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Economic Lotsize Model for a Price Dependent Demand System under Credit Policy

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Abstract- The main purpose of this paper is to study an inventory model for time dependent deterioration with selling price dependent demand. Here it is assumed that demand rate is a function of selling price. This model allows shortages, which is partially backlogged. It is also investigated under credit policy. The results are illustrated with the help of numerical examples. The sensitivity analysis for the model is carried out to study the effect of changes on the values of the parameters associated with the model.

Keywords: EOQ, Partially Backlogging, Trade Credit, Selling Price Dependent Demand, Deterioration

I. INTRODUCTION

In recent years, mathematical ideas have been infused in different areas in real life problems, particularly for controlling inventory. Inventory is a fundamental part of manufacturing, distribution, retail infrastructure and demand plays an important role in choosing the beneficial inventory strategy. Many researchers worked with different kind of demand like constant demand, linear time varying demand, parabolic demand, exponential demand etc. But in reality demand is not always dependent on time. A discount price attracts more customers to buy the product in a super market. In real world applications, the demand is influenced by the selling price. The dependence of the sale of any item on its selling price is not a new concept.

In this model, the optimal inventory model for time dependent deteriorating items and price dependent demand is considered in linear form. This paper is formulated with selling price dependent demand with partially backlogging under credit policy. The model is solved to optimize the total profit, the results of which are illustrated with numerical examples and comprehensive sensitivity analysis. These facts make this study interesting and unique.

Section I contains the introduction of the economic lotsize model for a price dependent demand system under credit policy. Section II contains the related work of the previous research works about problem, objectives. Section III contains the methodology including experimental design and techniques used along with the year of experimentation. Section IV contains results and discussion about the finding in few of the results obtained in this and in past studies on this topic. Section V explains about conclusion of the research work and also discuss about the future scope for improvement. Section VI describes results and discussion. Section VII contains the recommendation and references.

II. RELATED WORK

Abad (2001) considered a pricing and lot sizing problem for a product with a variable rate of deterioration allowing shortages and partial backlogging. Dye (2007) amended Abad (2001) model by adding both backorder cost and cost of lost sales into the total profit. Sridevi, et al. (2010) have developed a model with random replenishment rates with selling price dependent demand. Chang and Dye (2001) obtained the optimal order quantity of deteriorating items in the presence of trade credit. Roy (2008) proposed an EOQ model for deteriorating items in which deterioration rate and holding cost are considered as linearly increasing function of time, selling price dependent on demand rate and shortage is allowed and completely backlogged. Tripathy (2013) presented an inventory model for non-deteriorating items under permissible delay in payments where holding

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cost is a function of time. Karmakar (2014) developed an inventory model involving ramp-type demand for deteriorating items with partial backlogging with time varying holding cost.

III. METHODOLOGY

1. ASSUMPTIONS AND NOTATIONS

- The demand rate is a function of selling price which is f(s) = ($a-bs$) >0
- Shortages whenever allowed is partially backlogged i.e. $\frac{a-bs}{1+\delta(T-t)}$ s
- Deterioration rate is proportional to time which is $\theta=\alpha t$, where 0< α <1
- Replenishment is instantaneous and lead time is zero.
- T is the length of the cycle.
- Q : Ordering quantity in one cycle
- A: Ordering cost
- C: Cost per unit
- h: Inventory holding cost per unit per unit time
- π: Shortages cost per unit per unit time
- s : Selling price per unit
- M: Permissible delay in settling the accounts
- I_C, I_e: Interest charge and interest earned respectively where $I_C \geq I_e$
- During time t_1, inventory is depleted due to deterioration and demand of the time
- $P(T,s)$: Average profit rate function for $M \leq t_1$ and $M \geq t_1$

2. MATHEMATICAL MODEL

Let I(t) be the inventory level at time "t" ($0 \leq t \leq T$). The differential equations governing the system in the cycle time [0, T] are

$$\frac{d}{dt}I(t)+\alpha\, tI(t) = -(a-bs) \qquad 0\leq t\leq t_1 \qquad ....(1)$$

$$\frac{d}{dt}I(t) = -\frac{a-bs}{1+\delta(T-t)} \qquad t_1\leq t\leq T \qquad ....(2)$$

