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Application of Kernel Density Estimation in Chain Ladder Method for Claim Reserving

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***Abstract*—**The process of estimating the accurate reserves for incurred but not reported (IBNR) claims is the important task performed by the actuaries in non-life insurance business. Chain ladder (CL) method is probably the most commonly used method in loss reserving. But this method is not derived from any fundamental theory about the way the claims occur. So that theoretical justification of this method is rather difficult. To overcome this, we introduced a modified method using a well defined non parametric model called Kernel Density Estimation (KDE) for estimating the IBNR claims reserves with the help of percentiles. In this paper, we developed a procedure for the modification of the CL predictors (outstanding claims estimates) for the future cumulative claims (lower triangle values) with the use of KDE and showed that our procedure provides better results compare to the existing CL method for finding the IBNR claims reserves in terms of the Mack's standard error.

Keywords— Chain-ladder method,Kernel density estimation; Mack's standard error

I. INTRODUCTION

The process of estimating the accurate reserves for future is the important task faced by the actuaries. The insurance company possibly will come to a decision to set up the reserves to allot funds to the expected losses, but the ultimate amount is not known while the reserves have to be set. These types of reserves are said to be the incurred but not reported (IBNR) claims reserves. Depends upon the complexity of the damage, the claims we focus on require months or years to happen. The time-consuming legal processes or the complications of the claim size determination leads to the delay in payments. As a result, insurers need to stock up reserves enabling them to pay off the outstanding claims and to meet expected claims on the written agreements. In the claims reserving process in general insurance, chain ladder (CL) method is probably the most commonly used method, which is frame on historical loss development. Normally in CL triangle the observations exist merely in the upper portion of the development triangle and the lower portion of development triangle require to be estimated or predicted. The disadvantage of using a CL method is that it is a smart algorithm which considered the calculation of numbers instead of a distinct mathematical or statistical model based on a sound mathematical statistics, where the procedure is the computation of estimating the parameters of the model. The connections between CL method and statistical models have been made clear by the later evolutions in actuarial science. There have been some articles, which show that how the CL estimates can be associated with the maximum likelihood estimation (MLE). For instance, Mack [5] proved that CL estimators are the classical MLE estimators of a multiplicative Poisson model. Similarly Renshaw and Verral [10] showed that CL estimators are the classical MLE estimators of the over-dispersed Poisson model. This relationship was a progress in the path of validating the CL method such that the approaching with the help of statistical models could be taken in to account without losing the original perception and straightforwardness of CL method. Further reviews of chain ladder methods have been provided in [2, 3, 18, 20]. However, it is notable that the motivation behind these papers was to develop a statistical model that contributes the similar reserve estimates as the CL method. After some time Verdonck et al. [16] used a robust chain-ladder method, which is utilized to some run-off triangles having outliers and without outliers, depicting its tremendous functioning. But all these methods require the help of a classical or robust statistical estimation procedure for estimating the IBNR claim reserves.

The aim of this paper is not to commence from fundamental risk theory and develop a new model for the run-off triangle. The goal is to modify the existing chain ladder model in year wise (row wise) by applying a method based on the non-parametric statistics known as kernel density estimation (KDE) and replace the chain ladder predictors (estimates) using the estimators produced by this method. Modification of CL method using KDE provide the qualities of a statistical model to the reserve estimates, since it is a fundamental data smoothing techniques to obtain the distributional form using finite sample observations. Further in non-life insurance, it is advisable to use the KDE for modelling the claim size distribution due to the lack of symmetry of the data. But the replacement of chain ladder estimates with KDE estimates is quite difficult due to the nature of outcomes produced by both methods. CL method contains discrete outcomes and KDE method produces continuous outcomes. Thus we calculate the percentile values in origin year wise from the CL table and identify the percentiles corresponding to the lower triangle values. Subsequently carry out the KDE for the given data using R software and locate the above computed percentiles in KDE to complete the lower triangle matrix. Then calculated the IBNR claims reserves. Because the new estimation method replaces the CL estimates with KDE estimates and modifies the CL method, we call this new method the 'modified chain ladder method' (modified CL). In general, the suggested method is very flexible and can be simply utilized to many situations. The results are further improved by implementing different bandwidth selection techniques for smoothing the data.

