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On s-normal Circulant and con-s-normal Circulant Matrices

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ABSTRACT: s-normal Circulant and con-s-normal circulant matrices are introduced and the basic roperties,results, consepts and structured theorems with examples are discussed in this paper

KEYWORDS: *s-normal,con-s-normal,s-normaql circulant,con-s-normal circulant matrices.*

I. INTRODUCTION

The concept of s-normal matrix, (conjugate) con-s-normal matrix was introduced in [6], [7] and [8], some properties of s-normal matrix given in [1] .In this paper, our intention is to define s-normal circulant matrix, con-s normal circulant matrix also we discussed some properties and results on normal circulant matrix. Let A be circulant normal matrix, $\overline{A}$ is called conjucate of A, A^T is called transpose of A , A^S is called secondary transpose of A, A^Θ is called conjugate secondary transpose of A,A^{-1} is called inverse of A, $A^\dagger$ is called Moore Penrose of A

II. s-NORMAL CIRCULANT MATRICES

DEFINITION: 2.1

For any given $c_0, c_1, c_2, \ldots c_{n-1} \in C^{nx\,n}$ the Circulant matrix $A=(A_{i,j})_{n\,x\,n}$ is defined by

$(A_{i,j}) = A_{j-1(\mathrm{mod}\ n)}$

$$A = \begin{bmatrix} c_0 & c_1 & c_2 & \ldots & c_{n-1} \\ c_{n-1} & c_0 & c_1 & \ldots & c_{n-2} \\ c_{n-2} & c_{n-1} & c_0 & \ldots & c_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_1 & c_2 & c_3 & \ldots & c_0 \end{bmatrix}$$

DEFINITION: 2.2

A Circulant matrix $A \in C^{nx\,n}$ is said to be normal Circulant matrix if $AA^* = A^*A$

DEFINITION: 2.3

A matrix $A \in C^{nx\,n}$ is said to be s-normal Circulant matrix if $A\,A^\Theta = A^\Theta A$ where $A^\Theta = \overline{A^s}$

EXAMPLE: 2.4

$$A = \begin{bmatrix} 2 & 1+i & 1-i \\ 1-i & 2 & 1+i \\ 1+i & 1-i & 2 \end{bmatrix}$$

$$A\,A^\Theta = \begin{bmatrix} 4 & 6 & 6 \\ 6 & 4 & 6 \\ 6 & 6 & 4 \end{bmatrix} = A^\Theta A$$

RESULT:2.5

(i) If A is Circulant matrix then $A=A^S$

(ii) Let a Circulant matrix $A \in C^{nx\,n}$ is said to be s-normal Circulant matrix if $A\overline{A} = \overline{A}A$ ($\because A= A^S$)

THEOREM: 2.6

Let A, B $\in C^{nx\,n}$ are s-normal circulant matrices then A±B is also s-normal circulant matrices.

Proof:

Let A, B are s-normal circulant matrices then $AA^\Theta=AA$ and $BB^\Theta=B^\Theta B$

To prove A±B are s-normal circulant matrices. We will show that $(A\pm B)(A\pm B)^\Theta = (A\pm B)^\Theta (A\pm B)$

Now $(A\pm B)(A\pm B)^\Theta = (A\pm B)(A^\Theta \pm B^\Theta)$

$= (A\pm B)(A^\Theta \pm B^\Theta)$

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$= AA^{\Theta} \pm BA^{\Theta} \pm AB^{\Theta} \pm BB^{\Theta}$

$= A^{\Theta}A \pm B^{\Theta}A \pm A^{\Theta}B \pm B^{\Theta}B$

$= (A^{\Theta} \pm B^{\Theta})\, A \pm (A^{\Theta} \pm B^{\Theta})\, B$

$= (A^{\Theta} \pm B^{\Theta})\,(A \pm B)$

$= (A \pm B)^{\Theta}\,(A \pm B)$

Therefore A±B are s-normal circulant matrices.

