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$(1,2)^* - \alpha^* -$ Closed Maps in Bitopological Spaces

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Abstract— In this paper,we introduce the concept of $(1,2)^* - \alpha^* -$ closed maps and study its properties.Also we discuss about Strongly $(1,2)^* - \alpha^* -$ closed maps and obtain several characterizations of these maps. Futher we discuss about Almost- $(1,2)^* - \alpha^* -$ closed mappings and studied its bitopological properties.

Keywords— *$(1,2)^* - \alpha^* -$ closed $(1,2)^* - \alpha^* -$ open, $(1,2)^* - \alpha^* -$ closed map, $(1,2)^* - \alpha^* -$ open map , Strongly $(1,2)^* - \alpha^* -$ closed map ,Almost $(1,2)^* - \alpha^* -$ closed map.*

I. INTRODUCTION

Malghan [11] introduced and studied generalized closed mappings. Long[8],Gnanambal [6] and Arockiarani[1] introduced and studied the concepts of regular closed maps,gpr-closed maps and rg-closed maps in topological spaces.Lellis Thivagar and Ravi.O[10] initiated the study of the notion of a (1,2)*-g-closed map, (1,2)*-sg-closed map and(1,2)*-gs-closed map in bitopological spaces.Arockiarani and Mohana[3] introduced and studied $(1,2)^* - \pi g\alpha -$ closed maps.A.Devika and L.Elvina Mary [4]defined and studied the properties of $(1,2)^* - \alpha^* -$ closed set in bitopological spaces. In this paper,a new class of maps called $(1,2)^* - \alpha^* -$ closed maps,strongly $(1,2)^* - \alpha^* -$ closed maps and almost-$(1,2)^* - \alpha^* -$ closed maps.We also obtain some important results.

Throughout the present paper $(X, \tau_1, \tau_2), (Y, \sigma_1, \sigma_2), (Z, \eta_1, \eta_2)$ are represented by *X, Y,Z* be bitopological spaces.

II. PRELIMINARIES

Definition 2.1: [9] A subset S of a bitopological space X is said to be $\tau_{1,2}$ -open if S=A∪B where $\tau_1 \in$ A and $\tau_2 \in$ B .A subset S of X is said to be (i) $\tau_{1,2}$ - closed if the complement of S is $\tau_{1,2}$ -open. (ii) $\tau_{1,2}$ -clopen if S is both $\tau_{1,2}$ -open and $\tau_{1,2}$ -closed.

Definition 2.2 :[9] Let S be a subset of the bitopological space X. Then the $\tau_{1,2}$ - interior of S denoted by $\tau_{1,2}$ - int(S) is defined by ∪{G: G⊆S and G is $\tau_{1,2}$ - open} and $\tau_{1,2}$ - closure of S denoted by $\tau_{1,2}$ -cl(S) is defined by∩{F: S⊆F and F is $\tau_{1,2}$ - closed}.

Remark 2.3 :[9] $\tau_{1,2}$ open sets need not form a topology.

Definition 2.4 : A subset A of a bitopological space X is said to be

1. (1,2)*-regular open [9] if A= $\tau_{1,2} -$ int($\tau_{1,2} -$ cl(A)).
2. (1,2)*-α-open[9]if A $\subseteq \tau_{1,2} -$ int($\tau_{1,2} -$ cl($\tau_{1,2} -$ int(A))).
3. (1,2)*-generalized closed(briefly (1,2)*-g-closed) set[9] if $\tau_{1,2} -$ cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$ open set in X.
4. $(1,2)^* - \alpha^* -$ closed[4] if $\tau_{1,2} -$ cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $(1,2)^* - \alpha -$ open set in X.

The complement of the sets mentioned from (i) and (ii) are called their respective closed sets and the complements of the sets mentioned above (iii) is called the respective open set.

Definition 2.5: A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called

1. $(1,2)^*-$g-closed [12] if f(U) is $(1,2)^*-$ g-closed set in Y for every $\tau_{12}-$closed set U in X.
2. $(1,2)^*-$ g-open [12] if f(U) is $(1,2)^*-$ g-open set in Y for every $\tau_{12}-$open set U in X.
3. $(1,2)^*-$ πgα-closed [3] if for every $\tau_{12}-$closed set U of X, f(U) is $(1,2)^*-$πgα-closed in Y.
4. $(1,2)^*-\alpha^*-$ continuous [5] if the inverse image of every $\tau_{12}-$ closed set of Y is $(1,2)^*-\alpha^*-$closed set in X.
5. $(1,2)^*-\alpha^*-$-irresolute [5] if the inverse image of every $(1,2)^*-\alpha^*-$ closed set of Y is $(1,2)^*-\alpha^*-$closed set in X.

