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Operations on Graphs and its Minimum Diameter Spanning Tree

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***Abstract*:** In this paper we present dominating sets, minimum diameter spanning tree for operations on graphs. We find out the minimum diameter spanning tree through the graph of P_n *and* S_n which have a minimum diameter spanning tree such that both have same domination number. The spanning tree T of the simple connected graph G is said to be a minimum diameter spanning tree. if there is no other spanning tree T' of G such that $d(T') < d(T)$. The diameter of a graph G is length of the shortest path between the most distance node.

***Keywords*:** Domination, Diameter, Ladder graph, Grid graph, star graph, Path, Spanning tree, Cartesian product, co-normal product, lexico graphic product, and union, sum.

I. INTRODUCTION

Researches have defined several operations on graphs depending on their need. some popular binary operations on graphs are given below. A subset S of vertices from V is called a dominating set for G, if every vertex of G is either a member of S or adjacent to a member of S. A dominating set of G is called a minimum dominating set, if G has no dominating set of smaller cardinality. The cardinality of minimum dominating set of G is called the dominating number for G and it is denoted by γ (*G*) [4]. Chandrasekaran V.T and Rajasri.N (2018) Minimum Diameter Spanning Tree [1]. In this paper, we discuss path, star simple connected graphs for which the domination numbers of the graph and that of its minimum diameter spanning trees are the same.

Section I contains the sum of any two graphs, section II contains the operations of star graph, section III contains the oprations of path graph.

II. PRILIMINARIES

Let $G = (V,E)$ be a graph. A subset S of V is called dominating set if every vertex in $V - S$ is adjacent to a vertex in S. The minimum cardinality of a dominating set in G is called the domination number of G and it is denoted by $\gamma(G)$

Let $G = (v_1, \mathrm{x}_1)$ and $G = (v_2, \mathrm{x}_2)$ be two graphs with $v_1 \cap v_2 = \phi$ we define

(1) The **UNION** $G_1 \cup G_2$ to be (V, X) where $V = V_1 \cup V_2$ and $X = X_1 \cup X_2$.

(2) The **SUM** $G_1 + G_2$ as $G_1 \cup G_2$ together with all the lines joining points V_1 to points of V_2.

(3) The **CARTESIAN** $G_1 \square G_2$ as having $V = V_1 \times V_2$ and $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent if $u_1 = v_1$ and u_2 is adjacent to v_2 in G_2 or u_1 is adjacent to v_1 in G_1 and $u_2 = v_2$.

(4) **CO NORMAL PRODUCT:** For a graph G_1 having having the vertex set $\{u_1, u_2, u_3, \ldots, u_m\}$ and G_2 having having the vertex set $\{v_1, v_2, v_3, \ldots, v_n\}$, he graph having the vertex set $V(G_1) \times V(G_2) = \{u_i, v_j : u_i \in V(G_1) \text{ and } v_j \in V(G_2), 1 \le i \le m, 1 \le j \le n\}$

And the adjacency relation defined as $(u_i, v_j) \square (u_r, v_s)$, if $u_i \square u_r \text{ in } G_1 \text{ or } v_j \square v_s \text{ in } G_2$ is called the co normal product graph.It is denoted as $(G_1 * G_2)$.

(5) **LEXICO GRAPHIC PRODUCT:** For any two graphs G_1 and G_2 The graph obtained from the vertex set $V(G_1)\times V(G_2)$ by defining the adjacency relation as $(u_i,v_j)\square(u_r,v_s)$, if $u_i \square u_r$ *in* G_1 *or* $(u_i = u_r$ and $u_j \square u_s$ in G_2) is called the lexico graphic product graph and denoted as $G_1[G_2]$

[1] Sum of any two graphs

LEMMA:1.1 Let G_1 and G_2 be any graph and T be the minimum diameter spanning tree of the graph, then $\gamma(G_1+G_2)=\gamma(T)$

Proof:

(i) Either $\gamma(G_1)=1\, or\, \gamma(G_2)=1$ or both $\gamma(G_1)=1\, and\, \gamma(G_2)=1$. let $\gamma(G_1)=1$ then G_1 has a vertex $v\in V(G_1)$ *such that* $\deg v = |V(G_1)|-1$. In the graph G_1+G_2, $\gamma(G_1+G_2)=1$ and $\{v\}$ is the dominating set. Let T be a tree constructed by removing all edges except the edges incident to the vertex v. Clearly $\text{diam}(T)=2$ and hence it is a minimal sp

