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Life Test Sampling Plans Based on Marshall – Olkin Extended Exponential Distribution

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Abstract: Sampling inspection plans for life tests, termed as reliability sampling plans, are the special procedures that are employed to determine the acceptability or non-acceptability of a lot of finished items by carrying out tests of lifetime of the items drawn randomly from the lot and observing the number of failures of items. A specific life test sampling plan can be developed considering the lifetime of the products as the quality characteristic, which is modelled by an appropriate continuous-type probability distribution. Various procedures and rules for the design and evaluation of life test sampling plans based on the tests of sampled lifetime data have been developed and are found in the literature of product control. In this paper, evaluation of life test sampling plans under the conditions for application of Marshall – Olkin extended exponential distribution is considered. Three different criteria for designing life test plans when lot quality is evaluated in terms of mean life, hazard rate and reliability life are proposed. Factors for adapting MIL-STD-105D to life and reliability testing indexed by acceptable quality level under the assumption of Marshall – Olkin extended exponential distribution are also illustrated.

Keywords: *Acceptable mean life, Hazard rate, Marshall – Olkin Extended Exponential Distribution, Mean life, Reliable life, Reliability sampling.*

I. INTRODUCTION

Product control is the methodology that consists of procedures with which acceptability or non-acceptability of a lot of finished items can be determined by the inspection of one or more samples of items drawn randomly from the lot. Most often, the perception of producers as well as consumers about the quality of the items is greatly influenced by the efficacy of items for a certain length of time, and hence, the lifetime of an item becomes an important quality characteristic and its assessment turns into an imperative aspect in product control. The process of assessing the lifetime of the product or item through experiments is called a life-test. Sampling inspection procedures that are adopted for taking decisions on the disposition of the lot(s) of items based on the assessment of quality using lifetimes of the items as the quality variables are known as the life test or reliability sampling plans. Under a life test sampling plan, the test units, which are subjected to a set of test procedures, are randomly selected from the lot, and the lot is either accepted or rejected based on the information provided by the test results. A specific life test sampling plan can be developed considering the lifetime of the products as the quality characteristic, which is modelled by an appropriate continuous-type probability distribution. Life test sampling plans usually focus on the objective of determining whether the lifetimes of items reach the specified standard or not based on the observations made from the sampled lifetime data. A central feature of a life test plan is that it involves the lifetime characteristic as the random variable which can be more adequately described by the exponential, Weibull, lognormal, or gamma distribution rather than the normal distribution.

In this paper, a specific life test sampling plan is devised with reference to the life-time quality characteristic modelled by Marshall – Olkin extended exponential distribution (MOEED). Three different criteria for designing life test plans when lot quality is evaluated in terms of mean life, hazard rate and reliability life are proposed. Factors for adapting MIL-STD-105D [23] to life and reliability testing indexed by acceptable quality level under the assumption of MOEED are also illustrated.

The material in this paper is organized in four sections. Section I provides the conceptual framework of acceptance sampling and reliability sampling. Section II presents a review on reliability and life testing sampling plans citing varying references. Sampling inspection plans for life tests based on MOEED are introduced in Section III. The notion of MOEED and its application in life testing are also presented in this section. Section IV presents concluding remarks on the contents of this paper.

II. LITERATURE IN SAMPLING INSPECTION PLANS FOR LIFE TESTS

The relevance of many continuous distributions in the studies concerned with the design and evaluation of life test sampling plans has been significantly marked out in the literature of product control. While important contributions have been made during the past five decades in the development of life test sampling plans employing exponential, Weibull, lognormal and gamma distributions as the lifetime distributions, the literature also provides application of several compound distributions for modelling lifetime data. Epstein [6, 7], Handbook H-108 [13], and Goode and Kao [8, 9, 10] present the theory and development of reliability sampling plans based on exponential and Weibull distributions.

Gupta and Groll [12] discussed life-test sampling plans using gamma distribution. Life test sampling plans using normal and lognormal distribution have been discussed by Gupta [11]. A detailed account of such plans was provided by Schilling and Neubauer [21]. The recent literature in reliability sampling plans includes the works of Wu and Tsai [32], Wu, *et al*., [33], Kantam, *et al*., [16], Jun, *et al*., [15], Tsai and Wu [22], Balakrishnan, *et al*., [4], Aslam and Jun [1, 2], Aslam, *et al*., [3], Kalaiselvi and Vijayaraghavan [18], Kalaiselvi, *et al*., [17], Loganathan, *et al*., [19], Hong, *et al*., [14], Vijayaraghavan, *et al*., [28, 29] and Vijayaraghavan and Uma [30, 31].

