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Alpha Weakly Semi Closed Sets in Topological Spaces

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Abstract – N. Levine introduced the concept of generalized closed (briefly g-closed) sets in topology. Researches in topology studied several versions of generalized closed sets and they characterized that sets. In this paper, we introduce a new class of closed sets which is called Alpha weakly semi closed sets in topological spaces and we study the relationships of this set with some other generalized closed sets. Also we study some of its basic properties.

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Key words*:* αws-*closed*, ws-*open*, *α-closure.*

I. INTRODUCTION

In 1970, Levine [1] introduced generalized closed (briefly g-closed) sets in topology. Researches in topology studied several versions of generalized closed sets. In 2000, M. Sheik John [2] introduced and investigated w-closed sets in topology. In 2017, Veeresha A Sajjanar [3] introduced weakly semi closed sets and investigated some of their properties. In this paper, Section I contains the concept of Alpha Weakly semi-closed (briefly **αws**-closed) set is introduced and their properties are investigated. Section II contains the Certain preliminary concepts, Section III contain the concept of αws-closed set is studied and a diagram also included which states the relationships among the generalized closed sets in topological spaces and Section IV contains the conclusions and Section V contains the references.

II. PRELIMINARIES

Throughout this paper X and Y represents the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, *cl*A and *int*A denote the closure of A and the interior of A respectively. X – A denotes the complement of A in X. We recall the following definitions.

Definition 2.1: A subset A of a space X is called

(i) pre-open [4] if A ⊆ *int cl*A and pre-closed if *cl int*A ⊆ A.
(ii) semi-open [5] if A ⊆ *cl int*A and semi-closed if *int cl*A ⊆ A.
(iii) semi-pre-open [6] if A ⊆ *cl int cl*A and semi-pre-closed if *int cl int*A ⊆ A.
(iv) α-open [7] if A ⊆ *int cl int*A and α-closed if *cl int cl*A ⊆ A.
(v) regular open [8] if A = *int cl*A and regular closed if *cl int*A = A.
(vi) b-open [9] if A ⊆ *cl int*A ∪ *int cl*A and b-closed if *cl int*A ∩ *int cl*A.
(vii) π-open [10] if A is the union of regular open sets and π-closed if A is the intersection of regular closed sets.

The alpha-closure (resp. semi-closure, resp. semi-pre-closure, resp. pre-closure, resp. b-closure) of a subset A of X is the intersection of all alpha-closed (resp. semi-closed, resp. semi-preclosed, resp. pre-closed, resp. b-closed) sets containing A and is denoted by *αcl*A (resp. *scl*A, resp. *spcl*A, resp. *pcl*A, resp. *bcl*A).