(ii) anning tree also $\gamma(T)=1$. Thus $\gamma(G_1+G_2)=\gamma(T)$. Similarly we can prove when $\gamma(G_2)=1\, and\, both\, \gamma(G_1)=1, \gamma(G_2)=1.$

(ii) Both $\gamma(G_1)\neq 1\, and\, \gamma(G_2)\neq 1$ Let G_1 and G_2 be any graph. And let $|V(G_1)|=m$ and $|V(G_2)|=n$ then $\{u,v\}$ is a dominating set where $u\in V(G_1)\, and\, v\in V(G_2)$ hence $\gamma(G_1+G_2)=2$ all edge between u to G_2 and v to G_1 together for a minimum diameter spanning tree T and it is isomorphic to bi-star graph $B_{m,n}$ hence, $\gamma(T)=\gamma(B_{m,n})=2$.Therefore $\gamma(G_1+G_2)=\gamma(T)$.

[2] Oprations of star graph

THEOREM :2.1 Let S_m and S_n be a graph and T be the minimum diameter spanning tree of the graph. Then $\gamma(S_m\cup S_n)=\gamma(T)$

(i) $\gamma(S_m+S_n)=\gamma(T)$

(ii) $\gamma(S_m * S_n)=\gamma(T)$

(iii) $\gamma(S_m\square S_n)=\gamma(T)$

(iv) $\gamma(S_m[S_n])=\gamma(T)$

Proof

(i) $\gamma(S_m\cup S_n)=\gamma(T)$

Its obviously true

(ii) $\gamma(S_m+S_n)=\gamma(T)$

By lemma 01

(iii) $\gamma(S_m * S_n)=\gamma(T)$

We need to show that $\deg(u_{m+1,}v_{n+1}) = u_{m+1}+v_{n+1}-1$ by the definition of co-normal product (u_{m+1},v_{n+1}) is adjacent to all the vertex of $S_m * S_n$ as u_{m+1} is adjacent to all vertex $u_1,u_2,u_3,\ldots,u_m$ in S_m and v_{n+1} is adjacent to all the vertices of S_n.

Hence $\gamma(S_m * S_n)=1$ and $\dim(S_m * S_n)=2$ Let T be a spanning tree of $S_m * S_n$ obtain by removing the edge except the edge incident on the vertex (u_{m+1},v_{n+1}) in $S_m * S_n$. Clearly, $T = S_{mn}$ and hence $\gamma(T)=1$ and $\dim(T)=2$.Thus T is a minimal spanning tree with domination number 2. Hence $\gamma(S_m * S_n)=\gamma(T)$.

(iv) $\gamma(S_m\square S_n)=\gamma(T)$

Let $V(S_m)=\{u_1,u_2,u_3,\ldots,u_{m+1}\}$, and $\deg(u_{m+1})=\Delta(S_m)$.

let $V(S_n)=\{v_1,v_2,v_3,\ldots,v_{n+1}\}$,and $\deg(v_{n+1})=\Delta(S_n)$

Assume that m < n, let vertex set of $S_m\square S_n$ =

$$\{ (u_1,v_1),(u_1,v_2),\ldots,(u_1,v_n),(u_1,v_{n+1})$$
$$(u_2,v_1),(u_2,v_2),\ldots,(u_2,v_n),(u_2,v_{n+1})$$
$$\vdots$$
$$(u_m,v_1),(u_m,v_2),\ldots,(u_m,v_n),(u_m,v_{n+1})$$

$$(u_2,v_1),(u_2,v_2),\ldots,(u_2,v_n),(u_2,v_{n+1})$$
$$(u_{m+1},v_1),(u_{m+1},v_2),\ldots,(u_{m+1},v_n),(u_{m+1},v_{n+1}) \}$$

And egde set of

$$S_m \square S_n = \left\{ \begin{array}{l} (u_i,v_j) \text{ is adjacent to } (u_k,v_l) \text{ iff} \\ v_j = v_l \;\; u_i \text{ is adjacent to } u_k \text{ in } S_m \text{ or} \\ u_i = u_k \;\; v_j \text{ is adjacent to } v_l \text{ in } S_n. \end{array} \right\}$$