Marshall – Olkin extended exponential distribution (MOEED), introduced by Marshall and Olkin [20], is a generalization of the exponential distribution and has a monotone failure rate. In some applications of biological, agricultural and entomological studies, the failure rate function of the underlying distribution may be inverted bathtub – shaped hazard function or unimodal. When a probability distribution for life-time variable has a failure rate function that takes various shapes, it is the natural choice to adopt the distribution in practice. According to Marshall and Olkin [20], MOEED has the failure rate that decreases with time, fairly constant failure rate and failure rate that increases with time, indicative of infantile or early-failures, useful life or random failures and wear-out failures, respectively.

Due to the possibility of various shapes of failure rate function, MOEED is especially suitable for modelling life time of an item. It has also been shown that MOEED is used commonly for the inference of life information due to its various shapes of failure rate function, similar to gamma and Weibull distribution. Hence, the use of MOEED as a member of the lifetime continuous distributions can be considered much attractive for practitioners to adopt in real life situations and may be used as an alternative to the gamma, Weibull and other exponentiated family of distributions.

III. LIFE TEST SAMPLING INSPECTION PLANS BASED ON MARSHALL – OLKIN EXTENDED EXPONENTIAL DISTRIBUTION

Sampling plans for life tests consist of a set of sampling procedures and rules for making the decision of either accepting or rejecting a lot of items based on the sampled lifetime information about the items. Under such plans, sampled items are tested for a specified length of time. When all units are tested to failure, the standard plans can be utilized to assess the results against specified requirements, and the results can be used in an attribute sampling plan when the lifetimes are measured and the distributional assumption of the quality characteristics is satisfied. Further, the number of failures which occur before a required time can be used with standard attributes plans in determining the disposition of the material. (See, Schilling and Neubauer [21]).

A typical life test sampling plan can be formulated in the following manner: Suppose, n items are placed for a life test and the experiment is stopped at a predetermined time, T. The number of failures occurred until the time point T is observed, and let it be m. The lot is accepted if m is less than or equal to the acceptance number, say, c; otherwise, it is rejected. Thus, the life test sampling plan is represented by n, the number of units on test, and c, the allowable number of failures, called the acceptance number.

Life tests, terminated before all units have failed, may be classified into two types, namely, failure terminated and time terminated. In a failure terminated life test, a given sample of n items is tested until the r^{th} failure occurs and then the test is terminated. In time terminated life-test, a given sample of n items is tested until a pre-assigned termination time, T, is reached and then the test is terminated. Generally, these tests may be defined with reference to the specifications given in terms of one of the characteristics such as (a) mean life, that is, the expected life of the product, (b) hazard rate, that is, the instantaneous failure rate at some specified time, t, and (c) reliable life, that is, the life beyond which some specified proportion of items in the lot will survive.

One of the significant features of a life test plan is that it involves a random characteristic, called lifetime or time to failure, which can be more adequately described most often by skewed distributions. Application of continuous-type of distributions such as normal, exponential, Weibull, gamma and lognormal for lifetime variables in the studies concerned with the design and evaluation of life test sampling plans has been provided in the literature of sampling inspection. Accordingly, important contributions have been made in the development of life test sampling plans based on the above distributions. Application of Marshall – Olkin extended exponential distribution, which is one of the lifetime distributions, is now considered under life test sampling plan.

Marshall - Olkin Extended Exponential Distribution

Let T be a random variable representing the lifetime of the components. Assume that T follows Marshall – Olkin extended exponential distribution (MOEED). The probability density function and the cumulative distribution function of T are, respectively, defined by

$$f(t;\gamma,\theta)=\frac{\gamma e^{-t/\theta}}{\theta(1-\bar{\gamma}e^{-t/\theta})^2},\ t,\gamma,\theta>0 \qquad (1)$$

and

$$F(t;\gamma,\theta)=\frac{1-e^{-t/\theta}}{1-\bar{\gamma}e^{-t/\theta}},\ t,\gamma,\theta>0, \qquad (2)$$

where $\bar{\gamma}=1-\gamma$, γ is the shape parameter and θ is the scale parameter.