Silverman [14] discussed various significant applications of KDEs. A consistent observational bandwidth selection technique for KDE was suggested by Sheather and Jones [13]. Jones et al. [4] proved that the most faithful bandwidth selection method of KDE is the solve-the-equation plug-in bandwidth selector considering the ample functioning. In this paper, we used the direct plug in bandwidth selection method for estimating the KDE values. The bandwidth of the kernel is treated as a free parameter which reveals a substantial power on the ensuing approximation. Zambom and Dias [21] provided a complete appraisal which gives a summary of the most significant theoretical features of kernel density estimation and a widespread depiction of modern and classical data diagnostic techniques to calculate the smoothing parameter. Recently, Sakthivel and Rajitha [11] used kernel density estimation for obtaining density for univariate claim severity distributions with goodness fits analysis.

For measuring the variability of the outcome standard error calculation of the chain ladder reserve estimates is very helpful, for that we considered the standard error formula proposed by Mack [6]. Mack [7] offered a recursive approach of analyzing the standard error of CL reserve estimates. Other approaches are available in the literature for estimating the standard errors of reserve estimates. [15, 17, 1] used least square regression approach for the calculation of standard errors of reserve estimates. Recently for obtaining the standard deviation of the chain ladder resulting estimates, Peremans et al. [8] considered and applied numerous robust bootstrap methods in the claim reserving model and inspected and evaluated their functioning on both simulated and real data.

The goal of this paper is to compare our modified chain ladder (modified CL) method for estimating IBNR claims reserves with the CL method by computing the standard error from already existing formula in Mack [6]. Here we calculated the analytical Mack's standard error of the two models. And then compare the two models with the data taken from the Reinsurance Association of America (RAA) by comparing the standard errors.

This paper is organized as follows: section I contains the introduction of the study. Section 2 contains an overview of the CL model and the Mack's distribution free formula for the standard error (S.E) calculation of the CL reserve estimates. Section 3 discusses about the modification of chain ladder predictors of the future cumulative claims using KDE method. Section 4 contains the details of data analysis used for this study. Section 5 contains the calculation and comparison of both methods in terms of Mack's distribution free standard error. Section 6 contains conclusions of the study.

II. CHAIN LADDER METHOD

In Chain ladder model is one of the most admired claims reserving technique specifically for the estimation of the IBNR claims. It is considered as a complete computational algorithm for estimating the claims reserves by the traditional actuarial literature. In this method it is observed that claims developing from various origin years have formulated over succeeding development years and then make use of the significant ratios such as development factors or grossing-up factors to predict how succeeding claims from these years will progress. The origin year or accident year denotes the year in which the accident occurred and the development year denote the postponement in reporting of claims with reference to the origin year. Normally in CL triangle the observations exist merely in the upper portion of the development triangle and the lower portion of development triangle require to be estimated or predicted. A triangular representation of the data called delay triangle or run-off triangle is commonly used for representing the data as given in table 1. The CL method usually uses the cumulative data and assuming the following set of cumulative claims

$$C_{ik} \,/\, i = 1,2,..n; k = 1,2,\ldots,n - i + 1$$

The suffixes $i \,\&\, k$ refers to the accident year or origin year (row) and the development year (column). The values of C_{ik} for $i + k \le n + 1$ are known and $i + k > n + 1$ are need to be estimated or predicted based on the past data. The already made payments in the calendar year(c) are represented in the diagonal line $(i + k - 1 = c)$ of the CL table.

Table 1.Run-off triangle of Cumulative Claims

Development year / **Origin year or accident year**	1	2	...	$n-i+1$	...	$n-1$	n
1	C_{11}	C_{12}	...	$C_{1,n-i+1}$	...	$C_{1,n-1}$	C_{1n}
2	C_{21}	C_{22}	...	$C_{2,n-i+1}$	...	$C_{2,n-1}$	
$\vdots$	...	...	...	...	...		
i	C_{i1}	C_{i2}	...	$C_{i,n-i+1}$			
$\vdots$	...	...	...				
n	C_{n1}						

The aim of claim reserving is to estimate the ultimate claims amount and thereby the IBNR claims reserve. The IBNR claims reserve of accident year i is defined as

$$R_i = C_{in} - C_{i,n+1-i}, \quad 1 \le i \le n$$

Where C_{in} denote the ultimate claims amount for each accident year $i = 1, 2, ..., n$ and $C_{i,n+1-i}$ denotes the already paid claims. The basic assumptions of CL method are

- The accident years are independent (a)
- $E(C_{i,k+1} / C_{i1,....}C_{ik}) = C_{ik} f_{k+1}, 1 \le i \le n, 1 \le k \le n-1$ (b)

where $f_2, f_3, ...f_n$ denotes the development factors and these factors are estimated by CL method as

$$\hat{f}_k = \sum_{i=1}^{n-k+1} C_{ik} \Big/ \sum_{i=1}^{n-j+1} C_{i,k-1}, \quad 2 \le k \le n$$