$$E(S_m \square S_n) = \left\{ \begin{array}{l} (u_{m+1},v_1)(u_1,v_1),(u_{m+1},v_1)(u_2,v_1),\ldots,(u_{m+1},v_1)(u_m,v_1) \\ (u_{m+1},v_2)(u_1,v_2),(u_{m+1},v_2)(u_2,v_2),\ldots,(u_{m+1},v_2)(u_m,v_2) \\ \vdots \\ (u_{m+1},v_n)(u_1,v_n),(u_{m+1},v_n)(u_2,v_n),\ldots,(u_{m+1},v_n)(u_m,v_n) \\ (u_{m+1},v_{n+1})(u_1,v_{n+1}),(u_{m+1},v_{n+1})(u_2,v_{n+1}),\ldots,(u_{m+1},v_{n+1})(u_m,v_{n+1}) \\ (u_1,v_{n+1})(u_1,v_1),(u_1,v_{n+1})(u_1,v_2),\ldots,(u_1,v_{n+1})(u_1,v_n) \\ (u_2,v_{n+1})(u_2,v_1),(u_2,v_{n+1})(u_2,v_2),\ldots,(u_2,v_{n+1})(u_2,v_n) \\ \vdots \\ (u_m,v_{n+1})(u_m,v_1),(u_m,v_{n+1})(u_m,v_2),\ldots,(u_m,v_{n+1})(u_m,v_n) \\ (u_{m+1},v_{n+1})(u_{m+1},v_1),(u_{m+1},v_{n+1})(u_{m+1},v_2),\ldots,(u_{m+1},v_{n+1})(u_{m+1},v_n) \end{array} \right\}$$

Let us find the Diameter of $S_m \square S_n$: (u_i,v_j) *and* (u_k,v_l) two vertices of $S_m \square S_n$

Case(i) $u_i = u_k$ and $v_j \neq v_l$

If $v_j = v_{n+1}$ *or* $v_l = v_{n+1}$ then v_l is adjacent to v_{n+1} and its distance is 1.

If $v_j \neq v_{n+1}$ and $v_l \neq v_{n+1}$ then v_j is adjacent to v_{n+1} and its minimal path is $(u_i,v_j)-(u_i,v_{n+1})-(u_i,v_l)$ and hence distance is 2.

Case(ii) let $u_i \neq u_k$ *and* $v_j = v_l$ similar to previous case

Case (iii) $u_i \neq u_k$ and $v_j \neq v_l$

The minimal path between (u_i,v_j) to (u_k,v_l) is attained by travelling along the vertex (u_i,v_j) to (u_{m+1},v_j) *to* (u_{m+1},v_{n+1}) *to* (u_{m+1},v_l) *to* (u_k,v_l) and its distance 4.

Hence the diameter of $S_m \square S_n$ is 4.

$$\gamma(S_m \square S_n) = \min\{m+1, n+1\}$$
$$= m+1$$

Consider a spanning tree T of $S_m \square S_n$ where $m<n$ have the following set as edge set

$$\left\{ \begin{array}{l} (u_{m+1},v_{n+1})(u_1,v_{n+1}),(u_{m+1},v_{n+1})(u_2,v_{n+1}),\ldots,(u_{m+1},v_{n+1})(u_m,v_{n+1}) \\ (u_1,v_{n+1})(u_1,v_1),(u_1,v_{n+1})(u_1,v_2),\ldots,(u_1,v_{n+1})(u_1,v_n) \\ (u_2,v_{n+1})(u_2,v_1),(u_2,v_{n+1})(u_2,v_2),\ldots,(u_2,v_{n+1})(u_2,v_n) \\ \vdots \\ (u_m,v_{n+1})(u_m,v_1),(u_m,v_{n+1})(u_m,v_2),\ldots,(u_m,v_{n+1})(u_m,v_n) \\ (u_{m+1},v_{n+1})(u_{m+1},v_1),(u_{m+1},v_{n+1})(u_{m+1},v_2),\ldots,(u_{m+1},v_{n+1})(u_{m+1},v_n) \end{array} \right\}$$

Now let us find diameter of T, consider any vertex (u_i,v_j), (u_k,v_l)

Diameter of T is (u_i,v_j) *to* (u_i,v_{n+1}) to (u_{m+1},v_{n+1}) *to* (u_k,v_{n+1}) *to* (u_k,v_l)

Therefore diameter (T)= 4. Hence $diam(S_m \square S_n) = 4 = diam(T)$

T is minimum spanning tree.

Minimal Domination set of T is $\{(u_1,v_{n+1}),(u_2,v_{n+1}),\ldots,(u_m,v_{n+1}),(u_{m+1},v_{n+1})\}$

Since, m < n Hence $\gamma(T)$ = m+1

Therefore $\gamma(S_m \square S_n) = \gamma(T)$. Hence proved.

(v) $\gamma(S_m[S_n]) = \gamma(T)$

Let $V(S_m) = \{u_1,u_2,u_3,\ldots,u_{m+1}\}$, and $\deg(u_{m+1}) = \Delta(S_m)$.