The mean life time, the reliability function and hazard function for specified time t under MOEED are respectively given by

$$\mu=E(T)=-\frac{\gamma\theta\log\gamma}{1-\gamma}, \qquad (3)$$

$$R(t;\theta,\gamma)=\frac{\gamma}{e^{t/\theta}-\bar{\gamma}},\ t\geq 0 \qquad (4)$$

and

$$Z(t;\theta,\gamma)=\frac{e^{t/\theta}}{\theta(e^{t/\theta}-\bar{\gamma})},\ t\geq 0. \qquad (5)$$

The reliability life is the life beyond which some specified proportion of items in the lot will survive. The reliability life associated with MOEED is defined and denoted by

$$\rho=\theta\log\left(\frac{\gamma+\bar{\gamma}R}{R}\right), \qquad (6)$$

where R is the proportion of items surviving beyond life ρ .

Application of Marshall – Olkin Extended Exponential Distribution

The procedures for determining life test sampling plans based on Weibull distribution were provided in Technical Reports TR 3 [24], TR 4 [25] and TR 6 [26] utilizing mean life, hazard rate and reliability life as the criteria for acceptance of the lots. Analogous approaches are considered here to develop sampling plans using MOEED as the lifetime distribution. The mean life criterion consists in the determination of the ratio $t/E(t)$, which corresponds to the proportion, p, of product failing before time t.

When a life test sampling plan is dealt with in practice, acceptable mean life and unacceptable mean life, associated with the producer's risk and consumer's risk, are the two commonly specified requirements. With the specification of these values, a desired sampling plan can be determined. Corresponding to the specifications for the acceptable and unacceptable mean life, the quality levels are defined by p_0 and p_1 with associated risks, α and β , where p_0 is the acceptable proportion of the lot failing before the specified time, t, and p_1 is the unacceptable proportion of the lot failing before the specified time, t. A set of conversion factors has been constructed based on mean life, hazard rate and reliability life as the criteria for life test plans under the conditions of MOEED. These factors can be used for determining the plans for specified requirements.

It can be observed that the dimensionless ratio, t/μ , the hazard rate function, $Z(t)$, and the reliability rate function, ρ, depend on the parameters of MOEED. According to the Technical Report TR 7 [27], the quantities such as $t/\mu\times 100$, $tZ(t)\times 100$ and $t/\rho\times 100$ can be used to determine the life test sampling plans under the three reliability criteria, respectively. The values of the ratio $t/\mu\times 100$ can be computed from the expressions (2) and (3) corresponding to the selected the proportion, p, of product failing before time t for specified values of γ. In a similar manner, the values of $tZ(t)\times 100$ and $(t/\rho)\times 100$ can be determined utilizing expressions (5) and (6). Tables 1, 2 and 3 provide the conversion factors, namely, $t/\mu\times 100$, $tZ(t)\times 100$ and $t/\rho\times 100$ relating to the mean life, hazard rate and reliable life criteria for specified sets of values of γ and p. These tables can be used, respectively, to determine life test plans and their operating characteristics based on mean life, hazard rate and reliable life.

The conversion factors can also be used to obtain the mean life, hazard rate and reliability life when the test termination time is specified, and *vice versa*. The following illustrations based on the three criteria are provided to demonstrate the use of Tables 1, 2 and 3 in determining the operating characteristics of a given plan and in finding a sampling plan satisfying the requirements in terms of acceptable quality level (acceptable mean life) and limiting quality level (unacceptable mean life).

Numerical Illustration 1

It is desired to use a single sampling plan by attributes having the parameters n = 130 and c = 5 for the requirements $(p_0 = 0.02, \alpha = 0.05)$ and $(p_1 = 0.07, \beta = 0.10)$ when the life time of the item follows MOEED with the shape parameter $\gamma = 1.5$. It is assumed to employ a test termination time of 300 hours and to count the number of failures at that time. Then, the operating characteristics of this plan in terms of mean life along with the values of $k = t/\mu \times 100$ and $\mu = t/k \times 100$, where t = 300 hours, are obtained and are tabulated in Table 4 using the operating characteristics of the attributes plan and Table 1.