With I(t) =0 at $t=t_1$

Solving equation (1) and (2) , we have

$$I(t)=(a-bs)(1-\frac{\alpha t^2}{2})[(t_1-t)+\frac{\alpha}{6}(t_1^{\,3}-t^3)] \qquad ....(3)$$

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$$I(t)=(a-bs)[(1-\delta T)(t_1-t)+\frac{\delta}{2}(t_1^{\,2}-t^2)] \qquad \text{......................(4)}$$

Stock loss due to deterioration in the cycle of length T is

$$L(t)=(a-bs)(\frac{\alpha t_1^{\,3}}{6}) \qquad \text{......................(5)}$$

Order quantity Q in the cycle of length T is

$$Q=(a-bs)(\frac{\alpha t_1^{\,3}}{6})+(a-bs)t \qquad \text{........................(6)}$$

Holding cost is obtained by substituting the equation (3) and (4), we have

$$H=h(a-bs)[\int_0^{t_1} I(t)dt+\int_{t_1}^{T} I(t)dt] \qquad \text{......................(7)}$$

Shortage cost during the cycle is

$$S=(a-bs)[\frac{1}{2}(1-\delta T)(T-t_1)^2+\frac{\delta}{4}(T^2-t_1^{\,2})^2] \qquad \text{....................(8)}$$

Let $P(T,t_1,s)$ be the profit rate function. Since the profit rate function is the total revenue per unit minus total cost per unit, we have

$$P(T,t_1,s) = S(a-bs)-\frac{1}{T}(A+CQ+H+\pi S+IC_1-IE_1) \qquad \text{.......................(9)}$$

3.1 Case-1 (M>t_1)

Since M>t_1, the retailer pays no interest but earns interest at an annual rate I_e during the period (0,M). But during [0,T], the retailer sells product at selling price per unit

$IC_1=0$

$$IE_1=C.I_e[\int_0^{t_1}(t_1-t)\frac{a-bs}{1+\delta(T-t)}dt+\int_0^{t_1}(M-t_1)\frac{a-bs}{1+\delta(T-t)}dt] \qquad \text{................(10)}$$

Substituting the value of Q,H,S,IC_1 ,IE_1 and putting $t_1=\gamma t, 0<\gamma<1$, we have

$$P(T,s)=s(a-bs)-\frac{A}{T}-C\left[(a-bs)\frac{\alpha\gamma^3T^2}{6}+(a-bs)\gamma T\right]-h(a-bs)\left[\frac{\gamma^2T}{2}+\frac{\alpha\gamma^4T^3}{12}+(1-\delta T)T(2\gamma-\frac{1}{2}+\frac{\gamma^2}{2})\right]$$
$$-h(a-bs)\frac{\delta T^2}{2}(\gamma^2-\frac{1}{3}-\frac{4\gamma^3}{3})-\frac{\pi}{2}(a-bs)T\left[(1-\delta T)(1-\gamma)^2+\frac{\delta}{2}(1-\gamma^2)\right]+(a-bs)CI_e$$

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$$\left[(1-\delta T)\gamma^2 T-(1+\delta T+\delta\gamma T)\frac{\gamma^2 T}{2}+\frac{\delta\gamma^3 T^2}{3}+(M-\gamma T)\left\{(1-\delta T)\gamma+\frac{\delta\gamma^2 T}{2}\right\}\right]$$