Let $V(S_n) = \{v_1,v_2,v_3,\ldots,v_{n+1}\}$,and $\deg(v_{n+1}) = \Delta(S_n)$

By case (i) the diameter of $(S_m[S_n])$ is 4. $\gamma(S_m[S_n]) = \min\{m+1, n+1\} = m+1$ Consider a spanning tree T of $S_m[S_n]$ where $m<n$, the diameter of T is 4 and the minimum domination number of T is $m+1$,hence $\gamma(S_m[S_n]) = \gamma(T)$.

Example for $S_5 \square S_6$

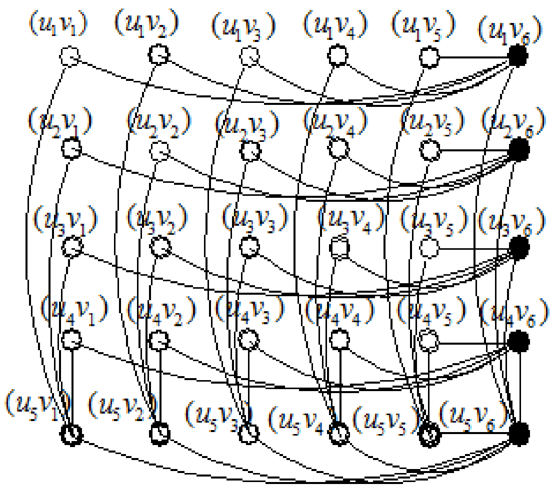

Figure 01 - Cartesian product graph for $S_5 \square S_6$

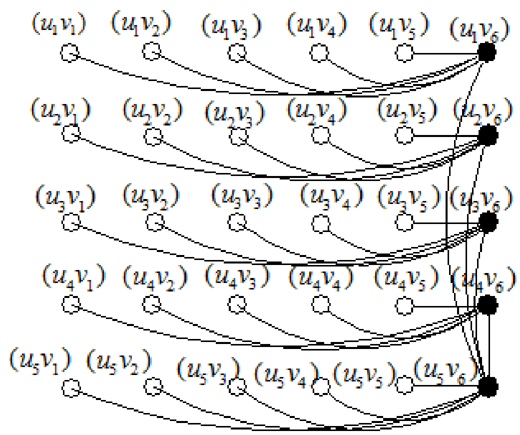

Figure 02 – Minimum diameter spanning tree for $S_5 \square S_6$

[3] Oprations of path graph

THEOREM : 3.1 Let P_n and P_m be path graph respectively and T be the minimum diameter spanning tree of the graph. Then $\gamma(P_m \cup P_n) = \gamma(T)$

(1) $\gamma(P_m + P_n) = \gamma(T)$

(2) $\gamma(P_m \square P_n) \le \gamma(T)$

(3) $\gamma(P_m[P_n]) = \gamma(T)$

PROOF:

(i) Let P_m and P_n graph respectively.

we know that $\gamma(P_m \cup P_n) = \gamma(P_m) + \gamma(P_n)$

$= \gamma(P_m) + \gamma(P_n)$ by definition

$\gamma(P_m \cup P_n) = \gamma(T)$.

(ii) Let G_1 and G_2 be P_n and P_m graph respectively. if $n \le 3$ *or* $m \le 3$ then $\gamma(G_1 + G_2) = 1$. Suppose $G_1 = P_n$, $n \le 3$ then there exist a $u \in V(G_1)$ such that u is adjacent to all vertices in G_1 and in $G_1 + G_2$. { u } is the dominating set hence u is a dominating vertex in $G_1 + G_2$, so $\gamma(G_1 + G_2) = 1 = \gamma(T)$ .and If $n > 3$ *or* $m > 3$ then $\gamma(G_1 + G_2) = \gamma(T)$ by lemma 01

(iii) Case(1): Let G_1 and G_2 be P_1 , P_n graph respectively, then $G_1 \square G_2$ is a P_n graph and hence $\gamma(G_1 \square G_2) = \gamma(P_1 \square P_n) = \gamma(P_n) = \gamma(T)$.

Case(2): Let G_1 and G_2 be P_2, P_n graph respectively, then $G_1 \square G_2$ is a L_n graph. [1] The domination number of the minimum diameter spanning tree of L_n is $\gamma(G_1 \square G_2) < \gamma(T)$, when n is odd. And when n is even The domination number of the minimum diameter spanning tree of L_n is $\gamma(G_1 \square G_2) = \gamma(T)$.