Definition 2.2: A subset A of a space X is called

(i) generalized closed [1] (briefly g-closed) if *cl*A ⊆ U whenever A ⊆ U and U is open.
(ii) regular generalized closed [11] (briefly rg-closed) if *cl*A ⊆ U whenever A ⊆ U and U is regular open.
(iii) α-generalized closed [12] (briefly αg-closed) if *αcl*A ⊆ U whenever A ⊆ U and U is open.
(iv) generalized α-closed [13] (briefly gα-closed) if *αcl*A ⊆ U whenever A ⊆ U and U is α-open.
(v) pre-semiclosed [14] if *spcl*A ⊆ U whenever A ⊆ U and U is g-open.
(vi) generalized semi-closed [15] (briefly gs-closed) if *scl*A ⊆ U whenever A ⊆ U and U is open.
(vii) generalized pre-closed [16] (briefly gp-closed) if *pcl*A ⊆ U whenever A ⊆ U and U is open.
(viii) generalized regular closed [17] (briefly gr-closed) if *rcl*A ⊆ U whenever A ⊆ U and U is open.
(ix) generalized semi-pre-closed [18] (briefly gsp-closed) if *spcl*A ⊆ U whenever A ⊆ U and U is open.
(x) π -generalized closed [19] (briefly πg -closed) if *cl*A ⊆ U whenever A ⊆ U and U is π-open.
(xi) weakly generalized closed [20] (briefly wg-closed) if *cl int*A ⊆ U whenever A ⊆ U and U is open.
(xii) $g^{\#}$-closed [21] if *cl* A ⊆ U whenever A ⊆ U and U is αg-open.
(xiii) $g^{\#}p^{\#}$-closed [22] if *cl* A ⊆ U whenever A ⊆ U and U is $g^{\#}$-open.
(xiv) generalized b-closed [23] (briefly gb-closed) if *bcl*A ⊆ U whenever A ⊆ U and U is open.
(xv) generalized αb-closed [24] (briefly gαb -closed) if *bcl*A ⊆ U whenever A ⊆ U and U is α-open.
(xvi) regular generalized b-closed [25] (briefly rgb-closed) if *bcl*A ⊆ U whenever A ⊆ U and U is regular open.
(xvii) weakly closed [2] (briefly w-closed) if *cl*A ⊆ U whenever A ⊆ U and U is semi open.
(xviii) weakly semi closed [3] (briefly ws-closed) if *scl* A ⊆ U whenever A ⊆ U and U is w-open.
(xix) strongly generalized closed [26] (briefly g*-closed) if *cl*A ⊆ U whenever A ⊆ U and U is g-open.
(xx) semi generalized-closed [27] (briefly sg-closed) if *scl*A ⊆ U whenever A ⊆ U and U is semi-open.
(xxi) (gsp)*-closed set [28] if *cl*A ⊆ U whenever A ⊆ U and U is gsp-open.
(xxii) genralized pre regular-closed [29] (briefly gpr-closed) if *pcl*A ⊆ U whenever A ⊆ U and U is regular-open.
(xxiii) alpha generalized regular-closed [30] (briefly αgr-closed) if *αcl*A ⊆ U whenever A ⊆ U and U is regular-open.
(xxiv) semi generalized b-closed [31] (briefly sgb-closed) if *bcl*A ⊆ U whenever A ⊆ U and U is semi-open.
(xxv) *g-closed [26] if *scl*A ⊆ U whenever A ⊆ U and U is semi-open.

The complements of the above mentioned closed sets are their respective open sets. For example a subset B of a space X is generalized open (briefly g-open) if X – B is g-closed.

Definition 2.3: A space X is called a
(i) T_b space [32] if every gs-closed set is closed.
(ii) α-space [7] if every α-closed set is closed.
(iii) door space [33] if every subset is either open or closed.

Lemm 2.4: [34] In an extremally disconnected space X,

(i) *pcl*A = *spcl*A.
(ii) α*cl*A = *scl*A.

Lemma 2.5: [34] In an extremally disconnected sub maximal space X,
*cl*A = α*cl*A = *scl*A = *pcl*A = *spcl*A.

Lemma 2.6: [6] For any subset A of X, the following results hold:
(i) *scl*A = A ∪ *int cl*A.
(ii) *pcl*A = A ∪ *cl int*A.
(iii) *spcl*A = A ∪ *int cl int*A.
(iv) α*cl*A = A ∪ *cl int cl*A

III. ALPHA WEAKLY SEMI CLOSED SETS

In this section, we introduce a new type of closed sets namely αws-closed sets in topological spaces and study some of their properties.

Definition 3.1: A subset A of a space X is called Alpha Weakly Semi closed (briefly αws-closed) if *αcl*A ⊆ U whenever A ⊆ U and U is ws-open.

Proposition 3.2:
(i) Every closed set is αws-closed.
(ii) Every α-closed set is αws-closed.
(iii) Every π-closed set is αws-closed.
(iv) Every regular closed set is αws-closed.
(v) Every (gsp)*-closed set is αws-closed.

Proof:
(i) Let A be a closed set in X. Let A ⊆ U and U is ws-open. Since A is closed, *cl*A = A. But *αcl*A ⊆ *cl*A . Therefore *αcl*A ⊆ U. Hence A is αws-closed in X.
(ii) Let A be a α-closed set in X. Let A ⊆ U and U is ws-open. Since A is α-closed, α*cl*A = A. Therefore *αcl*A ⊆ U. Hence A is αws-closed in X.
(iii) Let A be a π -closed subset of X. Since every π-closed set is closed [19] and by (i), we have A is αws-closed.
(iv) Let A be a regular-closed subset of X. Since every regular-closed set is closed [8] and By (i), we have A is αws-closed.
(v) Let A be a (gsp)*- closed set in X. Let A ⊆ U and U is ws-open. Since every ws-open set is gsp-open and A is (gsp)*-closed, *cl*A ⊆ A. But *αcl*A ⊆ *cl*A . Therefore *αcl*A ⊆ U. Hence A is αws-closed in X.