Our objective is to maximize the profit function P(T,S).The necessary conditions for maximizing the profit functions are

$$\frac{\partial P(T,s)}{\partial T}=0$$

$$\Rightarrow\frac{A}{T^2}-C(a-bs)\frac{\alpha\gamma^3 T}{3}+h(a-bs)[\frac{\alpha^2\gamma^6 T^3}{9}+(\frac{\gamma^2}{2}-\frac{3\alpha\gamma^4 T^2}{8})+(\frac{\alpha\gamma^4 T^2}{2}-\frac{5\alpha\gamma^6 T^4}{36})]$$

$$+\frac{\pi}{2}(a-bs)\{(1-\gamma)^2-0.5(1-\gamma^2)\}+(a-bs)CI_e\{(\gamma^2-2\delta\gamma^2 T)-(\frac{\gamma^2}{2}+\delta\gamma^2 T+\delta\gamma^3 T)$$

$$+\frac{2\delta\gamma^3 T}{3}\}+(\alpha\gamma^3 T-2\delta\gamma^2 T-\frac{\delta\gamma^2 M}{2}+\gamma^2+\delta\gamma M)=0$$

and $\frac{\partial P(T,s)}{\partial s}=0$

$$\Rightarrow a-2bs+\frac{bC\alpha\gamma^3 T^2}{6}-b+bh(\gamma-\frac{\alpha\gamma^3 T^2}{6})(\gamma+\frac{\alpha\gamma^3 T^2}{6})-bh(\frac{\gamma^2 T}{2}-\frac{\alpha\gamma^4 T^3}{8})$$

$$-\frac{hb\alpha}{6}(\gamma^4 T^3-\frac{\gamma^6 T^5}{6})+\frac{\pi b}{2}(1-\delta T)(1-\gamma)^2+\frac{\pi b\delta}{4}(T-\gamma^2 T)-bCI_e(\gamma^2 T-\delta\gamma^2 T^2)$$

$$+bCI_e(\frac{\gamma^2 T}{2}+\frac{\delta\gamma^2 T^2}{2}+\frac{\delta\gamma^3 T^2}{2})-bCI_e\frac{\delta\gamma^3 T^2}{3}=0$$

Provided they satisfy sufficient conditions

$$\frac{\partial^2 P(T,s)}{\partial T^2}<0 \ \ and \ \ \frac{\partial^2 P(T,s)}{\partial s^2}<0$$

3.2 Case-ll ($M\le t_1$)

For $M\le t_1$, the buyer has stock on hand beyond M and so he can use the sale revenue to earn interest at an annual rate I_e up to t_1. The interest earned denoted by IE_1, is therefore

$$IE_2=CI_e\int_0^{t_1}\frac{a-bs}{1+\delta(T-t)}tdt$$

$$IC_2=CI_C\int_M^{t_1}\frac{a-bs}{1+\delta(T-t)}dt$$

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Substituting the values of Q, H, S, IC_2, IE_2 and $t_1 = \gamma T$ in the previous equation, we have

$$P(s,T) = s(a-bs) - \tfrac{1}{T}\left[A + C\left\{(a-bs)\frac{\alpha\gamma^3T^3}{6} + (a-bs)T\right\} + h(a-bs)\left\{(\gamma T - \frac{\alpha\gamma^3T^3}{6})(\gamma T + \frac{\alpha\gamma^3T^3}{6})\right\}\right]$$

$$-\left\{\int_0^{\gamma T} te^{\frac{-\alpha t^2}{2}}dt + \frac{\alpha}{6}\int_{\gamma T}^{T}\gamma^3T^3e^{\frac{-\alpha t^2}{2}}dt\right\} + \pi(a-bs)\left\{0.5(1-\delta T)(T-\gamma T)^2 + \frac{\delta}{4}(T^2-\gamma^2T^2)\right\}$$

$$+CI_C\int_M^{\gamma T}\frac{a-bs}{1+\delta(T-t)}dt - CI_e\int_0^{\gamma T}\frac{a-bs}{1+\delta(T-t)}tdt$$