These development factors are applied to the latest cumulative claims corresponding to each accident year (each row) in the CL table to get the future estimates of cumulative claim amounts

$$\hat{C}_{i,n-i+2} = \hat{C}_{i,n-i+1} \hat{f}_{n-i+2}, \qquad 2 \le i \le n,$$

In general

$$\hat{C}_{i,k} = \hat{C}_{i,k-1} \hat{f}_k, \qquad 2 \le i \le n,\ n-i+3 \le k \le n$$

Also it is important to estimate the variability or standard error of the reserve estimate. For that several studies have done in the actuarial literature. Among these, the Mack's distribution free standard error calculation of the CL reserve estimate is one of the most important contributions in this field [6]. In this article we considered Mack's standard error for measuring the variability of the CL reserve estimate. Mack [6] developed a specialized model for the CL case (Mack CL model) and formulated an approximation to the S.E of the reserve estimate by adapting Schnieper's [12] idea. So in Mack CL model the S.E of the reserve estimate contains an additional estimate of the process variance and proved that development factors are uncorrelated and therefore the reserve estimate is unbiased. Moreover this model provided a formula for the S.E of the overall reserve estimator in addition to the S.E for each accident year. For calculating the S. E, at first Mack assumed the variance of C_{ik} as

$$Var(C_{i,k+1} / C_{i1,....}C_{ik}) = C_{ik}\sigma_k^2, 1 \le i \le n, 1 \le k \le n-1 \tag{3}$$

with unknown parameters σ_k^2, $1 \le k \le n-1$. And for $\hat{f}_k$ the unbiased estimator of σ_k^2 is derived as

$$\hat{\sigma}_k^2 = \frac{1}{n-k-1} \sum_{i=1}^{n-k} C_{ik} \left(\frac{C_{i,k+1}}{C_{ik}} - \hat{f}_k\right)^2, \quad 1 \le k \le n-2,$$

Then by the equations (1), (2) and (3) Mack derived the S.E as

$$(s.e(\hat{C}_{in}))^2 = \hat{C}_{in}^2 \sum_{k=n+1-i}^{n-1} \frac{\hat{\sigma}_k^2}{\hat{f}_k^2} \left(\frac{1}{\hat{C}_{ik}} + \frac{1}{\sum_{h=1}^{n-k} C_{hk}} \right) \tag{4}$$

where

$\hat{C}_{i,k} = \hat{C}_{i,k-1}\hat{f}_k, \qquad 2 \le i \le n,\ n-i+3 \le k \le n$ are the estimated values of the future $C_{i,k}$. $(s.e(\hat{C}_{in}))^2$ is at the same time is the standard error of the estimate $\hat{R}_i = \hat{C}_{in} - C_{i,n+1-i}$ for the claims reserve $R_i = C_{in} - C_{i,n+1-i}$. Mack [6] also derived an equation for the S.E of the overall reserve estimate

$$(s.e(\hat{R}) = \sum_{i=2}^{n} \left\{ \begin{array}{l} (s.e(\hat{C}_{in}))^2 + \hat{C}_{in}\left(\sum_{k=i+1}^{n} \hat{C}_{kn} \right) \\ \times \sum_{k=n+1-i}^{n-1} 2 \frac{\hat{\sigma}_k^2}{\hat{f}_k^2} \left(\frac{1}{\sum_{h=1}^{n-k} C_{hk}} \right) \end{array} \right\} \tag{5}$$

In this paper we used equation (4) for computing the origin year wise S.E of IBNR claims reserves and equation (5) is used for computing the overall S.E of IBNR claims reserves for CL method and modified CL method.

III. KERNEL DENSITY ESTIMATION (KDE)

In the proposed method (modified CL method), KDE is used for computing the outstanding claims reserves or future cumulative claims reserves.

3.1. Definition of KDE

Let $x_1,\dots,x_n$ *be* n independent and identically distributed random variables, and then the density estimator is given by

$$\hat{f}(x) = (n)^{-1} \sum_{i=1}^{n} K\{(x - X_i)/h\}$$

where k is the kernel and h is the bandwidth [19]. The bandwidth h is used for smoothing the estimated density curve. In this paper, we used the Normal (Gaussian) kernel and it is symmetric about its mean and it is given by

$$K(t) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right)$$

The choice of the bandwidth h has great significance for the sensible implementation of the KDE. The unique and best method to choose bandwidth parameters is still an ongoing research. In this paper we used the plug in selectors proposed by Sheather and Jones [13] is used for the bandwidth selection.