The reverse implications are not true as shown in Examples 3.3, 3.4, 3.5 and 3.6

Example 3.3: Let X = {a, b, c} with topology τ = {ϕ, {a}, {b, c}, X}. Then {b} is αws-closed but not regular closed.

Example 3.4: Let X = {a, b, c} with topology τ = {ϕ, {a}, {a, b}, X}. Then
{b} is αws-closed but not closed.
{b}is αws-closed but not (gsp)*-closed.

Example 3.5: Let X = {a, b, c, d} with topology τ = {ϕ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, X}. Then{c} is αws-closed but not π-closed.

Example 3.6: Let X = {a, b, c, d} with topology τ = {ϕ, {a, b}, {a, b, c}, X}. Then { a, c, d} is αws-closed but not α-closed.

Proposition 3.7:
(i) Every αws-closed set is αg-closed.
(ii) Every αws-closed set is gpr-closed.
(iii) Every αws-closed set is gb-closed.
(iv) Every αws-closed set is rgb-closed.

(v) Every αws-closed set is gp-closed.
(vi) Every αws-closed set is gs-closed.
(vii) Every αws-closed set is αgr-closed.
(viii) Every αws-closed set is gαb-closed.
(ix) Every αws-closed set is sg-closed.
(x) Every αws-closed set is sgb-closed.

Proof:

(i) Let A be a αws-closed subset of a space X. Let A $\subseteq$ U and U is open. Since every open set is ws-open in X and A is αws-closed, αclA $\subseteq$ U. Hence A is αg-closed.

(ii) Let A be a αws-closed set in X. Let A $\subseteq$ U and U is regular open. Since every regular open set is ws-open in X and since A is αws-closed, αclA $\subseteq$ U. But pclA $\subseteq$ αclA. Therefore pclA $\subseteq U$. Hence A is gpr-closed.

(iii) Let A be a αws-closed set in X. Let A $\subseteq$ U and U is open. Since every open set is ws-open in X & Since A is αws-closed, αclA $\subseteq$ U. But bclA $\subseteq$ αclA. Therefore bclA $\subseteq U$. Hence A is gb-closed in X.

(iv) Let A be a αws-closed set in X. Let A $\subseteq$ U and U is regular open. Since every regular open set is ws-open in X and since A is αws-closed, αclA $\subseteq$ U. But bclA $\subseteq$ αclA. Therefore bclA $\subseteq U$. Hence A is *rgb* -closed.

(v) Let A be a αws-closed set in X. Let A $\subseteq$ U and U is open. Since every open set is ws-open and since A is αws-closed, αclA $\subseteq$ U. But pclA $\subseteq$ αclA. Therefore pclA$\subseteq$ U. Hence A is gp-closed.

(vi) Let A be a αws-closed set. Let A $\subseteq$ U and U is open. Since every open set is ws-open and since A is αws-closed, α*cl*A $\subseteq$ U. But sclA $\subseteq$ αclA. Therefore sclA $\subseteq$ U. Hence A is gs-closed.

(vii) Let A be a αws -closed set. Let A $\subseteq$ U and U is regular open. Since every regular open set is ws-open and since A is αws-closed, αclA $\subseteq$ U. Hence A is αgr-closed.

(viii) Let A be a αws-closed set. Let A $\subseteq$ U and U is α-open. Since every α-open set ws-open and since A is αws-closed, αclA $\subseteq$ U. But bclA $\subseteq$ αclA. Therefore bclA $\subseteq$ U . Hence A is gαb-closed.

(ix) Let A be a αws-closed set. Let A $\subseteq$ U and U is semi-open. Since every semi-open set is ws-open and since A is αws-closed, α*cl*A $\subseteq$ U. But sclA $\subseteq$ αclA. Therefore sclA $\subseteq$ U. Hence A is sg-closed.

