International Journal of Scientific Research in Mathematical and Statistical Sciences
Vol.6, Issue.1, pp.198-202, February (2019)
DOI: https://doi.org/10.26438/ijsrmss/v6i1.198202

Review Paper

E-ISSN: 2348-4519

Hyers-Ulam Stability of Third Order System of Differential Equation

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Available online at: www.isroset.org

Received: 17/Jan/2019, Accepted: 08/Feb/2019, Online: 28/Feb/2019

***Abstract*—** In this paper, we study the Hyers-Ulam stability, Generalised Hyers-Ulam stability, Hyers-Ulam-Rassias stability and Generalised Hyers-Ulam-Rassias stability of the system of third order differential equation of the form $\boldsymbol{x'''(t) = f(t, x(t), x'(t), x''(t))}$with initial conditions $\boldsymbol{x(a) = x_0}$, $\boldsymbol{x'(a) = x_1}$ and $\boldsymbol{x''(a) = x_2}$ in Banach spaces.

***Keywords*—** *Hyers-Ulam stability; Generalised Hyers-Ulam stability; Hyers-Ulam-Rassias stability; Generalised Hyers-Ulam-Rassias stability; system of differential equations; Initial conditions*.

I. INTRODUCTION

In pure mathematics, differential equations are studied from several different perspectives, mostly concerned with their solutions. Only the simplest differential equations are solvable by explicit formulas. However, some properties of solutions of a given differential equation may be determined without finding their exact form. Equilibria are not always stable. Since stable and unstable equilibrium play quite different roles in the dynamics of a system, it is useful to be able to classify equilibrium points based on their stability.

Definition of Hyers-Ulam stability and Hyers-Ulam-Rassias stability have applicable significance since it means that if one is studying the Hyers-Ulam stable system then one does not have to reach the exact solution. (Which usually is quite difficult or time consuming). This is quite useful in many applications, for example Numerical Analysis, Optimization, Biology, and Economics etc., where finding the exact solution is quite difficult.

The stability problem for various functional equations was originated from a famous talk of S.M. Ulam [20]. In 1940, Ulam was raised the question: Suppose one has a function y(t) which is close to solve an equation. Is there an exact solution x(t) of the equation which is close to y(t) ? (See [4, 11]). In 1941, D. H. Hyers [4] gave an affirmative answer to the equation of Ulam for additive Cauchy equation in Banach Spaces. The result of Hyers was generalized by T. Aoki [22] and Th. M. Rassias [16]. After that many Mathematicians have extended Ulam's problem in various direction [3, 12, 15].

A generalization of Ulam's problem was recently proposed by replacing functional equations with differential equations: The differential equation

$$\varphi\left(f, x, x', x'', \dots x^{(n)}\right) = 0$$

has the Hyers-Ulam stability, if for a given $\varepsilon > 0$ and a function x such that

$$\left|\varphi\left(f, x, x', x'', \dots x^{(n)}\right)\right| \leq \varepsilon,$$

there exists a solution x_a of the differential equation such that

$$\|x(t) - x_a(t)\| \leq K(\varepsilon),$$

and $\lim_{n\to\infty} K(\varepsilon) = 0$.

If the preceding statement is also true when we replace ε and $K(\varepsilon)$ by $\varphi(t)$ and $\phi(t)$, where ϕ, φ are appropriate functions not depending on x and x_a explicitly, then we say that the corresponding differential equation has the generalized Hyers-Ulam stability or Hyers-Ulam-Rassias stability.

Obloza seems to be the first author who has investigated the Hyers – Ulam stability of linear differential equations [13, 14]. Thereafter, in 1998, C. Alsina and R. Ger [1] were the first authors who investigated the Hyers-Ulam stability of differential equations. They proved in [1] the following Theorem.

Theorem 1.1. Assume that a differentiable function $f : I \to \mathbb{R}$ is a solution of the differential inequality

$\|x'(t) - x(t)\| \leq \varepsilon$. Where I is an open sub interval of $\mathbb{R}$. Then there exists a solution $g: I \to \mathbb{R}$ of the differential equation $x'(t) = x(t)$ such that for any $t \in I$, we have, $\|f(t) - g(t)\| \leq 3\varepsilon$.