3.2. Modified chain ladder method using Kernel Density Estimation

In this method, the CL estimates of the future cumulative claims are modified with KDE estimates and estimated the IBNR claim reserves. The following procedure is employed for obtaining the modified claim reserve estimates.

Step1: Compute full run off triangle matrix using CL method.

Step2: Calculating the percentile values (origin) year wise from the chain ladder table.

Step3: Identify the percentiles of the lower triangle from CL table.

Step4: Perform KDE for the given data and locate the above computed percentiles in KDE to complete the full triangle matrix.

Step5: Estimating the IBNR claim reserves by subtracting the latest cumulative claim amounts from ultimate claim amounts obtained from the KDE estimates.

Statistically for a given run-off triangle the cumulative loss amount arising from accident year $i = 1,2,\dots,n$ be C_{ik} paid at the end of the development year (age) $k = 1,\dots,n$. The loss amounts C_{ik} have been observed for $k \le n+1-i$ where as the other amounts; especially the ultimate amounts $C_{in}, i > 1$ have to be predicted. Let P_{ik} denote the percentile values corresponding to the cumulative loss amounts C_{ik} of accident year $i = 1,2,\dots,n$, and $k > n+1-i$. A_{ik} denotes the loss amounts corresponding to the percentile values P_{ik} which have been observed for $k > n+1-i$ from the chain ladder full triangle. And $K_{ik}, k > n+1-i$ denote the KDE values corresponding to $C_{ik}, k > n+1-i$ values in the CL table where $i = 1,2,\dots,n$. Let $Q_{ik}, k > n+1-i$ denote the cumulative loss amounts from KDE values K_{ik} corresponding to the percentiles P_{ik}. That is here we replace the cumulative loss amounts C_{ik}, $i = 1,2,\dots,n$, and $k > n+1-i$ with the corresponding KDE values $Q_{ik}, k > n+1-i$. The IBNR claims reserves can be computed by subtracting the latest cumulative claim amounts from the ultimate cumulative claim amounts, since the ultimate claims amounts is the combination of paid claim amounts, outstanding reported claim amounts and the IBNR claims amounts.

IV. DATA ANALYSIS

It should include important findings discussed briefly. Wh In this paper we used the data from the Historical loss development study, Reinsurance Association of America [9], Automatic Facultative General Liability data, is a matrix with 10 accident years and 10 development years. It presents claims developments for the origin years from 1981 to 1990 in a cumulative form where cumulative loss was given in $1,000. Table 2 shows the run-off triangle of RAA data in the cumulative form. It is also accessible in the R software. In CL method we observe that in what way claims coming from different origin years have risen over subsequent development years, and then use significant ratios to predict how future claims from these years will progress. These ratios are called development factors. For instance the development factor for the development year 1-2 can be calculated by d_1/d_2,where

$$d_1 = 5012 + 106 + 3410 + 5655 + 1092 + 1513 + 557 + 1351 + 3133$$

$$d_2 = 8269 + 4285 + 8992 + 11555 + 9565 + 6445 + 4020 + 6947 + 5395$$

V. EXPERIMENTAL RESULTS

In this study it is observed that IBNR claims reserves are computed using CL method and modify the CL full run-off triangle matrix using KDE and compute the IBNR claims reserves using the modified method. Before computing the IBNR claims reserves, it needs to calculate outstanding claims estimates (lower triangle values) using the above mentioned methods. Then compare the efficiency of these methods in terms of Mack's distribution free formula for standard error. Table 3 presents the development factors of the run-off triangle of the RAA data using CL method. The estimates of the expected future claims or outstanding claims estimates and IBNR reserves for each origin year obtained using CL method is given in table 4.

Table 5 shows outstanding claims estimates (with IBNR reserves for each rows) using modified CL method. For example in table 5 percentile P_{ij} is calculated from the chain ladder triangle for the accident year 1982 and development age 10 is 96% and the corresponding KDE value for the cumulative loss amount Q_{ij} is 16769 instead of 16858 (lower triangle value or outstanding claim reserve) in CL method. The diagrammatic representation of the origin year wise estimation of outstanding claims reserves using KDE is given in figure 1. It is observed that IBNR claims reserves can be obtained by deducting the latest cumulative claim amounts from the ultimate cumulative claim amounts. As we know the standard error calculation is very helpful to know the variability of the result, the smaller the variability the more accurate the result is said to be. Table 6 shows the origin year wise standard error of CL method and the modified CL method for calculating IBNR claims reserves using equation (4). Table 7 shows the overall S.E of the CL reserve estimates obtained by equation (5).