THEOREM: 2.7

Let A, B ∈ $C^{nx\ n}$ are s-normal circulant matrices and AB=BA then AB is also s-normal circulant matrix

Proof:

Let A, B are s-normal circulant matrices then $AA^{\Theta}=A^{\Theta}A$ and $BB^{\Theta}=B^{\Theta}B$ given AB=BA

To prove AB is s-normal circulant matrix. We will show that $(AB)\,(AB)^{\Theta} = (AB)^{\Theta}\,(AB)$

Now $(AB)\,(AB)^{\Theta} = AB\,A^{\Theta}B^{\Theta}$

$= BA\,A^{\Theta}B^{\Theta}$

$= BA^{\Theta}AB^{\Theta}$

$= BA^{\Theta}B^{\Theta}A$

$= A^{\Theta}BB^{\Theta}A$

$= A^{\Theta}B^{\Theta}BA$

$= (AB)^{\Theta}(AB)$

Therefore AB is s-normal circulant matrix.

THEOREM: 2.8

Let A, B ∈ $C^{nx\ n}$ are s-normal circulant matrices and AB = BA then AB^{Θ} and $A^{\Theta}B$ are also s-normal circulant matrices.

Proof:

Let A, B are s-normal circulant matrices then $AA^{\Theta} = A^{\Theta}A$ and $BB^{\Theta} = B^{\Theta}B$ given AB = BA

To prove AB^{Θ} is s-normal circulant matrix. We will show that $(AB^{\Theta})\,(AB^{\Theta})^{\Theta} = (AB^{\Theta})^{\Theta}\,(AB^{\Theta})$

Now $(AB)\,(AB)^{\Theta} = (AB)^{\Theta}(AB)$

$AB\,B^{\Theta}A^{\Theta} = (BA)^{\Theta}(BA)$ Where AB=BA

$AB\,B^{\Theta}A^{\Theta} = A^{\Theta}B^{\Theta}BA$

$AB^{\Theta}BA^{\Theta} = A^{\Theta}BB^{\Theta}$ Where $B^{\Theta}B=BB^{\Theta}$

$AB^{\Theta}\,(B^{\Theta})^{\Theta}A^{\Theta} = A^{\Theta}(B^{\Theta})^{\Theta}B^{\Theta}A$ Where $(B^{\Theta})^{\Theta}=B$

$(AB^{\Theta})(AB^{\Theta})^{\Theta} = (AB^{\Theta})^{\Theta}(AB^{\Theta})$

Therefore AB^{Θ} is s-normal circulant matrices.

Similarly we can prove $A^{\Theta}B$ is s-normal circulant matrices.

THEOREM: 2.9

Let A ∈ $C^{nx\ n}$ be s-normal circulant matrix then

(i) iA is s-normal circulant matrix

(ii) –iA is s-normal circulant matrix

Proof:

Let A be a s-normal circulant matrix then $AA^{\Theta}=A^{\Theta}A$

To prove (i) iA is s-normal circulant matrix. We will show that $(iA)\,(iA)^{\Theta}= (iA)^{\Theta}\,(iA)$

Now $AA^{\Theta}=A^{\Theta}A$

$-i^2\,AA^{\Theta} = -i^2\,A^{\Theta}A$

$(iA)(-i)A^{\Theta} = (-i)A^{\Theta}(iA)$

$(iA)(\bar{i})\,A^{\Theta} = (\bar{i})A^{\Theta}(iA)$ where $(–i) = \bar{i}$

$(iA)(\bar{i})^{S}\,A^{\Theta} = (\bar{i})^{S}\,A^{\Theta}(iA)$ where $(\bar{i})^{S} = \bar{i}$

$(iA)\,i^{\Theta}A^{\Theta} = i^{\Theta}A^{\Theta}(iA)$ where $i^{\Theta}= (\bar{i})^{S}$

$(iA)\,(iA)^{\Theta} = (iA)^{\Theta}(iA)$

Therefore iA is s-normal circulant matrix.

(ii) -iA is s-normal circulant matrix. We will show that $(-iA)\,(-iA)^{\Theta}=(-iA)^{\Theta}\,(-iA)$

Now $AA^{\Theta}=A^{\Theta}A$

$-i^2\,AA^{\Theta} = -i^2\,A^{\Theta}A$

$(-i)\,i\,A\,A^{\Theta} = i\,(-i)A^{\Theta}A$

$(-iA)\,(i)\,A^{\Theta} = (i)\,A^{\Theta}(-iA)$

$(-iA)(\overline{-i})A^{\Theta} = (\overline{-i})A^{\Theta}(-iA)$ where $i = \overline{-i}$

$(-iA)(\overline{-i})^{S}\,A^{\Theta} = (\overline{-i})^{S}A^{\Theta}(-iA)$

$(-iA)\,-i^{\Theta}A^{\Theta} = -i^{\Theta}A^{\Theta}(-iA)$ where $-i^{\Theta}= (\overline{-i})^{S}$

$(-iA)\,(-iA)^{\Theta} = (-iA)^{\Theta}(-iA)$

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Therefore -iA is s-normal circulant matrix.