This result of C. Alsina and R. Ger [1] has been generalized by Takahasi [12]. They proved in [12] that the Hyers - Ulam stability holds true for the Banach Space valued differential equation $x'(t) = \lambda x(t)$. After that, many Mathematicians have extended Ulam's problem in various direction (See [2, 4, 5, 8, 10]).

Those previous results were extended to the Hyers-Ulam stability of linear differential equation of first order and higher order with constant coefficients in [6, 7, 8, 9, 10, 19, 23] and in [2], [11], [12], [13], [14], [17], [18], [21] and [26-29] respectively. Furthermore, Jung has proved the Hyers-Ulam stability of linear differential equations (See [5-9]). Rus investigated the Hyers-Ulam stability of linear differential equation and integral equations using the Gronwall's lemma and the technique of weakly Picard's operators (See [24], [25]).

In 2014, Q.H. Alqifiary and J.K. Miljanovic [30] are examine the relation between practical stability and Hyers-Ulam-stability and Hyers-Ulam-Rassias stability as well. In addition, by practical stability we gave a sufficient condition in order that the first order nonlinear Systems of differential equations has local generalized Hyers-Ulam stability and local generalized Hyers-Ulam-Rassias stability.

Recently, R. Murali and A. Ponmana Selvan [31] are studied the Hyers-Ulam stability, Generalized Hyers-Ulam stability, Hyers-Ulam-Rassias stability and Generalised Hyers-Ulam-Rassias stability of the system of second order differential equation of the form

$$x''(t) = f(t, x(t), x'(t))$$

with initial conditions $x(a) = x_0$ *and* $x'(a) = x_1$ in Banach spaces.

Encouraged by the above ideas, the purpose of this paper is to study the Hyers-Ulam stability, Generalised Hyers-Ulam stability, Hyers-Ulam-Rassias stability and Generalized Hyers-Ulam-Rassias stability of the system of third order differential equation of the form

$$x'''(t) = f(t, x(t), x'(t), x''(t)) \qquad (1.1)$$

in Banach space with initial conditions

$$x(a) = x_0,\ x'(a) = x_1 \text{ and } x''(a) = x_2 \qquad (1.2)$$

for all $t \in I$, $x(t) \in C^3(I, \mathbb{B})$ and where $I = [a, b)$, $a \in \mathbb{R}$, $b \in \overline{\mathbb{R}}$, $-\infty < a < b < +\infty$.

II. PRELIMINARIES

Let $(\mathbb{B}, \|.\|)$ be a Banach Space (Real or Complex), and $I = [a, b)$, $a \in \mathbb{R}$, $b \in \overline{\mathbb{R}}$, $a < b \leq +\infty$, ε be a positive real number, $f: I \times \mathbb{B} \to \mathbb{B}$ continuous operator and $\phi: I \to \mathbb{R}_+$ be a continuous function. Let us consider the system $x''(t) = f(t, x(t), x'(t))$ for all $t \in I$, where f is defined and continuous on $I \times \mathbb{B}$. Let G be a closed and bounded set of $\mathbb{B}$ containing the origin then there exists a real number $M > 0$ such that $G = \{x: \|x\| \leq M\}$ and let G_0 be a subset of G.

Definition 2.1 Let $x^*(t, x_0, x_1, x_2)$ be the solution of the system (1.1) satisfying the intial conditios:

$$x^*(a) = x_0, x^{*\prime}(a) = x_1 \text{ and } x^{*\prime\prime}(a) = x_2.$$

If for every $\varepsilon > 0$, $x_0, x_1, x_2 \in G_0$ and each $x^*(t, x_0, x_1, x_2)$ is in G for all $t \in I$, then the origin is said to be (G_0, G, ε)-practically stable. The solution which starts initially in G_0 remain thereafter in G.