VI. CONCLUSION

In this paper we compared the standard error of the IBNR claims for CL method and the modified CL method for computing the chain ladder predictors or outstanding claims estimates using KDE. It is observed that standard error for the modified method using KDE is smaller compared to that of CL method for computing the IBNR reserves. From this result we can conclude that our modified method using KDE works and suits well when compared to CL method for estimating the IBNR reserves.

TABLES AND FIGURES

Table 2. Cumulative claims loss of RAA data as run-off triangle

Devt / Origin	1	2	3	4	5	6	7	8	9	10
1981	5012	8249	10907	11805	13539	16181	18009	18608	18662	18834
1982	106	4285	5396	10666	13782	15599	15496	16169	16704	...
1983	3410	8992	13873	16141	18735	22214	22863	23466	...	...
1984	5655	11555	15766	21266	23425	26083	27067	...	...	...
1985	1092	9565	15836	22169	25955	26180	...	...	...	...
1986	1513	6445	11702	12935	15852	...	...	...	...	...
1987	557	4020	10946	12314	...	...	...	...	...	...
1988	1351	6947	13112	...	...	...	...	...	...	...
1989	3133	5395	...	...	...	...	...	...	...	...
1990	2063	...	...	...	...	...	...	...	...	...

Table 3. Development factors of the RAA Run-off triangle using CL method

2.999	1.624	1.271	1.172	1.113	1.042	1.033	1.017	1.009	1.000

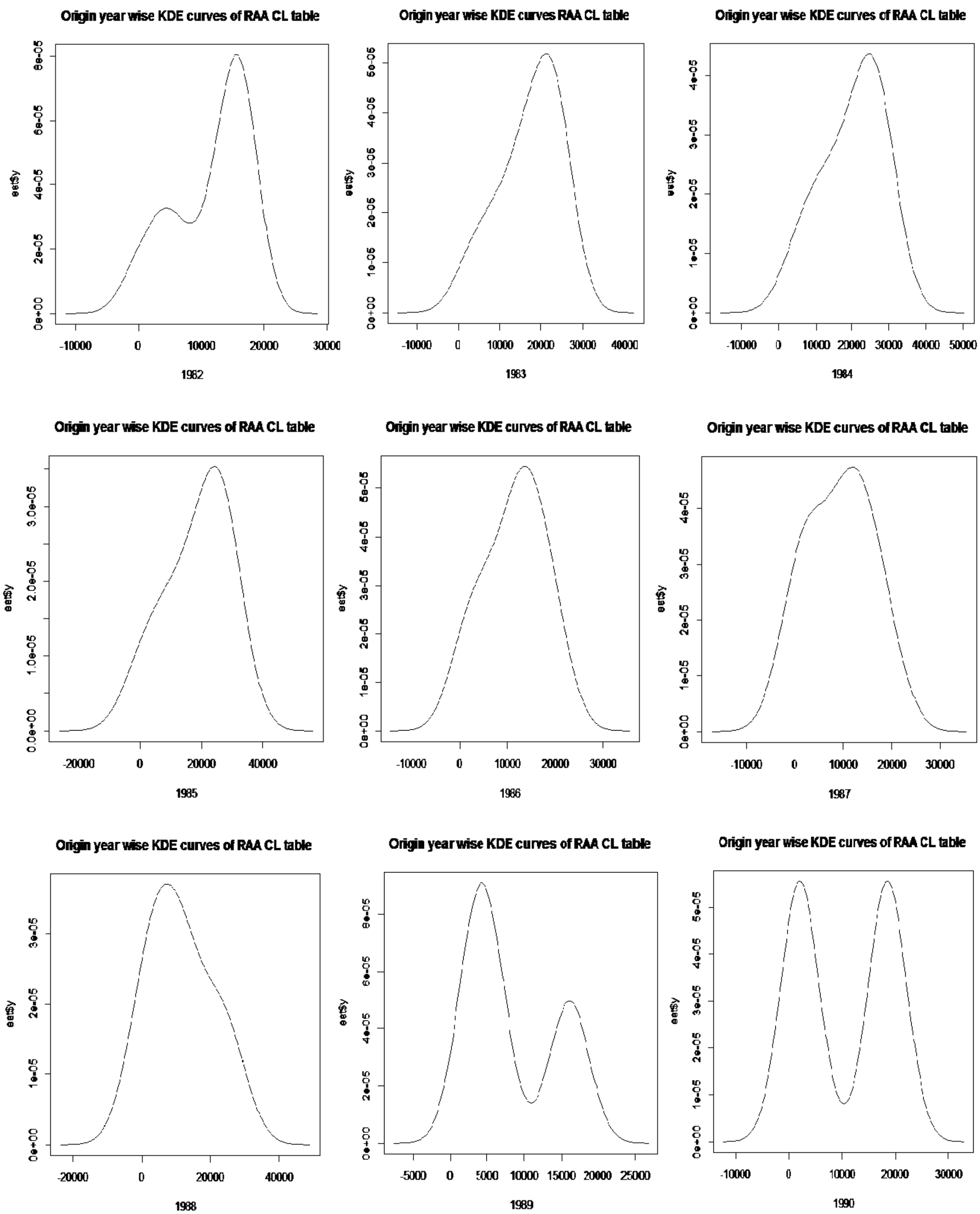

Figure 1. Origin year wise KDE curves for the RAA CL data

Table 4. Outstanding claims estimates and IBNR reserves obtained using CL method.

Origin year	**Development year**										**IBNR**
	1	2	3	4	5	6	7	8	9	10	
1981	5012	8249	10907	11805	13539	16181	18009	18608	18662	18834	0
1982	106	4285	5396	10666	13782	15599	15496	16169	16704	16858	154
1983	3410	8992	13873	16141	18735	22214	22863	23466	23863	24083	617
1984	5655	11555	15766	21266	23425	26083	27067	27967	28441	28703	1636
1985	1092	9565	15836	22169	25955	26180	27278	28185	28663	28927	2747