THEOREM: 2.9

Let $A \in C^{nx\,n}$ are s-normal circulant matrix then

(i) $\overline{A}$ is s-normal circulant matrix

(ii) A^S is s-normal circulant matrix

(iii) A^Θ is s-normal circulant matrix

(iv) λA is s-normal circulant matrix (Where λ is a non zero real no)

Proof:

Let A be a s-normal circulant matrix then $AA^\Theta = A^\Theta A$

Proof of (i) $AA^\Theta = A^\Theta A$

$$\overline{AA^\theta} = \overline{AA^\theta}$$

$$\overline{A^\theta}\ \overline{A} = \overline{A}\ \overline{A^\theta}$$

$$\overline{(A)}^\Theta \overline{A} = \overline{A}\ \overline{(A)}^\Theta$$

$$\overline{A}\ \overline{(A)}^\Theta = \overline{(A)}^\Theta\ \overline{A}$$

Therefore $\overline{A}$ is s-normal circulant matrix

Proof of (ii) $AA^\Theta = A^\Theta A$

$$(AA^\Theta)^S = (A^\Theta A)^S$$

$$(A^\Theta)^S A^S = (A)^S (A^\Theta)^S$$

$$(A^S)^\Theta A^S = (A)^S (A^S)^\Theta$$

$$(A)^S (A^S)^\Theta = (A^S)^\Theta A^S$$

Therefore A^S is s-normal circulant matrix

Proof of (iii) $AA^\Theta = A^\Theta A$

$$(AA^\Theta)^\Theta = (A^\Theta A)^\Theta$$

$$(A^\Theta)^\Theta A^\Theta = A^\Theta (A^\Theta)^\Theta$$

$$A^\Theta (A^\Theta)^\Theta = (A^\Theta)^\Theta A^\Theta$$

Therefore A^Θ is s-normal circulant matrix

Proof of (iv) $AA^\Theta = A^\Theta A$

$$\lambda^2 AA^\Theta = \lambda^2 A^\Theta A$$

$$(\lambda A)(\lambda A)^\Theta = (\lambda A)^\Theta (\lambda A) \text{ where } \lambda=\lambda^\Theta$$

Therefore λA is s-normal circulant matrix

THEOREM: 2.10

Let $A \in C^{nx\,n}$ and A^{-1} be an inverse of A then A is s-normal circulant matrix iff A^{-1} is s-normal circulant matrix

Proof:

Let A be a s-normal circulant matrix then $AA^\Theta = A^\Theta A$

To prove A^{-1} is k-normal circulant matrix. We will show that $(A^{-1}) (A^{-1})^\Theta = (A^{-1})^\Theta (A^{-1})$

Now $A A^\Theta = A^\Theta A$

$$(AA^\Theta)^{-1} = (A^\Theta A)^{-1}$$

$$(A^\Theta)^{-1} A^{-1} = A^{-1} (A^\Theta)^{-1}$$

$$(A^\Theta)^{-1} A^{-1} = A^{-1} (A^\Theta)^{-1}$$

$$(A^{-1})^\Theta A^{-1} = A^{-1} (A^{-1})^\Theta$$

$$A^{-1} (A^{-1})^\Theta = (A^{-1})^\Theta A^{-1}$$

Therefore A^{-1} is s-normal circulant matrix.

Let as assume that, A^{-1} is s-normal circulant matrix.

To prove A is s-normal circulant matrix. We will show that $A A^\Theta = A A^\Theta$

Now $A^{-1} (A^{-1})^\Theta = (A^{-1})^\Theta A^{-1}$

$$(A^{-1} (A^{-1})^\Theta)^{-1} = ((A^{-1})^\Theta A^{-1})^{-1}$$

$$A^\Theta (A^{-1})^{-1} = (A^{-1})^{-1} A^\Theta$$

$$A^\Theta A = A A^\Theta$$

$$A A^\Theta = A^\Theta A$$

Therefore A is s-normal circulant matrix

THEOREM: 2.11

Let $A \in C^{nx\,n}$ and $A^\dagger$ be the Moore Penrose inverse of A then A is s-normal circulant matrix iff $A^\dagger$ is s-normal circulant matrix