Our objective is to maximize the profit function $P(T,s)$ .The necessary conditions for maximizing the profit functions are

and $\dfrac{\partial P(T,s)}{\partial s} = 0$

$$\Rightarrow a - 2bs + bc\frac{\alpha\gamma^3T^2}{6} - b + bh(\gamma - \frac{\alpha\gamma^3T^2}{6})(\gamma + \frac{\alpha\gamma^3T^2}{6}) + hb(\frac{\gamma^2T}{2} - \frac{\alpha\gamma^4T^3}{8}) + \frac{hb\alpha\gamma^3T^3}{6}$$

$$(1-\gamma-\frac{\alpha T^2}{6}+\frac{\alpha\gamma^3T^2}{6}) + \pi b/2(1-\delta T)(1-\gamma)^2 + \frac{\pi b\delta}{4}(T-\gamma^2T) + bCI_{e)}$$

$$(\lambda - \delta\gamma T + \frac{\delta\gamma^2T}{2} - \frac{M}{T} - \frac{\delta M^2}{2T} + \delta M) - bCI_e(\gamma - \delta\gamma T + \frac{\delta\gamma^2T}{2}) = 0$$

Provided they satisfy the sufficient condition

$$\frac{\partial^2 P(T,s)}{\partial T^2} < 0 \ \ and \ \ \frac{\partial^2 P(T,s)}{\partial s^2} < 0$$

IV REAULTS AND DISCUSSIONS

NUMERICAL EXAMPLE

Case l:(M> t_1)

Let A=100,C=10,h=2,π=10,α=0.5,γ=0.4,a=100,b=2,I_C=0.15,I_e=0.11

Based on above data and using the software Mathematica5.1, we calculate the optimal value M=35, s=25.464,T=7.9156, $P(T,s)$ =1591.2791

Case ll: (M≤t_1)

Based on above input data and using the software Mathematica5.1, we calculate the optimal value M=30, s=33.217,T=6.1881, $P(T,s)$ =1457.1525

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Table-1(Case-I)

M	s	T	$P(T,s)$
5	26.4234	6.07393	1075.4195
10	26.0611	6.41252	941.742
15	25.7833	6.73702	985.6718
20	25.5872	7.04866	1102.7974
25	25.4704	7.34844	1104.9854
30	25.4304	7.63721	1230.5643
35	25.4649	7.91569	1531.2751
40	25.5717	8.18454	1021.7632
45	25.7488	8.44432	994.0465

Table-2(Case-II)

M	s	T	$P(T,s)$
5	28.978	3.07297	868.1983
10	30.1538	3.77613	1056.2521
15	31.3549	4.44256	1121.2085
20	32.5948	5.06177	1329.7001
25	33.8811	5.64126	1394.8143
30	35.2171	6.18814	1457.1525
35	49.6831	4.28826	1032.0140
40	49.7195	5.15218	926.8559

3. SENSITIVITY ANALYSIS

To know, how the optimal results affected by the values of parameters we derive the sensitivity analysis for some parameters. The particular values of some parameters are increased or decreased by +20%,-20% and +10%,-10%. After that, we derive the value of with the $s,T,P(T,s)$ help of increased or decreased values of a, b, α, δ. The results are shown in Table -1 and Table -2 for M> t_1 and M≤t_1 respectively.

The relationship between the parameters and the optimal values are shown in figure

TABLE - 1 (CASE - I)						
Sensitivity analysis of the model (M>t_1)						
Variation Parameters	**Optimal Policies**	**Change in parameters**				
		-20%	**-10%**	**0%**	**10%**	**20%**
a	s	20.9989	23.4962	25.4649	28.4923	30.9909
	T	8.6877	8.69208	7.91569	8.69834	8.70065
	$P(T,s)$	1027.0332	1070.988	1531.2751	1086.7071	1092.3131
b	s	32.2439	28.7718	25.4649	23.7214	21.8275
	T	8.69578	8.69566	7.91569	8.69542	8.69529
	$P(T,s)$	1028.9591	1128.6567	1531.2751	1080.3734	1052.3413
α	s	25.6513	25.8192	25.4649	26.1764	26.3667
	T	8.4048	8.547	7.91569	8.85102	9.01405