1986	1513	6445	11702	12935	15852	17649	18389	19001	19323	19501	3649
1987	557	4020	10946	12314	14428	16064	16738	17294	17587	17749	5435
1988	1351	6947	13112	16644	19525	21738	22650	23403	23800	24019	10907
1989	3133	5395	8759	11132	13043	14521	15130	15634	15898	16045	10650
1990	2063	6188	10046	12767	14959	16655	17353	17931	18234	18402	16339

Table 5. Outstanding claims (lower triangle) estimation and IBNR reserves using modified CL method

Origin Year	Development year										IBNR
	1	2	3	4	5	6	7	8	9	10	
1981	5012	8269	10907	11805	13539	16181	18009	18608	18662	18834	0
1982	106	4285	5396	10666	13782	15599	15496	16169	16704	*96%* 16769	*65*
1983	3410	8992	13873	16141	18735	22214	22863	23466	*94%* 22956	*98%* 23788	*322*
1984	5655	11555	15766	21266	23425	26083	27067	*93%* 27223	*97%* 28151	*99%* 28615	*1548*
1985	1092	9565	15836	22169	25955	26180	*90%* 27274	*95%* 27673	*98%* 28512	*99%* 28793	*2613*
1986	1513	6445	11702	12935	15852	*89%* 17610	*93%* 18333	*97%* 19056	*98%* 19237	*100%* 19599	*3747*
1987	557	4020	10946	12314	*85%* 15245	*92%* 16455	*95%* 16974	*98%* 17492	*99%* 17665	*100%* 17838	*5524*
1988	1351	6947	13112	*77%* 18897	*86%* 20949	*93%* 22544	*96%* 23227	*98%* 23683	*99%* 23911	*100%* 24139	*11067*
1989	3133	5395	*66%* 11708	*77%* 13137	*86%* 14306	*93%* 15216	*95%* 15475	*98%* 15865	*99%* 15995	*100%* 16125	*10730*
1990	2063	*25%* 6171	*49%* 10115	*65%* 12744	*78%* 14880	*89%* 16687	*93%* 17345	*97%* 18002	*99%* 18331	*100%* 18495	*16432*

Table 6: Comparison of Standard Error of CL method and Modified CL method

Year	**Mack's S.E**	
	CL	**Modified CL method**
1981	0	0
1982	206	199
1983	623	617
1984	747	749
1985	1469	1464
1986	2002	2012
1987	2209	2190
1988	5358	5346
1989	6333	6078
1990	24566	24604

Table 7: Overall Standard Error of IBNR claims reserves for CL method and Modified CL method

Overall Mack's S.E	
CL	26,909.01
Modified CL method	25994.79

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