Definition 2.6[2] The finite union of $(1,2)^*-$ regular open sets is said to be $\tau_{12}-\pi-$ open. The complement of $\tau_{12}-\pi-$open is said to be $\tau_{12}-\pi-$closed.

Definition 2.7[7] A space (X,τ) is called a $\alpha-$ normal space , if for every pair of disjoint closed subsets H,K ,there exist disjoint $(1,2)^*-\alpha-$open sets U,V of X, H$\subseteq$ α intU and K$\subseteq$ α intV and α intU $\cap$ α intV=φ

Definition2.8[2] A space (X,τ_1,τ_2) is called a $(1,2)^*-$ quasi $\alpha-$ normal space , if for every pair of disjoint $\tau_{12}-\pi-$closed subsets H,K ,there exist disjoint $(1,2)^*-\alpha-$open sets U,V of X, H$\subseteq$ U and K$\subseteq$ V.

Definition2.9[3] A space (X,τ_1,τ_2) is called a $(1,2)^*-\alpha-$ normal space , if for every pair of disjoint closed subsets H,K ,there exist disjoint $\tau_{12}-\alpha-$open sets U,V of X, H$\subseteq$ $(1,2)^*-\alpha$ intU and K$\subseteq$ $(1,2)^*-\alpha$ intV and $(1,2)^*-\alpha$ intU $\cap$ $(1,2)^*-\alpha$ intV=φ

Definition2.10[13] A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be almost continuous if $f^{-1}(V)$ is closed in X for every regular closed in Y.

Definition 2.11:[3] A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be $(1,2)^*-\pi-$ continuous if $f^{-1}(V)$ is $\tau_{12}-\pi-$closed in X for every $\sigma_{12}-$-closed set V in Y.

Definition 2.12:[3] A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be almost $(1,2)^*-$ continuous if $f^{-1}(V)$ is $\tau_{12}-$closed in X for every $(1,2)^*-$ regular closed in Y.

III. $(1,2)^*-\alpha^*-$CLOSED MAPS

Definition 3.1: A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be $(1,2)^*-\alpha^*-$ closed if for every $\tau_{12}-$ closed F of X, f(F) is $(1,2)^*-\alpha^*-$ closed in Y.

Proposition 3.2: Every $\tau_{12}-$ closed map is $(1,2)^*-\alpha^*-$closed map.

Proof: It is straight forward since every $\tau_{12}-$closed set is $(1,2)^*-\alpha^*-$closed.

The converse of the above theorem need not be true.

Example3.3:Let
$X=\{a,b,c\}=Y$, $\tau_1=\{\varphi,X,\{b\},\{a,b\}\},\tau_2=\{\varphi,X,\{b\},\{a,c\}\}$ and $\sigma_1=\{\varphi,Y,\{b\}\},\sigma_2=\{\varphi,Y,\{a,c\}\}$. Let $f:X\to Y$ be an identity mapping.Then f is $(1,2)^*-\alpha^*-$ closed map but not $\tau_{12}-$closed map since $f(\{c\},\{b,c\})=\{c\},\{b,c\}$ is not $\sigma_{12}-$closed in Y**.**

Proposition3.4: If a mapping $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is $(1,2)^*-\alpha^*-$ closed,then for every subset A of X , $(1,2)^*-\alpha^*-cl(f(A))\subset f(\sigma_{12}-cl(A))$.

Proof: Let A⊂X.Since f is $(1,2)^*-\alpha^*-$ closed, $f(\sigma_{12}-cl(A))$ is $(1,2)^*-\alpha^*-$ closed in Y.Now $f(A)\subset f(\sigma_{12}-cl(A))$. Also, $f(A)\subset(1,2)^*-\alpha^*-cl(f(A))$ .By definition ,we have $(1,2)^*-\alpha^*-cl(f(A))\subset f(\sigma_{12}-cl(A))$.