Proof:

Let A be a s-normal circulant matrix then $AA^\Theta=A^\Theta A$

To prove $A^\dagger$ is s-normal circulant matrix. We will show that $(A^\dagger) (A^\dagger)^\Theta= (A^\dagger)^\Theta (A^\dagger)$

Now $A A^\Theta = A^\Theta A$

$$(AA^\Theta)^\dagger = (A^\Theta A)^\dagger$$

$$(A^\Theta)^\dagger A^\dagger = A^\dagger (A^\Theta)^\dagger$$

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$(A^{\dagger})^{\Theta} A^{\dagger} = A^{\dagger} (A^{\dagger})^{\Theta}$

$A^{\dagger} (A^{\dagger})^{\Theta} = (A^{\dagger})^{\Theta} A^{\dagger}$

Therefore $A^{\dagger}$ is s-normal circulant matrix.

Let as assume that, $A^{\dagger}$ is s-normal circulant matrix.

To prove A is s-normal circulant matrix. We will show that $A A^{\Theta} = A A^{\Theta}$

Now $A^{\dagger} (A^{\dagger})^{\Theta} = (A^{\dagger})^{\Theta} A^{\dagger}$

$(A^{\dagger} (A^{\dagger})^{\Theta})^{\dagger} = ((A^{\dagger})^{\Theta} A^{\dagger})^{\dagger}$

$A^{\Theta} (A^{\dagger})^{\dagger} = (A^{\dagger})^{\dagger} A^{\Theta}$

$A^{\Theta} A = A A^{\Theta}$

$A A^{\Theta} = A^{\Theta} A$

Therefore A is s-normal circulant matrix.

III. CONJUGATE s-NORMAL CIRCULANT MATRICES

DEFINITION: 3.1

A Circulant matrix $A \in C^{nx\,n}$ is said to be con k-normal Circulant matrix if $AA^{\Theta} = A^{S}\,\overline{A}$

EXAMPLE: 3.2

$$A = \begin{bmatrix} 2 & 1+i & 1-i \\ 1-i & 2 & 1+i \\ 1+i & 1-i & 2 \end{bmatrix}$$

$$A A^{\Theta} = \begin{bmatrix} 4 & 6 & 6 \\ 6 & 4 & 6 \\ 6 & 6 & 4 \end{bmatrix} = A^{S}\,\overline{A}$$

THEOREM: 3.3
Let $A, B \in C^{nx\,n}$ are con s-normal circulant matrices then A±B are also con s-normal circulant matrices.

Proof:
Let A , B are con s-normal circulant matrices then $AA^{\Theta} = A^{S}\,\overline{A}$ and $BB^{\Theta} = B^{S}\,\overline{B}$

To prove A±B are con s-normal circulant matrices.

We will show that $(A \pm B)(A \pm B)^{\Theta} = (A \pm B)^{S}\,\overline{(A \pm B)}$.

Now $(A \pm B)(A \pm B)^{\Theta} = (A \pm B)(A^{\Theta} \pm B^{\Theta})$

$= (A \pm B)(A^{\Theta} \pm B^{\Theta})$

$= AA^{\Theta} \pm AB^{\Theta} \pm BA^{\Theta} \pm BB^{\Theta}$

$= A^{S}\overline{A} \pm B^{S}\overline{A} \pm A^{S}\overline{B} \pm B^{S}\,\overline{B}$

$= A^{S}\,[\overline{A \pm B}] \pm B^{S}\,[\overline{A \pm B}]$

$= [A^{S} \pm B^{S}]\,[\overline{A \pm B}]$

$= [A \pm B]^{S}\,[\overline{A \pm B}]$ Where $[A \pm B]^{S} = [A^{S} \pm B^{S}]$, $[\overline{A \pm B}] = \overline{A} \pm \overline{B}$

THEOREM: 3.4
Let $A, B \in C^{nx\,n}$ are con s-normal circulant matrices and AB=BA then AB is also con s-normal circulant matrix
Proof:
Let A, B are con s-normal circulant matrices then $AA^{\Theta} = A^{S}\,\overline{A}$ and $BB^{\Theta} = B^{S}\,\overline{B}$

To prove AB is con s-normal circulant matrix. We will show that $(AB)(AB)^{\Theta} = (AB)^{S}\,\overline{(AB)}$