Definition 2.2 We say that the differential equation (1.1) has the Hyers-Ulam stability with initial condition (1.2), if there exists a constant $S_f > 0$ such that for every $\varepsilon > 0$ and for each solution $x(t) \in C^3(I, \mathbb{B})$ satisfying the inequality $\left\|x'''(t) - f\big(t, x(t), x'(t), x''(t)\big)\right\| \leq \varepsilon$, for all $t \in I$. Then there exists some $y \in C^3(I, \mathbb{B})$ satisfies the differential equation (1.1) with initial conditions (1.2) such that $\|x(t) - y(t)\| \leq S_f \varepsilon$, for all $t \in I$. We call such S as the Hyers-Ulam stability constant for (1.1) with (1.2).

Definition 2.3 We say that the differential equation (1.1) has the Hyers-Ulam-Rassias stability with initial condition (1.2) with respect to ϕ if there exists a constant $S_{f,\phi} > 0$ such that for every $\varepsilon > 0$ and for each solution $x(t) \in C^3(I, \mathbb{B})$ satisfies the inequality $\left\|x'''(t) - f\big(t, x(t), x'(t), x''(t)\big)\right\| \leq \varepsilon\phi(t)$, for all $t \in I$. Then there exists some $y \in C^3(I, \mathbb{B})$ satisfies the differential equation (1.1) with initial conditions (1.2) such that $\|x(t) - y(t)\| \leq S_{f,\phi}\varepsilon\phi(t)$, for all $t \in I$. We call such S as the Hyers-Ulam-Rassias stability constant for (1.1) with initial condition (1.2).

Definition 2.4 The differential equation (1.1) is said to have the Generalized Hyers-Ulam stability with initial condition (1.2), if there exists $\theta_f \in C(\mathbb{R}_+, \mathbb{R}_+)$ with $\theta_f(0) = 0$ such that $x \in C^3(I, \mathbb{B})$ satisfying the inequality $\left\| x'''(t) - f\big(t, x(t), x'(t), x''(t)\big) \right\| \leq \varepsilon$, for all $t \in I$. Then there exists some $y \in C^3(I, \mathbb{B})$ satisfies the differential equation (1.1) with initial conditions (1.2) such that $\|x(t) - y(t)\| \leq \theta_f(\varepsilon)$, for all $t \in I$.

Definition 2.5 The differential equation (1.1) is said to have the Generalized Hyers-Ulam-Rassias stability with initial condition (1.2) with respect to ϕ if there exists a constant $S_{f,\phi} > 0$ such that for every $\varepsilon > 0$ and for each solution $x(t) \in C^3(I, \mathbb{B})$ satisfies the inequality $\left\| x'''(t) - f\big(t, x(t), x'(t), x''(t)\big) \right\| \leq \phi(t)$, for all $t \in I$. Then there exists some $y \in C^3(I, \mathbb{B})$ satisfies the differential equation (1.1) with initial conditions (1.2) such that $\|x(t) - y(t)\| \leq S_{f,\phi}\phi(t)$, for all $t \in I$. We call such S as the Generalized Hyers-Ulam-Rassias stability constant for (1.1) with initial condition (1.2).

Remark 2.6 Consider the system of differential equation (1.1) with initial condition

$$x(a) = x_0 \in G_0, \;\; x'(a) = x_1 \in G_0 \text{ and } x''(a) = x_2 \in G_0 \tag{2.1}$$

where f is defined and continuous on $I \times \mathbb{B}$ and equilibrium state is at the origin: $f(t, 0,0,0) = 0$, for all $t \in I$. The system (1.1) to be (G_0, G, ε)-practically stable it is sufficient that there exists a continuous non increasing on the system (1.1) solutions Lyapunov function $V(x, t)$ such that

$$\wp = \left\{x \in \mathbb{B}: V\big(t, x(t), x'(t), x''(t)\big) \leq 1\right\} \subseteq G, \quad t \in I, \tag{2.2}$$

$$G_0 \subseteq \left\{x \in \mathbb{B}: V\big(t, x(t), x'(t), x''(t)\big) \leq 1\right\}. \tag{2.3}$$

Proof. We prove this remark by contradiction. Suppose that the conditions (2.2) and (2.3) are satisfied but there are $\mu \in I$ and $x_0, x_1, x_2 \in G_0$ such that the solution $x(t) = x(a, x_0, x_1, x_2)$ of (1.1) leaves the set G. From (2.2) follows the inequality $V\big(\mu, x(\mu), x'(\mu), x''(\mu)\big) > 1$ which contradicts the condition (2.3). Hence the equilibrium of the system (1.1) is (G_0, G, ε)-practically stable.