Proposition3.5:If for every subset A of X, $\tau_{1,2}-cl(\tau_{1,2}-\mathrm{int}(\tau_{1,2}-cl(A)))\subset f(\sigma_{1,2}-cl(A))$,then a mapping $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is $(1,2)^*-\alpha^*-$closed

Proof:Let A be closed in X. Since $\tau_{1,2}-cl(\tau_{1,2}-\mathrm{int}(\tau_{1,2}-cl(A)))\subset f(\sigma_{1,2}-cl(A))\subset f(A)$.

f(A) is $(1,2)^* - \alpha -$ closed and hence $(1,2)^* - \alpha^* -$ closed.

Definition 3.6:A space (X,τ_1,τ_2) is called $(1,2)^* -_{\alpha} T_{\alpha^*} -$ space if every $(1,2)^* - \alpha -$ closed set is $(1,2)^* - \alpha^* -$ closed.

Proposition3.7:If a mapping $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(1,2)^* - \alpha^* -$ closed and Y is $(1,2)^* -_{\alpha} T_{\alpha^*} -$ space ,then $\tau_{1,2} - cl(\tau_{1,2} - \text{int}(\tau_{1,2} - cl(A))) \subset f(\sigma_{1,2} - cl(A))$.

Proof: Let $A \subset X$. Then $\tau_{1,2} - cl(A)$ is closed in X.Since f is $(1,2)^* - \alpha^* -$ closed, $f(\sigma_{1,2} - cl(A))$ is $(1,2)^* - \alpha^* -$ closed in Y and so $(1,2)^* - cl(f(\tau_{1,2} - cl(A))) \subset f(\sigma_{1,2} - cl(A))$.

Hence $\tau_{1,2} - cl(\tau_{1,2} - \text{int}(\tau_{1,2} - cl(A)))$
$\subset \tau_{1,2} - cl(\tau_{1,2} - \text{int}(\tau_{1,2} - cl(f(\tau_{1,2} - cl(A)))))$
$\subset f(\sigma_{1,2} - cl(A))$.

Theorem 3.8. A surjection $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(1,2)^* - \alpha^* -$ closed iff for each subset S of Y and each $\tau_{12} -$ open set U containing $f^{-1}(S)$,there exists $(1,2)^* - \alpha^* -$ open set V of Y such that S ⊂ V and $f^{-1}(V) \subset U$.

Proof. Necessity: Suppose that f is $(1,2)^* - \alpha^* -$ closed. Let S be a subset of Y and U be an $\tau_{12} -$ regular open subsets of X containing $f^{-1}(S)$.If V = Y − f(X − U),then V is a $(1,2)^* - \alpha^* -$ open set of Y,such that S ⊂ V and $f^{-1}(V) \subset U$.

Sufficiency : Let F be any regular closed set of X.Then $f^{-1}(Y - f(F)) \subset X - F$ and X − F is $\tau_{12} -$ open in X. There exists $(1,2)^* - \alpha^* -$ open set of V of Y such that Y −f(F) ⊂ V and $f^{-1}(V) \subset X - F$. Therefore , $V \subset f(F) \subset f(X - f^{-1}(V)) \subset Y - V$. Hence we obtain f(F) = Y −V and f(F) is $(1,2)^* - \alpha^* -$ closed in Y which shows that f is $(1,2)^* - \alpha^* -$ closed.

Theorem 3.9: If $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(1,2)^* - \alpha^* -$ closed and A is $\tau_{12} -$ closed subset of X, then $f \mid A:(A) \to (Y)$ is $(1,2)^* - \alpha^* -$ closed.

Proof: Let B ⊂ A be $\tau_{12} -$ closed in A. Then B is $\tau_{12} -$ closed in X.Since f is $(1,2)^* - \alpha^* -$ closed, f(B) is $(1,2)^* - \alpha^* -$ closed in Y.But f(B) = (f |A)(B) .So f|A is $(1,2)^* - \alpha^* -$ closed.

Proposition 3.10: If $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $\tau_{12} -$ closed and $g:(Y,\sigma_1,\sigma_2) \to (Z,\eta_1,\eta_2)$ be the $(1,2)^* - \alpha^* -$ closed, then $g \circ f:(X,\tau_1,\tau_2) \to (Z,\eta_1,\eta_2)$ is $(1,2)^* - \alpha^* -$ closed.