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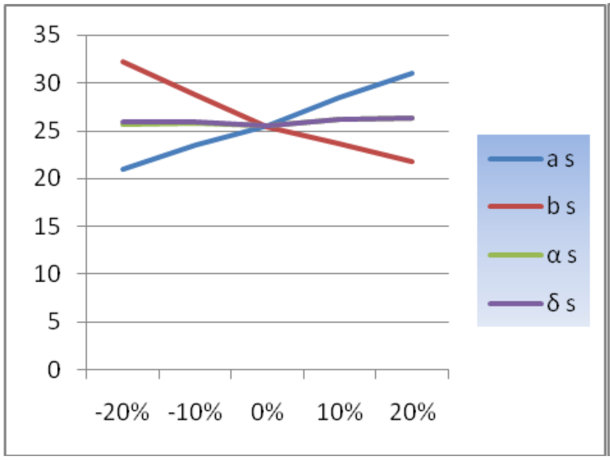
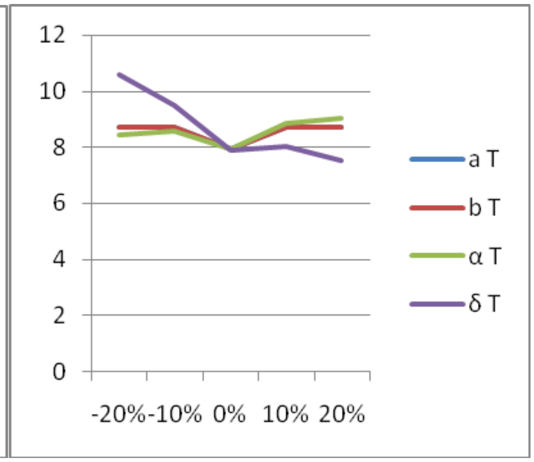
	$P(T,s)$	1063.955	1078.9497	1531.2751	1112.5988	1131.5855
δ	s	25.9418	25.9133	25.4649	26.1439	26.3398
	T	10.5667	9.50204	7.91569	8.05899	7.54095
	$P(T,s)$	1369.273	1130.0031	1531.2751	1089.5624	999.0071





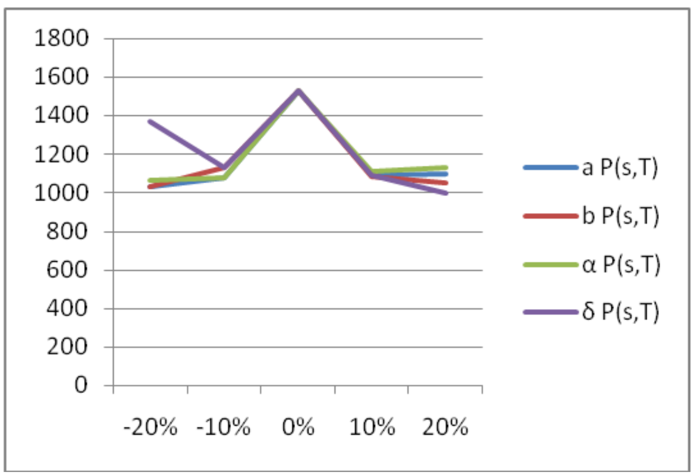


TABLE - 2 (CASE - II)						
Sensitivity analysis of the model ($M \leq t_1$)						
Variation Parameters	**Optimal Policies**	**Change in parameters**				
		-20%	**-10%**	**0%**	**10%**	**20%**
a	s	30.3021	32.7505	33.2171	37.6936	40.1762
	T	6.16691	6.17979	6.11814	6.119401	6.19836
	$P(T,s)$	1321.6	1231.62	1457.1525	1221.574	1352.87
b	s	41.4474	37.9844	33.2171	32.9565	31.0768
	T	6.19306	6.19076	6.11814	6.1851	6.18155
	$P(T,s)$	1121.43	1276.96	1457.1525	1275.341	1120.54
α	s	35.2931	35.3552	33.2171	35.1787	35.1399