(x) Let A be a αws-closed set. Let A $\subseteq$ U and U is semi-open. Since every semi-open set ws-open and since A is αws -closed, αclA $\subseteq$U. But bclA $\subseteq$ αclA. Therefore bclA $\subseteq$ U . Hence A is sgb-closed.

The reverse implications are not true as shown in Example 3.8, 3.9 And 3.10

Example 3.8: Let X = {a, b, c,} with topology τ ={ϕ,{a},{a, b},X}. Then
{ a, c} is αg-closed but not αws-closed.
{a} is gpr-closed but not αws-closed.
{ a, c} is gb-closed but not αws-closed.
{ a} is rgb-closed but not αws-closed.
{a ,c} is gp-closed but not αws-closed.
{a, c} is gs-closed but not αws-closed.
{a} is αgr-closed but not αws -closed.

Example 3.9: Let X={a, b, c, d}with topology τ ={ϕ, {a, b},{a, b, c},X}. Then
{a} is gαb-closed but not αws-closed.

Example 3.10: Let X={a, b, c, d}with topology τ ={ϕ,{a},{b},{a, b},{b ,c},{a, b, c},X}. Then
{a} is sg-closed but not αws -closed.

{a} is sgb-closed but not αws -closed.

The concept "αws-closed" is independent from the concepts "g-closed", "gr-closed", "g∗-closed", "rg-closed", "$g^{\#}p^{\#}$-closed", "∗g-closed", "πg-closed" as seen in the following Examples 3.11 & 3.12

Example 3.11: Let X = {a, b, c} with topology τ ={ϕ,{a},{a, b},X}.
{b} is αws-closed but not g-closed and {a, c} is g-closed but not αws -closed.
{b} is αws-closed but not gr -closed and {a, c} is gr -closed but not αws -closed.
{b} is αws - closed but not g∗ -closed and {a, c} is g∗ -closed but not αws -closed.

Example 3.12: Let X = {a, b, c, d} with topology τ = {ϕ,{a},{b},{a, b},{b, c},{a, b, c},X}.
{c} is αws-closed but not rg-closed and {a, b} is rg-closed but not αws-closed.
{c} is αws-closed but not $g^{\#}p^{\#}$-closed and {b, d} is $g^{\#}p^{\#}$-closed but not αws-closed.
{c} is αws-closed but not ∗g-closed and {a, b, d} is ∗g-closed but not αws- closed.
{c} is αws-closed but not πg-closed and {b, d} is πg-closed but not αws-closed.

Thus the above discussions lead to the following diagram. In this diagram, "A→B" means A implies B but not conversely and "A ↮ B" means A and B are independent of each other.