Proof: Let A be $\tau_{12} -$ closed in X,then f(A) is $\sigma_{12} -$ closed in Y.Since g is $(1,2)^* - \alpha^* -$ closed,g(f(A)) is $(1,2)^* - \alpha^* -$ closed in Z.Hence g ∘ f is $(1,2)^* - \alpha^* -$ closed.

Theorem 3.11. Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2) \to (Z,\eta_1,\eta_2)$ be two mappings and let $g \circ f:(X,\tau_1,\tau_2) \to (Z,\eta_1,\eta_2)$ be $(1,2)^* - \alpha^* -$ closed.Then

1. If f is $(1,2)^* -$ continuous and surjection,then g is $(1,2)^* - \alpha^* -$ closed.
2. If g is $(1,2)^* - \alpha^* -$ irresolute and injective,then f is $(1,2)^* - \alpha^* -$ closed .

Proof. 1).Let A be $\sigma_{12} -$ closed in Y .Since f is $(1,2)^* -$ continuous, $f^{-1}(A)$ is $\tau_{12} -$ closed in X.Since g ∘ f is $(1,2)^* - \alpha^* -$ closed, $g \circ f(f^{-1}(A)) = g(f(f^{-1}(A))) = g(A)$ is $(1,2)^* - \alpha^* -$ closed.

2)Let A be $\tau_{12} -$ closed in X .Since g ∘ f is $(1,2)^* - \alpha^* -$ closed, $(g \circ f)(A)$ is $(1,2)^* - \alpha^* -$ closed in Z. Since g is $(1,2)^* - \alpha^* -$ continuous , $g^{-1}((g \circ f)(A)) = f(A)$ is $(1,2)^* - \alpha^* -$ closed in Y.Hence f is $(1,2)^* - \alpha^* -$ closed.

Proposition 3.12:For any bijection $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ the following statements are equivalent.

1. f is a $(1,2)^* - \alpha^* -$ open map.
2. f is a $(1,2)^* - \alpha^* -$ closed map.

3. $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is $(1,2)^*-\alpha^*-$ continuous.

Proof: $(i)\Rightarrow(ii)$ Let f be a $(1,2)^*-\alpha^*-$ open map. Let U be $\tau_{1,2}$ -closed in X. Then X-U is $\tau_{1,2}$ -open in X. By assumption, f(X-U) is a $(1,2)^*-\alpha^*-$ open map and it implies Y-f(U) is $(1,2)^*-\alpha^*-$ open map and hence f(U) is $(1,2)^*-\alpha^*-$ closed.

$(ii)\Rightarrow(iii)$ Let V be $\tau_{1,2}$ -closed in X. By (ii), $f(V)=(f^{-1})^{-1}(V)$ is $(1,2)^*-\alpha^*-$ closed in Y.

$(iii)\Rightarrow(i)$ Let V be $\tau_{1,2}$ -open in X. By (iii), $(f^{-1})^{-1}(V)=f(V)$ is $(1,2)^*-\alpha^*-$ closed in Y.

Definition 3.13. A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be strongly $(1,2)^*-\alpha^*-$ closed if the image f(A) is $(1,2)^*-\alpha^*-$ closed in Y for every $(1,2)^*-\alpha^*-$ closed A in X.

Theorem 3.14. Every strongly $(1,2)^*-\alpha^*-$ closed map is $(1,2)^*-\alpha^*-$ closed map.

Proof:The proof follows from the definition.

Theorem 3.15 Every strongly $(1,2)^*-\alpha^*-$ closed map is $(1,2)^*-\pi g\alpha-$ closed map.

Proof: The proof is straight forward.

The converse of the theorem need not be true as seen in the following example.

Example 3.16. Let X = {a, b, c, d} = Y, τ_1 = { φ ,X, {d}, {c, d}}, τ_2 = { φ ,X, {a, b, d}}
and σ_1 = { φ, Y, {a}, {d}, {a, d}, {c, d}, {a, c, d}}, σ_2 = { φ, Y, {a, c}}. Let f : X → Y be an identity mapping. Then f is $(1,2)^*-\pi g\alpha-$ closed map but not strongly $(1,2)^*-\alpha^*-$ closed map.