III. Main Results

In this section, we prove the Hyers-Ulam stability, Generalised Hyers-Ulam stability, Hyers-Ulam-Rassias stability and Generalised Hyers-Ulam-Rassias stability of the differential equation (1.1) with (1.2). Firstly, we prove the Hyers-Ulam stability of (1.1) with (1.2).

Theorem 3.1 Assume that there exists a constant $S_f > 0$ such that for every $\varepsilon > 0$ and for each solution $x(t) \in C^3(I, \mathbb{B})$ satisfying the inequality $\left\| x'''(t) - f\big(t, x(t), x'(t), x''(t)\big) \right\| \leq \varepsilon$, with (1.2) for all $t \in I$. Then there exists some $y \in C^3(I, \mathbb{B})$ satisfies (1.1) with initial condition $y(a) = y_0$ $y'(a) = y_1$ and $y''(a) = y_2$ such that

$$\|x(t) - y(t)\| \leq S_f \varepsilon, \quad \forall \; t \in I.$$

Proof. Given that for every $\varepsilon > 0$, and for each solution $x(t) \in C^3(I, \mathbb{B})$ satisfying

$$\left\| x'''(t) - f\big(t, x(t), x'(t), x''(t)\big) \right\| \leq \varepsilon, \tag{3.1}$$

for all $t \in I$. Now, we are going to prove that there exists a real number $S_f > 0$ and for some y in $C^3(I, \mathbb{B})$ satisfies the inequality (1.1) with $y(a) = y_0$, $y'(a) = y_1$ and $y''(a) = y_2$ such that

$$\|x(t) - y(t)\| \leq S_f \varepsilon,$$

for all $t \in I$. Let G be a closed and bounded set, then there exists a real number $M > 0$ such that

$$G = \{x: \|x\| \leq M\}.$$

Now, let $x^* = f(a, x_0, x_1, x_2)$ satisfies the inequality (3.1) for arbitrary ε, then x^* be the solution of the differential equation (1.1) with (1.2). Since the equilibrium of (1.1) is (G_0, G, ε)-practically stable, then $x^* \in G$.

Hence $\|x^*\| \leq M$, since $M > 0$, and $\varepsilon > 0$ then there exists $S_f > 0$ such that $M = S_f \varepsilon$. Then we have

$$\|x^*\| \leq S_f \varepsilon,$$

for all $t \in I$. Obviously, $y(t) \equiv 0$ be the solution of (1.1) with (1.2) such that

$$\|x^*(t) - y(t)\| \leq S_f \varepsilon,$$

for all $t \in I$. Hence by the virtue of Definition 2.2, the system of second order differential equation has the Hyers-Ulam stability.

The following corollary shows the Generalized Hyers-Ulam stability of the system (1.1) with (1.2).

Corollary 3.2 Assume that there exists $\theta_f > 0$ such that for every $\varepsilon > 0$ and for each solution $x(t) \in C^3(I, \mathbb{B})$ satisfies $\left\| x'''(t) - f\big(t, x(t), x'(t), x''(t)\big) \right\| \leq \varepsilon$, with initial condition $x(a) = x_0$ $x'(a) = y_1$ and $x''(a) = y_2$ for all $t \in I$. Then there exists some $y \in C^3(I, \mathbb{B})$ satisfies (1.1) with initial condition $y(a) = y_0$ $y'(a) = y_1$ and $y''(a) = y_2$ such that

$$\|x(t) - y(t)\| \leq \theta_f(\varepsilon), \quad \forall \ t \in I.$$

Proof. We can prove this corollary by the similar way as in the proof of Theorem 3.1.

Next, we prove the Hyers-Ulam-Rassias stability of the system (1.1) with initial condition (1.2).