Case(3): Let G_1 and G_2 be any P_3, P_n graph respectively where n>3. then $G_1 \square G_2$ is a $G_{3,n}$ graphSo, $\gamma(P_3 \square P_n) = \gamma(G_{3,n}) = \left\lfloor \frac{3n+4}{4} \right\rfloor \ldots (1)$. Let T be a spanning tree of $P_3 \square P_n$ obtain by removing the edge except the **LINE** and **I**-shape that is from the Figure(2A) the line is formed in first column and I - shape formed in third column continue this process line and **I** – shape are formed alternatively in odd columns. Lines are drawn continuously when there is not possibility to draw **I**-shape. The domination number of the minimum diameter spanning tree is $\gamma(T) = \left\lfloor \frac{3n+4}{4} \right\rfloor$, clearly from the Figure(2A) $\gamma(P_n \square P_m) = \gamma(T)$ and since from the above $\gamma(G_1 \square G_2) \le \gamma(T)$.

(iii) For any P_m, P_n graph, by the definition of lexico graphic product $P_m[P_n]$ where $m > 3$ *and* $n > 3$. $P_m[P_n]$ has a minimum diameter spaning tree T. if $m \equiv 0$ *or* $3 (\mathrm{mod}\ 4)$ and let γ_1 be the minimum dominating set of $P_m[P_n]$, where $\gamma_1 =$

$\left\{(u_{4i-2}v_2),(\mathrm{u}_{4i-1}\,v_n)\right\}$, $i \le \left\lceil \frac{m}{4} \right\rceil$ then $\gamma(\mathrm{P}_m[\mathrm{P}_n]) = \left\lceil \frac{m}{2} \right\rceil$. delete all the edges except the edges incident with $(u_{4i-2}v_2)$ and $(u_{4i-1}v_n)$ such that the cycle does not exist, thus T is obtained then $\gamma(T) = \left\lceil \frac{m}{2} \right\rceil$. If $m \equiv 1 \pmod 4$ and the dominating set is $\gamma_1 \cup \{\mathrm{u}_{m-1}\mathrm{v}_n\}$ then $\gamma(P_m[\mathrm{P}_n]) = \left\lceil \frac{m}{2} \right\rceil$. delete all the edges except the edges incident with $(u_{4i-2}v_2)$, $(u_{4i-1}v_n)$ and $(u_{m-1}v_n)$ such that the cycle does not exist, thus T is obtained then $\gamma(T) = \left\lceil \frac{m}{2} \right\rceil$. if $m \equiv 2 \pmod 4$ and the dominating set is $\gamma_1 \cup \{(\mathrm{u}_{m-2}\,v_n),(\mathrm{u}_{m-1}\,v_n)\}$ then $\gamma(\mathrm{P}_m[\mathrm{P}_n]) = \left(\frac{m}{2}\right)+1$ delete all the edges except the edges incident with $(u_{4i-2}v_2)$, $(u_{4i-1}v_n)$, $(u_{m-2}v_n)$ and $(u_{m-1}v_n)$ such that the cycle does not exist, thus T is obtained then $\gamma(T) = \frac{m}{2}+1$.

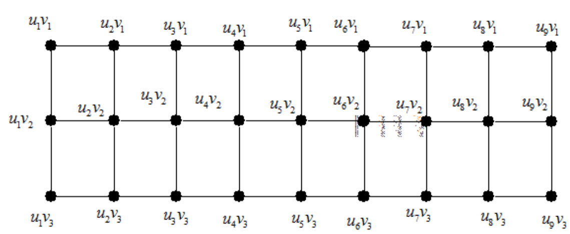

FIGURE 3-$G_{3,9}$

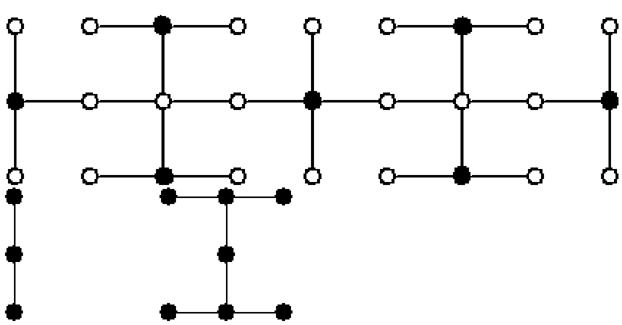

FIGURE 4 (line) (I-shape)

III. CONCLUSION

In this article, minimum diameter spanning tree for operations on graphs. We have analysed the minimum diameter spanning tree through the graph of P_n *and* S_n which have a minimum diameter spanning tree such that both have same domination number. Further research can be done in exploring various graphs and various domination with the same property. The condition for which a graph does not Posses such spanning tree may also be explored.

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