Now $(AB)(AB)^{\Theta} = A B B^{\Theta} A^{\Theta}$

$= (AB^{\Theta}) B A^{\Theta}$

$= (AB^{\Theta})\, A^{S}\,\overline{B}$

$= (AB^{\Theta})\, A^{S}\,\overline{B}$

$= B^{S}\,\overline{A}\, A^{S}\,\overline{B}$

$= B^{S} A^{S}\,\overline{A}\,\overline{B}$

THEOREM: 3.5

Let $A \in C^{nx\,n}$ be con s-normal circulant matrix then
(i) iA is con s-normal circulant matrix
(ii) –iA is con s-normal circulant matrix
Proof:
Let A is con s-normal circulant matrix then $AA^{\Theta} = A^{S}\,\overline{A}$

To prove (i) iA is con s-normal circulant matrix. We will show that $(iA)(iA)^{\Theta} = (iA)^{S}\,\overline{(iA)}$

Now $AA^{\Theta} = A^{S}\,\overline{A}$

$i^{2} AA^{\Theta} = (i^{2}) A^{S}\,\overline{A}$

$(iA)(iA^{\Theta}) = (i) A^{S} (i\,\overline{A})$

$(iA)(-i)^{\Theta} A^{\Theta} = (i^{S}) A^{S}$ **(-** $\overline{i\,A}$)

$-(iA)(iA)^{\Theta} = -(iA)^{S}(\overline{iA})$

$(iA)(iA)^{\Theta} = (iA)^{S}(\overline{iA})$

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Therefore iA is con s–normal circulant matrix.

(ii) - iA is con s-normal circulant matrix. We will show that $(-iA)(-iA)^{\Theta} = (-iA)^S(-\overline{iA})$

$$\text{Now } AA^{\Theta} = A^S\overline{A}$$

$$-i^2 AA^{\Theta} = (-i)^2 A^S\overline{A}$$

$$(-iA)(iA^{\Theta}) = (-i A^S)(i\overline{A})$$

$$(-iA)(-i^{\Theta}A^{\Theta}) = (-i^S A^S)(-\overline{i\,A})$$

$$(-iA)(-iA)^{\Theta} = (-iA)^S(-\overline{iA})$$

Therefore -iA is con s-normal circulant matrix.

THEOREM: 3.6

Let $A \in C^{nx\,n}$ be con s-normal circulant matrix then

(i) $\overline{A}$ is con s-normal circulant matrix

(ii) A^S is con s-normal circulant matrix

(iii) A^{Θ} is con s-normal circulant matrix

(iv) λA is con s-normal circulant matrix (Where λ is a non zero real no)

Proof:

Let A be a con s-normal circulant matrix then $AA^{\Theta} = A^S\overline{A}$

Proof of (i) $AA^{\Theta} = A^S\overline{A}$

$$\overline{AA^{\theta}} = \overline{A^S\overline{A}}$$

$$\overline{A^{\theta}}\ \overline{A} = AA^{\theta}$$

$$A^T\overline{A} = AA^{\theta}$$

$$A A^{\theta} = A^S\overline{A}$$

Therefore $\overline{A}$ is con s-normal circulant matrix.

Proof of (ii) $AA^{\Theta} = A^S\overline{A}$

$$(AA^{\Theta})^S = (A^S\overline{A})^S$$

$$(A^{\Theta})^S A^S = (\overline{A})^S (A)$$

$$(\overline{A}) A^S = A^{\Theta}A$$

$$A^S\overline{A} = AA^{\Theta}$$

$$AA^{\Theta} = A^S\overline{A}$$

Therefore A^S is con s-normal circulant matrix.

Proof of (iii) $AA^{\Theta} = A^S\overline{A}$

$$(AA^{\Theta})^{\Theta} = (A^S\overline{A})^{\Theta}$$

$$(A^{\Theta})^{\Theta}A^{\Theta} = (\overline{A})^{\Theta}(A^S)^{\Theta}$$

$$AA^{\Theta} = A^S\overline{A}\ S$$

$$AA^{\Theta} = A^S\overline{A}$$

Therefore A^{Θ} is con s-normal circulant matrix

Proof of (iv) $AA^{\Theta} = A^S\overline{A}$

$$\lambda^2 AA^{\Theta} = \lambda^2 A^S\overline{A}$$

$$(\lambda A)(\lambda A)^{\Theta} = (\lambda A^S)(\lambda\overline{A}) \text{ where } \lambda = \lambda^{\Theta}$$

Therefore λA is con s-normal circulant matrix

THEOREM: 3.7

Let $A \in C^{nx\,n}$ and A^{-1} be an inverse of A then A is con s-normal circulant matrix iff A^{-1} is con s-normal circulant matrix.