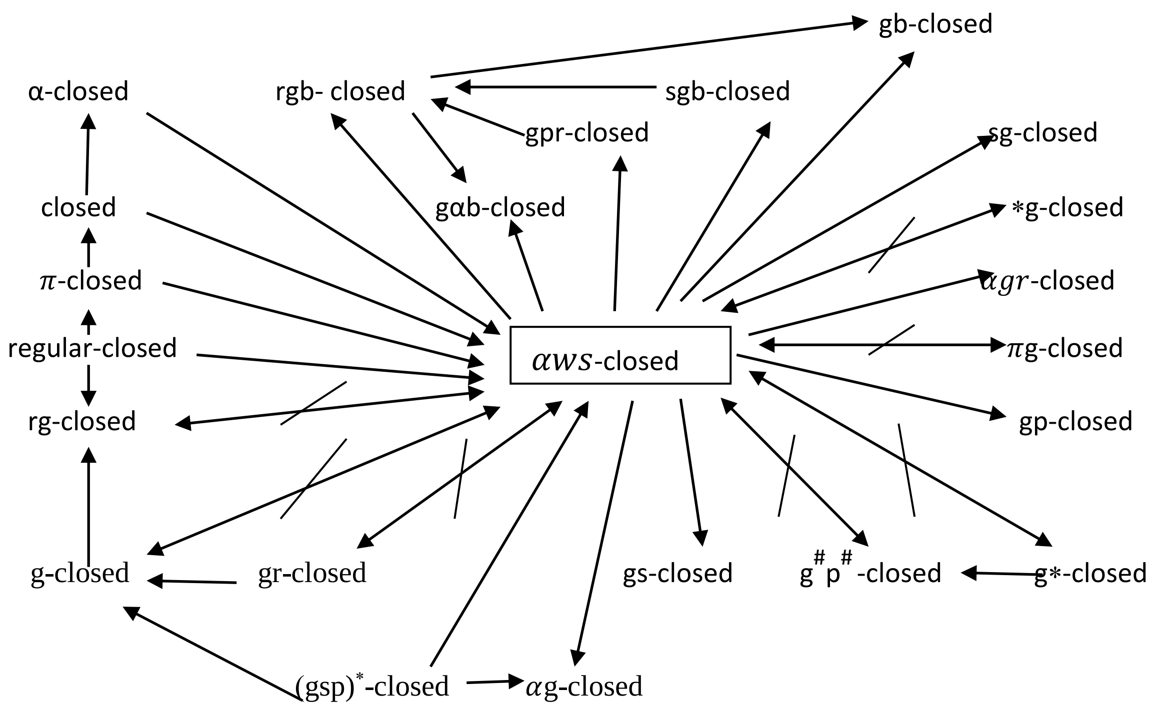

Figure.1

Theorem: 3.13

The union of two αws-closed subsets of X is αws-closed set.

Proof:

Let A and B be any two αws -closed sets in X. Let A ⊆ U & U is ws-open, B ⊆ U & U is ws-open. Then αclA ⊆ U and αcl B ⊆ U. since A ⊆ U and B ⊆ U, then A∪B ⊆ U

⇒ αcl (A) ∪ αcl (B) ⊆ U. we know that αcl (A∪B) = αcl (A) ∪ αcl (B) [35].

Hence αcl (A∪B) = αcl(A) ∪ αcl (B) ⊆ U. Hence αcl (A∪B) ⊆ U. Therefore A∪B is αws-closed in X.

Theorem: 3.14

If a subset A of X is αws-closed in X, then αcl A − A does not contain any non- empty ws-closed set in X.

Proof:

Let A be a αws-closed set in X and F be a ws-closed subset of αclA – A.

Then F ⊆ αclA ∩ (X−A) ⇒ F ⊆ αclA & F ⊆ X – A ⇒ A ⊆ X – F

Since A is αws- closed set and X−F is ws-open, then αclA ⊆ X – F (ie) then F ⊆ X – αcl A

We have F ⊆ αclA. Therefore, F ⊆ (X – αclA) ∩ αclA = ϕ. Thus F ⊆ ϕ.

Hence αclA – A does not contain any non-empty ws-closed set in X.

Theorem: 3.15

If a subset A is αws-closed set in X and, A ⊆ B ⊆ αclA, then B is also αws-closed set.

Proof:

Let A be a αws-closed set in X such that A ⊆ B⊆ αcl (A). To prove B is also αws-closed set in X. It is enough to prove αcl (B) ⊆ U. Let U be a ws-open set in X such that B ⊆ U.

Since A⊆ B, A ⊆ U. Also since A is αws-closed, αcl (A) ⊆ U. Now, B ⊆ αcl (A)

⇒αcl (B) ⊆ αcl [αcl (A)]= αclA ⊆ U [36]. (ie) αcl (B) ⊆ U. Therefore, B is αws-closed set in X.

Theorem: 3.16

For every point x in a space X, X – {x} is αws-closed or ws-open.

Proof: **<u>Case (i)</u>**

suppose X – {x} is not ws-open. Then X is the only ws-open set containing X – {x}

Then using Definition 3.1 αcl (X – {x}) ⊆ X. Hence X – {x} is αws-closed.

<u>Case (ii)</u>

Suppose X – {x} is not αws-closed. Then there exists a ws-open set U containing X– {x} such that αcl (X– {x}) ⊈ U.