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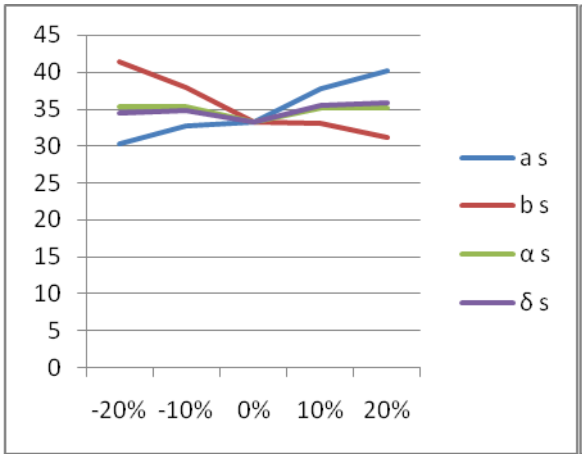
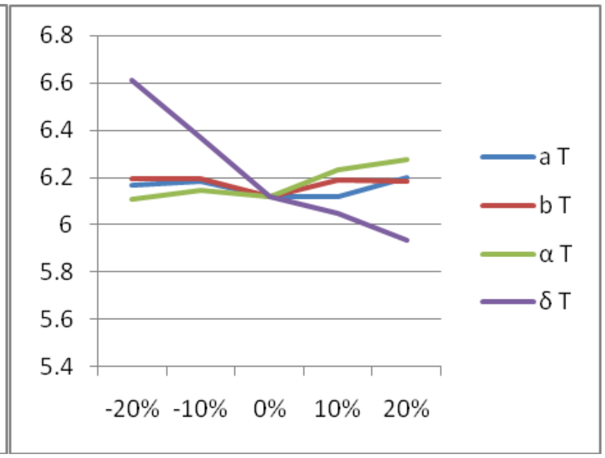
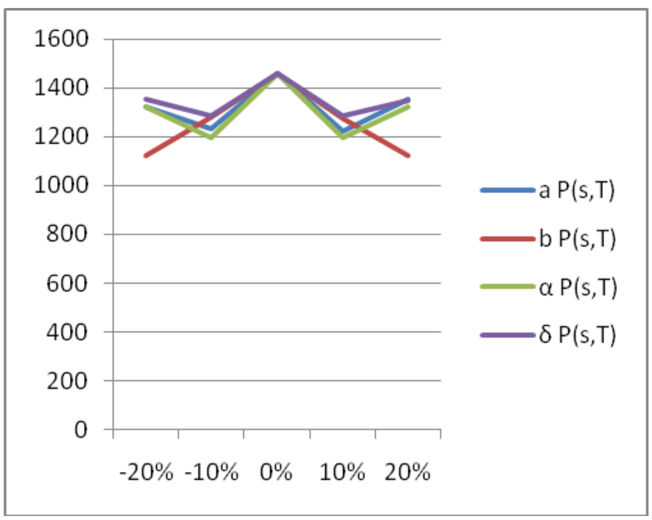
	T	6.1074	6.14728	6.11814	6.23011	6.27324
	$P(T,s)$	1321.43	1198.68	1457.1525	1197.542	1320.76
δ	s	34.5417	34.8734	33.2171	35.567	35.9201
	T	6.61363	6.36971	6.11814	6.04782	5.93617
	$P(T,s)$	1351.24	1287.38	1457.1525	1286.318	1350.23







Observation for Table 1: (Case-I)

- It has been observed that selling price has no significant effect on changing the parameters α and δ, but in case of parameters a and b there is a positive and negative relationship with the selling price respectively.
- It is found that optimal time period T has a reverse relationship with the parameter δ but it increase for other parameters a, b, α.
- Average profit rate function has decreased in nature with the change of parameter a, b, α, δ.

Observation for Table 1: (Case-II)

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- It is found that there is no change on selling price by changing the parameters α and δ. But the selling price has the reverse relationship with b and positive and negative relationship with a.
- The optimal time period has positive and negative relationship with parameter δ and very minute changes of T (which is negligible) for changing the parameters a, b, α.
- It is observed that for changing the parameters a, α, δ, profit rate function has also changed but for changing b, the average total profit of an inventory system decreases.