Proof:

Let A be a con s-normal circulant matrix then $AA^{\Theta} = A^S\overline{A}$

To prove A^{-1} is con s-normal circulant matrix. We will show that $(A^{-1})(A^{-1})^{\Theta} = (A^{-1})^S(\overline{A}^{-1})$

$$\text{Now}\quad A A^{\Theta} = A^S\overline{A}$$

$$(AA^{\Theta})^{-1} = (A^S\overline{A})^{-1}$$

$$(A^{\Theta})^{-1}A^{-1} = (\overline{A})^{-1}(A^S)^{-1}$$

$$(A^{-1})^{\Theta}(A^{-1}) = \left(\overline{A^{-1}}\right)(A^{-1})^S$$

$$A^{-1}(A^{-1})^{\Theta} = (A^{-1})^S\left(\overline{A^{-1}}\right)$$

Therefore A^{-1} is con s-normal circulant matrix.

Let as assume that, A^{-1} is con s-normal circulant matrix.

To prove A is con s-normal circulant matrix. We will show that $A A^{\Theta} = A^S\overline{A}$

$$\text{Now}\quad A^{-1}(A^{-1})^{\Theta} = (A^{-1})^S\left(\overline{A^{-1}}\right)$$

$$[A^{-1}(A^{-1})^{\Theta}]^{-1} = [(A^{-1})^S\left(\overline{A^{-1}}\right)]$$

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$$A^{\Theta}(A^{-1})^{-1} = \overline{A}\,A^{S}$$

$$A^{\Theta}A = \overline{A}\,A^{S}$$

$$A\,A^{\Theta} = A^{S}\,\overline{A}$$

Therefore A is con s-normal circulant matrix.

THEOREM: 3.8

Let $A \in C^{n \times n}$ and $A^{\dagger}$ be the Moore Penrose inverse of A then A is con s-normal circulant matrix iff $A^{\dagger}$ is con s-normal circulant matrix

Proof:

Let A be a con s-normal circulant matrix then $AA^{\Theta} = A^{S}\,\overline{A}$

To prove $A^{\dagger}$ is con s-normal circulant matrix. We will show that $(A^{\dagger})(A^{\dagger})^{\Theta} = (A^{\dagger})^{S}(\overline{A}^{\dagger})$

Now $A\,A^{\Theta} = A^{S}\,\overline{A}$

$$(AA^{\Theta})^{\dagger} = (A^{S}\,\overline{A})^{\dagger}$$

$$(A^{\Theta})^{\dagger}A^{\dagger} = (\overline{A})^{\dagger}(A^{S})^{\dagger}$$

$$(A^{\dagger})^{\Theta}A^{\dagger} = (\overline{A}^{\dagger})(A^{\dagger})^{S}$$

$$A^{\dagger}(A^{\dagger})^{\Theta} = (A^{\dagger})^{S}(\overline{A}^{\dagger})$$

herefore $A^{\dagger}$ is con s-normal circulant matrix.

Let as assume that, $A^{\dagger}$ is con s-normal circulant matrix.

To prove A is con s-normal circulant matrix. We will show that $AA^{\Theta} = A^{S}\,\overline{A}$

Now $A^{\dagger}(A^{\dagger})^{\Theta} = (A^{\dagger})^{S}(\overline{A}^{\dagger})$

$$[A^{\dagger}(A^{\dagger})^{\Theta}]^{\dagger} = [(A^{\dagger})^{S}(\overline{A}^{\dagger})]^{\dagger}$$

$$A^{\Theta}(A^{\dagger})^{\dagger} = (\overline{A}^{\dagger})\,A^{S}$$

$$A^{\Theta}A = \overline{A}\,A^{S}$$

$$A\,A^{\Theta} = A^{S}\,\overline{A}$$

Therefore A is con s-normal circulant matrix.

IV.CONCLUSION

The s-normal circulant and con-s-normal circulant matrices defined and proof of structure theorem is shown by example. We can find the determinant, inverse and some properties of the s-normal circulant matrices through the result.In future the various circulant matrices similar to unitary circulant, polynomial circulant matrices may be consider this study

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