Numerical Illustration 2

Suppose that a single sampling plan by attributes with parameters n and c is to be defined when the requirements are specified in terms of acceptable mean life of 12110 hours and unacceptable mean life of 4050 hours with associated risks of 5 percent and 10 percent, respectively. Assume that the individual items are to be tested for 350 hours and that the lifetime of the items is distributed as MOEED with the shape parameter fixed as $\gamma = 2$. Then, at the specified levels, the values of k are determined as

$$k_0 = t/\mu \times 100 = (350/12110) \times 100 = 2.89$$

and

$$k_1 = t/\mu \times 100 = (350/4050) \times 100 = 8.64.$$

Entering Table 1 with these values, one obtains the proportion of product failing corresponding to acceptable and unacceptable mean life as $p_0 = 0.02$ and $p_1 = 0.06$, respectively. The operating ratio, which is the measure of discriminating good and bad lots, is defined by $OR = 0.06/0.02 = 3$, corresponding to which a single sampling plan can be chosen from Schilling and Neubauer [21] as $(n = 193, c = 7)$ or from Cameron [5] as $(n = 194, c = 7)$.

In a similar manner, Tables 2 and 3 can be used to determine conversion factors so as to obtain the life test plans and the corresponding hazard and reliability rates.

Numerical Illustration 3

The acceptable mean life under the life test sampling plans based on MOEED can be determined using the ratio $k = t/\mu \times 100$ for any specified value of acceptable quality level, AQL, shown in MIL-STD-105D [23]. When AQL = 3% at $P_a(AQL) = 0.95$, the test termination time, t = 300 hours and the shape parameter, $\gamma = 1.5$, the expected mean life, μ, is determined as $\mu = t/k \times 100 =$ 8047 hours, which can be considered as an acceptable mean life. Accordingly, if a lot consisting of items which have the acceptable mean life specified at 8047 hours, the probability of acceptance of the lot would be 95%. Corresponding to the fixed value of AQL = 3% with 95% acceptance probability, the conversion factors for mean life and hazard rate criteria for the case in which the shape parameter is specified as $\gamma = 1.5$ under MOEED are as given below:

Table 1

Criterion	Conversion Factor	Value of the Factor	AQL
Percent Nonconforming as per MIL-STD-105D	$p \times 100$	3	0.03
Expected Mean Life	$k = (t/\mu) \times 100$	3.728	8047
Hazard Rate	$tZ(t) \times 100$	3.068	0.000102
Reliable Life	$(t/\rho) \times 100$	1.46702	20450

IV. CONCLUDING REMARKS

Sampling inspection plans for life-tests are proposed when the lifetime quality characteristic is modelled by a Marshall – Olkin extended exponential distribution. The procedures and tables for the selection of life test sampling plans when lot quality is evaluated in terms of three different criteria based on mean life, hazard rate and reliability life are developed. The tables presented for the selection of parameters of the life test sampling plans are restricted for the values of shape parameter fixed as $\gamma = 0.5, \gamma = 1, \gamma = 2$ and $\gamma = 3$. The practitioners can generate the required sampling plans for other choices of γ adopting the procedure described in this paper. Conversion factors are provided which would help to determine the mean life and hazard rate for a specified test termination time and *vice versa*. The factors for adapting MIL-STD-105D to life and reliability testing indexed by acceptable quality level are provided.

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Table 1: Values of $t/\mu \times 100$ Based on MOEED for the Specified Values of γ