Theorem 3.17. If $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is $(1,2)^*-\pi-$ continuous and strongly $(1,2)^*-\alpha^*-$ closed in X,then $f(A)$ is $(1,2)^*-\alpha^*-$ closed in Y for every $(1,2)^*-\alpha^*-$ closed set A of X.

Proof: Let A be any $(1,2)^*-\alpha^*-$ closed set of X and V be any $\sigma_{12}-\pi-$ open set of Y containing f(A).Since f is $(1,2)^*-\pi-$ continuous, $f^{-1}(V)$ is $\tau_{12}-\pi-$ open in X and $A\subset f^{-1}(V)$. Therefore $(1,2)^*-\alpha cl(A)\subset f^{-1}(V)$,and hence $f((1,2)^*-\alpha cl(A))\subset V$ .Since f is strongly $(1,2)^*-\alpha^*-$ closed, $f((1,2)^*-\alpha cl(A))$ is $(1,2)^*-\alpha^*-$ closed in Y and hence we obtain $(1,2)^*-\alpha cl(f(A))\subset$ $(1,2)^*-\alpha cl(f((1,2)^*-\alpha cl(A)))\subset V$ .Hence $f(A)$ is $(1,2)^*-\alpha^*-$ closed in Y.

Proposition 3.18:For any bijection $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ the following statements are equivalent.

1. $f^{-1}:(Y,\sigma_1,\sigma_2)\to(X,\tau_1,\tau_2)$ is $(1,2)^*-\alpha^*-$ irresolute.
2. f is a strongly $(1,2)^*-\alpha^*-$ open map.
3. f is a strongly $(1,2)^*-\alpha^*-$ closed map.

Proof: $(i)\Rightarrow(ii)$ Let U be a $(1,2)^*-\alpha^*-$ open in X.By (i), $(f^{-1})^{-1}(U)=f(U)$ is $(1,2)^*-\alpha^*-$ open in Y.Hence (ii) holds.

$(ii)\Rightarrow(iii)$ Let V be $(1,2)^*-\alpha^*-$ closed in X. By (ii), $f(X-V)=Y-f(V)$ is $(1,2)^*-\alpha^*-$ open in Y. That is, $f(V)$ is $(1,2)^*-\alpha^*-$ closed in Y and so f is a strongly $(1,2)^*-\alpha^*-$ closed map.

$(iii)\Rightarrow(i)$ Let V be $(1,2)^*-\alpha^*-$ closed in X.By (iii), $f(V)=(f^{-1})^{-1}(V)$ is $(1,2)^*-\alpha^*-$ closed in Y. Hence (i) holds.

IV. ALMOST $(1,2)^*-\alpha^*-$ CLOSED MAPS

Definition 4.1. A map $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be almost $(1,2)^*-\alpha^*-$ closed if for every $(1,2)^*-$ closed F of X, f(F) is $(1,2)^*-\alpha^*-$ closed in Y.

Theorem 4.2.
Every almost $\tau_{1,2}$ -closed map is almost $(1,2)^*-\alpha^*-$ closed.

Proof: The proof is straight forward.

Theorem 4.3: Every strongly $(1,2)^*-\alpha^*-$ closed map is almost $(1,2)^*-\alpha^*-$ closed.

Proof: The proof is straight forward.

The converse of the above theorem need not be true as shown in the following example.

Example 4.4. Let X = {a, b, c, d} = Y,
τ_1 = { φ ,X, {d}, {c,d}}, τ_2 = { φ ,X, {a, b, d}}.
σ_1 = { φ , Y, {a}, {d}, {a, d}, {c, d}, {a, c, d}},
σ_2 = { φ , Y, {a, c}}.Let f be a identity mapping. It is almost $(1,2)^* - \alpha^* -$ closed map but not strongly $(1,2)^* - \alpha^* -$ closed map.

Theorem 4.5. A surjection $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is almost $(1,2)^* - \alpha^* -$ closed iff for each subset S of Y and U ∈ $(1,2)^* -$ regular open of X containing $f^{-1}(S)$, there exists a $(1,2)^* - \alpha^* -$ open set V of Y ,such that S ⊂ V and $f^{-1}(S) \subset U$.