Theorem 3.3 Assume that there exists a constant $S_{f,\phi} > 0$ such that for every $\varepsilon \in (0, \varepsilon_0]$ and for each solution $x(t) \in C^3(I, \mathbb{B})$ satisfying the inequality

$$\left\| x'''(t) - f\big(t, x(t), x'(t), x''(t)\big) \right\| \leq \varepsilon\phi(t),$$

with initial condition (1.2) for all $t \in I$. Then there exists some $y \in C^3(I, \mathbb{B})$ satisfies (1.1) with initial condition $y(a) = y_0, y'(a) = y_1$ and $y''(a) = y_2$ such that

$$\|x(t) - y(t)\| \leq S_{f,\phi}\varepsilon\phi(t), \quad \forall \ t \in I.$$

Proof. Given that for every $\varepsilon \in (0, \varepsilon_0]$, and for each solution $x(t) \in C^3(I, \mathbb{B})$ satisfying

$$\left\| x'''(t) - f\big(t, x(t), x'(t), x''(t)\big) \right\| \leq \varepsilon\phi(t), \tag{3.2}$$

for all $t \in I$. Now, we prove that there exists real number $S_{f,\phi} > 0$ and for some $y \in C^3(I, \mathbb{B})$ satisfying the inequality (1.1) with $y(a) = y_0$, $y'(a) = y_1$ and $y''(a) = y_2$ such that

$$\|x(t) - y(t)\| \leq S_{f,\phi}\varepsilon\phi(t),$$

for all $t \in I$. Let G be a closed and bounded set, then there exists a real number $M > 0$ such that

$$G = \{x: \|x\| \leq M\}.$$

If the equilibrium is (G_0, G, ε_0)-practically stable and there exists $\varepsilon_1 > 0$ such that $\varepsilon_1 \leq \phi(t)\varepsilon \leq \varepsilon_0$ for all $t \in I$. Now, let $x^* = f(a, x_0, x_1, x_2)$ satisfies the inequality (3.2) for arbitrary ε_1, then x^* be the solution of the differential equation (1.1) with (1.2). Since the equilibrium of (1.1) is (G_0, G, ε_0) - practically stable, then $x^* \in G$.

Hence $\|x^*\| \leq M$, since $M > 0$, and $\varepsilon_1 > 0$ then there exists $S_{f,\phi} > 0$ such that $M = S_{f,\phi}\varepsilon_1$. Then we have $\|x^*\| \leq S_{f,\phi}\varepsilon_1$, for all $t \in I$, hence $\|x^*\| \leq S_{f,\phi}\varepsilon\phi(t)$.

Obviously, $y(t) \equiv 0$ be the solution of (1.1) with (1.2) such that $\|x^*(t) - y(t)\| \leq S_{f,\phi}\varepsilon\phi(t)$, for all $t \in I$. Hence by the virtue of Definition 2.3, the system of second order differential equation has the Hyers-Ulam stability.

The following corollary shows the Generalized Hyers-Ulam-Rassias stability of the system (1.1) with (1.2).

Corollary 3.4 Assume that there exists a constant $S_{f,\phi} > 0$ such that for every $\varepsilon \in (0, \varepsilon_0]$ and for each solution $x(t) \in C^3(I, \mathbb{B})$ satisfying the inequality

$$\left\| x'''(t) - f\big(t, x(t), x'(t), x''(t)\big) \right\| \leq \phi(t),$$

with (1.2) for all $t \in I$. Then there exists some $y \in C^3(I, \mathbb{B})$ satisfies (1.1) with initial condition $y(a) = y_0$, $y'(a) = y_1$ and $y''(a) = y_2$ such that

$$\|x(t) - y(t)\| \leq S_{f,\phi}\phi(t), \quad \forall \ t \in I.$$

Proof. We can prove this corollary by the similar way as in the proof of Theorem 3.3.

IV. CONCLUSION

In this paper, we studied the Hyers-Ulam stability, Hyers-Ulam-Rassias stability, Generalized Hyers-Ulam stability and Generalized Hyers-Ulam-Rassias stability of the system of second order differential equation. That is, we obtained the sufficient criteria for the Hyers-Ulam stability, Hyers-Ulam-

Rassias stability, Generalized Hyers-Ulam stability and Generalized Hyers-Ulam-Rassias stability of the system of second order differential equation.

ACKNOWLEDGMENT

The authors are very grateful to the editors for their valuable Comments.

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