Therefore αcl (X – {x}) is either X – {x} or X. Therefore Take

αcl (X− {x}) = X – {x}, then X−{x} is α-closed. By Preposition 3.2 (i) every α-closed set is αws-closed, X– {x} is αws closed. This is contradiction to our assumption. Therefore

αcl (X– {x}) = X. To prove X– {x} is ws-open. Suppose X – {x} is not ws-open. By case (i) X– {x} is αws-closed. Which is contradiction to our assumption. Therefore X – {x} is ws-open.

Theorem: 3.17

Let X and Y are topological spaces and A ⊆ Y ⊆ X. Suppose that A is αws-closed set in X then A is αws-closed relative to Y.

Proof:

Given A⊆ Y⊆ X and A is αws-closed in X. To prove that A is αws-closed relative to Y.

Let A $\subseteq$ Y $\cap$ U, where U is ws -open in X. Since A is αws-closed, then αcl A $\subseteq$ U. This implies Y $\cap$ αclA $\subseteq$ Y$\cap$ U, whereY $\cap$ αcl A is the α-closure of A in Y and Y$\cap$ U is ws -open in Y. Therefore αclA $\subseteq$ Y $\cap$ U in Y. Hence, A is αws-closed set relative to Y.

Theorem: 3.18

Let A be αws-closed in X. Then A is α-closed iff αclA – A is ws-closed.

Proof:

Suppose A is a α-closed set. Then αcl A = A$\Rightarrow$ αclA – A = ϕ which is ws-closed.

Conversely, suppose αclA – A is ws-closed. Since A is αws-closed, Then by Theorem 3.14, αclA – A = ϕ, (ie) αclA = A. Hence A is α-closed.

Theorem: 3.19

Suppose A is ws-open and A is αws-closed. Then A is α-closed.

Proof:

Given that A is ws-open and A is αws-closed. Then A $\subseteq$ A $\Rightarrow$ αclA $\subseteq$ A

Hence A is α-closed.

Theorem: 3.20

In a topological space if X αO(X) = {X, ϕ} then every subset of X is a αws-closed set.

Proof:

Given that X is a topological space and αO(X) = {X, ϕ}. Let A be a subset of X.

Suppose A = ϕ, then by Theorem 3.4, ϕ is αws-closed set. Suppose A ≠ ϕ, then X is only α-open set containing A. Therefore αclA $\subseteq$ X. Hence A is αws-closed set in X.

Theorem: 3.21

If A is regular open and αgr-closed set then A is αws-closed set in X.

Proof:

Suppose A is a regular open set and αgr-closed. Let U be any ws-open set in X ∋: A $\subseteq$ U.

Since A is regular open and αgr-closed set in X, by Definition αclA $\subseteq$ A. then αclA $\subseteq$ A $\subseteq$ U. Hence A is αws-closed.

Definition: 3.22

The intersection of all ws-open subsets of X containing A is called the ws-kernel of A and is denoted by ws-ker (A).

Theorem: 3.23

If A is a subset of X is αws-closed iff αclA $\subseteq$ ws-ker (A).

Proof:

Suppose A is αws closed. Then αclA $\subseteq$ U whenever A $\subseteq$ U & U is ws-open.

To prove αcl(A) $\subseteq$ ws-Ker (A) .Take $x \in \alpha cl$(A). To prove $x \in$ ws-ker (A)

Suppose $x \notin$ ws-ker(A) then there exist a ws-open set U containing A such that $x \notin$ U. Since A is αws-closed, then αclA $\subseteq$ U $\Rightarrow x \notin \alpha cl$(A), Which is a contradiction to our assumption. Therefore αcl A $\subseteq$ ws-ker (A). Conversely, Suppose αcl A $\subseteq$ ws-ker (A). To prove A is αws-closed. If U is any ws-open set containing A, then ws-ker A$\subseteq$ U $\Rightarrow$ αclA $\subseteq$ U. Hence A is αws-closed in X.

Note: 3.24 [37]

Let x be a point of X. Then $\{x\}$ is either nowhere dense or pre-open.

Remark: 3.25 [37]

By the above note we take the following decomposition of a given topology X, namely

$X = X_1 \cup X_2$

Where $X_1 = \{x \in X; \{x\}$ is nowhere dense$\}$

$X_2 = \{x \in X; \{x\}$ is pre-open $\}$

This is called Jankovic-Reilly Decomposition.

