (2k+1)^{-n}y) = 0$ for n= 0,1,2,....,m-1.Using definition of functions f(x) and $\Phi(x)$, we can easily calculate that

$$|D_f(x,y)| \leq 2\sum_{n=m}^{\infty}(\vartheta/(2k+1)^{3n}) + \frac{(\sum_{n=m}^{\infty}(\vartheta/(2k+1)^{3n}))^3[2(k^3+(k+1)^3)+6k(k+1)(2k+1)]}{(2k(k+1)+2k(k+1)+1)^3(\sum_{n=m}^{\infty}(\vartheta/(2k+1)^{3n}))^2}$$
$$\leq 2\sum_{n=m}^{\infty}(\vartheta/(2k+1)^{3n})(1+\frac{1}{(2k+1)^3}) \leq \frac{2\vartheta((2k+1)^3+1)}{(2k+1)^3-1}(|x|^{-3}+|y|^{-3})$$

for all $x \in R^*$.Hence (17) is proved.Next we claim that the cubic reciprocal functional equation is not stable for p= -3 in corollary (3.3).Assume that there exists a cubic reciprocal functional equation $Q: R^* \to R$ satisfying (18).Therefore ,we have

$$|f(x)| \leq (1+\rho)|x|^{-3} \quad (20)$$

Next,we can choose a positive integer r with $r\vartheta > \rho + 1$.If $x \in (1, 2^{r-1})$ then $3^{-n}x \in (1,\infty)$ for all n=0,1,2,....r-1 and therefore

$$|f(x)| = |\sum_{n=0}^{\infty}\frac{\Phi((2k+1)^{-n}x)}{(2k+1)^{3n}}| \geq \sum_{n=0}^{r-1}\frac{(2k+1)^{3n}\vartheta/x^3}{(2k+1)^{3n}} = \frac{r\vartheta}{x^3} > (\rho+1)x^{-3}$$

which is contradiction to (20),which completes the proof.

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