Proof:Necessity: Suppose that f is almost-$(1,2)^* - \alpha^* -$ closed.Let S be a subset of Y and U ∈ $(1,2)^* -$ regular open of X containing $f^{-1}(S)$. If V = Y − f(X − U),then V is a $(1,2)^* - \alpha^* -$ open set of Y ,such that S ⊂ V and $f^{-1}(V) \subset U$.
Sufficiency: Let F be any $(1,2)^* -$ regular closed set of X.Then $f^{-1}(Y - f(F)) \subset X - F$ and X − F is $(1,2)^* -$ regular open in X.There exists $(1,2)^* - \alpha^* -$ open set V of Y such that Y − f(F) ⊂ V and $f^{-1}(V) \subset X - F$. Therefore, $Y - V \subset f(F) \subset f(X - f^{-1}(V)) \subset Y - V$ .Hence we obtain f(F) = Y −U and f(F) is $(1,2)^* - \alpha^* -$ closed in Y which shows that f is almost $(1,2)^* - \alpha^* -$ closed.

Definition 4.6. A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost-$(1,2)^* - \alpha^* -$ continuous if $f^{-1}(V)$ is $(1,2)^* - \alpha^* -$ closed in X for every $(1,2)^* -$ regular closed in Y .

Preservation Theorem
Theorem 4.7. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an almost- $(1,2)^* - \alpha^* -$ continuous , $\tau_{12} - \pi -$ closed injection and Y is $(1,2)^* -$ quasi−α−normal ,then X is $(1,2)^* -$ quasi−α−normal.

Proof. Let A and B be any disjoint $\tau_{12} - \pi -$ closed sets of X.Since f is a $\tau_{12} - \pi -$ closed injection f(A) and f(B) are disjoint $\sigma_{12} - \pi -$ closed sets of Y .Since Y is $(1,2)^* -$ quasi −α-normal, there exist disjoint $(1,2)^* - \alpha -$ open sets U and V of Y such that f(A) ⊂ U and f(B) ⊂ V .Now if $G = \tau_{12} - \text{int}(\tau_{12} - cl(U))$ and $H = \sigma_{12} - \text{int}(\sigma_{12} - cl(V))$ .Then G and H are disjoint $(1,2)^* -$ regular open sets such that f(A) ⊂ G and f(B) ⊂ H.Since f is almost $(1,2)^* - \alpha^* -$ continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $(1,2)^* - \alpha^* -$ open sets containing A and B which shows that X is $(1,2)^* -$ quasi − α−normal.

Theorem 4.8. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an $(1,2)^* - \pi -$ continuous ,almost $(1,2)^* - \alpha -$ closed surjection and X is $(1,2)^* -$ quasi − α−normal ,then Y is $(1,2)^* -$ quasi −α−normal.

Proof: Let A and B be any disjoint $\sigma_{12} -$ closed sets of Y .Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\tau_{12} - \pi -$ closed sets of X.Since X is $(1,2)^* -$ quasi−α−normal, there exist disjoint $(1,2)^* - \alpha -$ open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$ .Now if $G = \tau_{12} - \text{int}(\tau_{12} - cl(U))$ and $H = \sigma_{12} - \text{int}(\sigma_{12} - cl(V))$ .Then G and H are disjoint $(1,2)^* -$ regular open sets such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$ .Set K = Y − f(X − G),L = Y − f(X − H).Then K and L are $(1,2)^* -$ α−open sets of Y ,such that A ⊂ K,B ⊂ L, $f^{-1}(K) \subset G$, $f^{-1}(L) \subset H$ .Since G and H are disjoint,K and L are disjoint .Since K and L are $(1,2)^* -$ α-open and we obtain A ⊂ $(1,2)^* -$ α - int K,B ⊂ $(1,2)^* -$ α - int L and $(1,2)^* -$ α−int K∩ $(1,2)^* -$ α−intL =ϕ.Therefore,Y is $(1,2)^* -$ quasi − α− normal.

Corollary 4.9. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is almost $(1,2)^* -$ continuous ,almost $\tau_{12} -$ closed surjection and X is $(1,2)^* -$ normal space,then Y is $(1,2)^* -$ quasi −α−normal.
Proof. Since every almost $(1,2)^* -$ closed map is almost $(1,2)^* - \alpha^* -$ closed ,then by theorem 3.8,Y is $(1,2)^* -$ quasi − α−normal.

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