p%	Shape Parameter, γ					
	0.5	0.75	1.5	2	2.5	3
1	0.72680	0.87449	1.23627	1.44274	1.63304	1.81156
2	1.46468	1.76006	2.47889	2.88578	3.25845	3.60597
3	2.21390	2.65698	3.72805	4.32938	4.87680	5.38425
4	2.97473	3.56547	4.98396	5.77386	6.48861	7.14738
5	3.74747	4.48580	6.24681	7.21950	8.09442	8.89628
6	4.53240	5.41822	7.51682	8.66658	9.69472	10.63184
7	5.32982	6.36301	8.79419	10.11541	11.29004	12.35492
8	6.14005	7.32045	10.07915	11.56628	12.88087	14.06633
9	6.96342	8.29083	11.37193	13.01949	14.46768	15.76687
10	7.80025	9.27444	12.67275	14.47533	16.05096	17.45730
11	8.65090	10.27160	13.98185	15.93412	17.63119	19.13837
12	9.51572	11.28261	15.29949	17.39616	19.20882	20.81079
13	10.39510	12.30782	16.62592	18.86177	20.78433	22.47525
14	11.28941	13.34757	17.96139	20.33126	22.35818	24.13244
15	12.19905	14.40221	19.30619	21.80496	23.93082	25.78302
16	13.12445	15.47210	20.66059	23.28318	25.50270	27.42764
17	14.06604	16.55763	22.02488	24.76626	27.07428	29.06692
18	15.02426	17.65919	23.39936	26.25455	28.64601	30.70150
19	15.99959	18.77719	24.78433	27.74839	30.21834	32.33197
20	16.99250	19.91206	26.18012	29.24813	31.79173	33.95895
21	18.00350	21.06422	27.58706	30.75412	33.36663	35.58302
22	19.03312	22.23415	29.00549	32.26675	34.94352	37.20478
23	20.08190	23.42232	30.43576	33.78640	36.52283	38.82481
24	21.15041	24.62921	31.87825	35.31344	38.10503	40.44369
25	22.23924	25.85534	33.33333	36.84828	39.69062	42.06199
50	58.49625	64.84193	75.32837	79.24812	82.03267	84.12397
60	80.73550	87.33852	96.89736	100.00002	102.02949	103.44858
70	111.54772	117.21282	123.65038	125.12502	125.84298	126.18595
80	158.49625	160.62805	159.97350	158.49625	157.01755	155.64784
90	245.94315	237.26337	219.84206	212.39636	206.72479	202.20688

Table 2: Values of $tZ(t)\times 100$ Based on MOEED for the Specified Values of γ

$p\%$	Shape Parameter, γ					
	0.5	0.75	1.5	2	2.5	3
1	1.00252	1.00378	1.00754	1.01003	1.01252	1.01500
2	2.01017	2.01523	2.03031	2.04027	2.05017	2.06001
3	3.02308	3.03453	3.06854	3.09093	3.11310	3.13506
4	4.04138	4.06186	4.12247	4.16222	4.20145	4.24019
5	5.06522	5.09741	5.19237	5.25438	5.31540	5.37546
6	6.09474	6.14138	6.27848	6.36765	6.45512	6.54095
7	7.13010	7.19396	7.38108	7.50227	7.62079	7.73676
8	8.17145	8.25538	8.50043	8.65850	8.81262	8.96300
9	9.21895	9.32583	9.63682	9.83662	10.03083	10.21979
10	10.27277	10.40555	10.79055	11.03689	11.27563	11.50728
11	11.33310	11.49477	11.96191	12.25961	12.54728	12.82564
12	12.40010	12.59373	13.15123	13.50508	13.84602	14.17505
13	13.47397	13.70267	14.35882	14.77360	15.17211	15.55570
14	14.55491	14.82185	15.58502	16.06552	16.52585	16.96781
15	15.64312	15.95154	16.83017	17.38115	17.90753	18.41161
16	16.73881	17.09202	18.09464	18.72086	19.31745	19.88734
17	17.84220	18.24357	19.37879	20.08500	20.75595	21.39530
18	18.95353	19.40649	20.68302	21.47396	22.22337	22.93575
19	20.07303	20.58109	22.00770	22.88812	23.72006	24.50901
20	21.20095	21.76770	23.35327	24.32791	25.24641	26.11540
21	22.33755	22.96664	24.72015	25.79373	26.80280	27.75528
22	23.48310	24.17827	26.10878	27.28604	28.38967	29.42902
23	24.63789	25.40294	27.51963	28.80530	30.00743	31.13699
24	25.80221	26.64104	28.95316	30.35199	31.65653	32.87963
25	26.97637	27.89294	30.40988	31.92660	33.33747	34.65736
50	60.81976	65.28851	76.35756	82.39592	87.69341	92.41962
60	78.34621	85.42748	102.15011	110.90356	118.41901	125.01486
70	100.51469	111.27609	135.36696	147.44109	157.58862	166.35532
80	131.83348	147.87140	181.61829	197.75021	211.01480	222.29562
90	187.52226	211.59489	258.50101	279.72168	296.75800	311.00574

Table 3: Values of $(t/\rho)\times 100$ Based on MOEED for the Specified Values of γ