V CONCLUSIONS & FUTURE SCOPE

We developed an economic order quantity model for time dependent deterioration with selling price dependent demand i.e. the demand rate is a function of selling price. The shortages are allowed which are partially backlogged and also supplier provides a permissible delay in payments. The models are developed under two different policies (i) $M>t_1$ and (ii) $M \le t_{1.}$ Manager of the industry always take care of selling price parameters and the credit period which affect the profit quickly. Numerical illustration and sensitivity analysis with respect to different parameters are also presented in the model. It is interesting to observe that the permissible delay period 30 and 35, the average total profit is maximum for case-I and case-II respectively. So case-I gives better result comparing to case-II. These results are very applicable for real life business scenario. This model is very much realistic and practical.

REFERENCES

[1] P.L. ABAD (2001), "*Optimal price and order size for a reseller under partial backordering*", Computational Operational Research. Vol. 28, pp. 53-65.

[2] P.L. ABAD, and C.K. JAGGI, (2003), *"A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive."* International Journal of Production Economic, Vol. 83, pp. 115-122.

[3] H.J. CHANG, and C.Y. DYE (2001), *"An inventory model for deteriorating items with partial backlogging and permissible delay in payment,"* International Journal of system Science. Vol. 32, pp. 345-352.

[4] K.J. CHUNG, and C.K. HUANG, (2009), *"An ordering policy with allowable shortage and permissible delay in payments."* Applied Mathematical modelling, Vol. 33, pp. 2518-2525.

[5] C.Y. DYE, (2007), *"Joint pricing and ordering policy for a deteriorating inventory with partial backlogging"*, Omega. Vol. 35, pp. 184-189.

[6] S.K. GOYAL, and B.C. GIRI, (2001). *"Recent trends in modelling of deteriorating inventory."* European Journal of operational Research. Vol. 134, pp. 1-16.

[7] A.K. JALAN, B.C. GIRI and K.S. CHADHURI, (2001),*"EOQ model for items with weibull distribution deterioration, shortages and ramp type demand"* **,**Recent Development in Operations Research. pp. 207-223.

[8] B. KARMAKAR, and K.D. CHOUDHURY, (2014), *"Inventory models with ramp-type demand rate for deteriorating items with partial backlogging and time varying holding cost"*, Yugoslav Journal of Operations Research. Vol. 24, pp. 249-266.

[9] A. ROY, (2008), *"An inventory model for deteriorating items with price dependent demand and time varying holding cost"* ,Advanced Modelling and Optimization, Vol. 10, pp. 25-37.

[10] K. SKOURI, and I. KONSTANTARAS, (2009),*"Order level inventory models for deteriorating seasonable products with time dependent demand and shortages"*, Hyundai publishing corporation mathematical problem in engineering article, Vol. ID679736.

[11] G. SRIDEVI, K. NIRUPAMA DEVI, and K. SRINIVAS RAO, (2010). *"Inventory model for deteriorating items with weibull rate of replenishment and selling price dependent demand."* International Journal of Operational Research. Vol. 9, pp. 329-349.

[12] G.P. SAMANTA, and A. ROY, (2004), *"A production inventory model with deteriorating items and shortages"*, Yugoslav Journal of Operation Research. Vol. 14, pp. 219-230.

[13] R.P. TRIPATHY, (2013). *"Inventory model with cash flow oriented and time dependent holding cost under permissible delay in payments."* Yugoslav Journal of Operations Research. Vol. 23, pp. 419-429.

[14] P.K. TRIPATHY, and S. PRADHAN (2012), *"Inventory model for ramp type items with Trade credit under extra ordinary purchase."* IOSR Journal of Mathematics, Vol. 2, pp. 2278-5728.

[15] J.T. TENG (2005), *"Optimal pricing and ordering policy under permissible delay in payments."* International Journal of Production Economics, Vol. 97, pp. 121-129.

[16] K.S. WU, and L.Y. OUYANG (2000), *"A replacement policy for deteriorating items with ramp-type demand rate."* Proceedings of the National Science Council, Republic of China, Part A: Physical Science and Engineering, Vol. 24 , pp. 279–286.