$p\%$	Shape Parameter, γ							
	1.5	2	2.5	3	3.5	4	4.5	5
1	0.49628	0.98523	1.46702	1.94181	2.40976	2.87101	3.32572	3.77403
2	0.98523	1.94181	2.87101	3.77403	4.65200	5.50598	6.33696	7.14590
3	1.46702	2.87101	4.21608	5.50598	6.74413	7.93367	9.07750	10.17827
4	1.94181	3.77403	5.50598	7.14590	8.70114	10.17827	11.58318	12.92117
5	2.40976	4.65200	6.74413	8.70114	10.53605	12.26023	13.88365	15.41507
6	2.87101	5.50598	7.93367	10.17827	12.26023	14.19703	16.00365	17.69307
7	3.32572	6.33696	9.07750	11.58318	13.88364	16.00365	17.96406	19.78257
8	3.77403	7.14590	10.17827	12.92117	15.41507	17.69307	19.78257	21.70645
9	4.21608	7.93367	11.23843	14.19703	16.86227	19.27655	21.47434	23.48396
10	4.65200	8.70114	12.26023	15.41507	18.23216	20.76394	23.05237	25.13144
11	5.08193	9.44908	13.24579	16.57923	19.53087	22.16387	24.52794	26.66287
12	5.50598	10.17827	14.19703	17.69307	20.76394	23.48396	25.91087	28.09024
13	5.92428	10.88941	15.11576	18.75986	21.93628	24.73096	27.20973	29.42395
14	6.33696	11.58318	16.00365	19.78257	23.05237	25.91087	28.43207	30.67303
15	6.74413	12.26023	16.86227	20.76394	24.11621	27.02903	29.58454	31.84537
16	7.14590	12.92117	17.69307	21.70645	25.13144	28.09024	30.67303	32.94792
17	7.54238	13.56659	18.49741	22.61242	26.10138	29.09879	31.70278	33.98678
18	7.93367	14.19703	19.27655	23.48396	27.02903	30.05855	32.67848	34.96738
19	8.31989	14.81302	20.03169	24.32303	27.91714	30.97302	33.60431	35.89451
20	8.70114	15.41507	20.76394	25.13144	28.76821	31.84537	34.48405	36.77248
21	9.07750	16.00365	21.47434	25.91087	29.58454	32.67848	35.32107	37.60512
22	9.44908	16.57923	22.16387	26.66287	30.36824	33.47496	36.11845	38.39589
23	9.81598	17.14223	22.83345	27.38888	31.12126	34.23720	36.87896	39.14789
24	10.17827	17.69307	23.48396	28.09024	31.84537	34.96737	37.60512	39.86391
25	10.53605	18.23216	24.11621	28.76821	32.54224	35.66750	38.29923	40.54651
50	18.23216	28.76821	35.66750	40.54651	44.18328	47.00036	49.24765	51.08256
60	20.76394	31.84537	38.77655	43.53181	47.00036	49.64369	51.72565	53.40825
70	23.05237	34.48405	41.35623	45.95323	49.24765	51.72565	53.65781	55.20686
80	25.13144	36.77248	43.53181	47.95731	51.08256	53.40825	55.20686	56.63955
90	27.02903	38.77655	45.39175	49.64369	52.60931	54.79652	56.47660	57.80779

Table 4: Operating Characteristics for a Specific Life Test Sampling Plan
($n = 130$, $c = 5$ and $\gamma = 1.5$.)

p	P_a	$\gamma = 1.5$	
		$k = \frac{t}{\mu} \times 100$	$\mu = \frac{300}{k} \times 100$
0.010	0.997923	1.24	24266.62
0.015	0.985937	1.86	16157.10
0.020	0.952742	2.48	12102.22
0.025	0.891422	3.10	9669.19
0.030	0.803138	3.73	8047.10
0.035	0.695775	4.36	6888.40
0.040	0.580216	4.98	6019.31
0.045	0.46684	5.61	5343.30
0.050	0.363431	6.25	4802.45
0.055	0.274527	6.88	4359.89
0.06	0.20175	7.52	3991.05
0.07	0.101285	8.79	3411.34
0.08	0.046734	10.08	2976.44
0.09	0.020067	11.37	2638.07
0.10	0.008094	12.67	2367.28