Theorem: 3.26

For any subset A of X, $X_2 \cap \alpha cl$ A $\subseteq$ ws- ker (A)

Proof:

To Prove $X_2 \cap \alpha cl$ (A) $\subseteq$ ws-ker(A). Consider $x \in X_2 \cap \alpha cl$ (A).To prove $x \in$ ws-ker(A)

Suppose $x \notin$ ws-ker (A), then there is a ws-open set U containing A such that $x \notin$ U.

If F = X – U, then F is ws-closed**.** Now, $x \in \alpha cl$ (A) $\Rightarrow \alpha cl$ ($\{x\}$) $\subseteq$ αcl (αcl (A) $\subseteq$ αcl (A)

Since α *cl* ($\{x\}$) $\subseteq \alpha cl$ (A), we get *int* (αcl ({x})) $\subseteq$ *int* (αcl (A)) $\subseteq$ A $\cap$ *int* (αcl (A))

Therefore*int* (αcl ($\{x\}$)) $\subseteq$ A $\cap$ *int* (*cl* (A)). Now, take $x \in X_2$. We have $x \notin X_1$ and so

int (*cl* ({x}))$\neq \phi$. Let $y \in$ *int* (*cl* {x}). Consider a point $y \in$ A $\cap$ *int* (*cl* ($\{x\}$))

$\Rightarrow y \in$ A $\cap$ *cl* ($\{x\}$) $\Rightarrow y \in$ A $\cap$ F which is a contradiction to $x \notin$ ws- ker(A) [38]**.** Therefore $x \in$ ws-ker(A). Hence $X_2 \cap \alpha cl$ (A) $\subseteq$ ws-ker(A).

Theorem 3.27:

A subset A of X is αws-closed iff $X_1 \cap \alpha cl$ (A) $\subseteq$ A

Proof:

Consider A is αws-closed . To prove $X_1 \cap \alpha cl$ (A) $\subseteq$ A. Let $x \in X_1 \cap \alpha cl$ (A), Then $x \in X_1$ and $x \in \alpha cl$ (A) .Since $x \in X_1$, *int* (*cl* ({x})) = ϕ. Hence $\{x\}$ is semi-closed. Every semi closed set is ws-closed in X [15], $\{x\}$ is ws-closed. If $x \notin$ A, then U = X – {x} ws-open set containing A and So αcl A $\subseteq$ U. Since $x \in \alpha cl$ (A), $x \in$ U which is a contradiction to $x \notin$U.

Hence $X_1 \cap \alpha cl$ (A) $\subseteq$ A. Conversely, let $X_1 \cap \alpha cl$ (A) $\subseteq$ A. To Prove A is αws-closed

Since $X_1 \cap \alpha cl$ (A) $\subseteq$ A, $X_1 \cap \alpha cl$ (A) $\subseteq$ ws-ker(A).

Now, αcl (A) = X $\cap$ αcl (A) = ($X_1 \cup X_2$) $\cap \alpha cl$ (A) = ($X_1 \cap \alpha cl$ (A)) $\cup$ ($X_2 \cap \alpha cl$ (A))

αcl (A) $\subseteq$ ws-ker (A)**.** Then by Theorem 3.23 A is αws-closed.

Theorem: 3.28

Arbitrary intersection of αws-closed set is αws-closed.

Proof:

Let $\{A_i\}$ be the collection αws-closed sets of X. Let A = $\cap A_i$. Since A $\subseteq A_i$ for each i,

then αcl (A) $\subseteq \alpha cl$ (A_i) $\Rightarrow X_1 \cap \alpha cl$ (A) $\subseteq X_1 \cap \alpha cl$ (A_i) for each i.

Since each A_i is αws-closed, then by theorem 3.28, $X_1 \cap \alpha cl\,(A_i) \subseteq A_i$ for each i.

Thus $X_1 \cap \alpha cl\,(A) \subseteq X_1 \cap \alpha cl\,(A_i) \subseteq A_i \subseteq A$ for each i. By Theorem 3.27, A is αws-closed.

Theorem: 3.29

In a door space X, every αws-closed set is α -closed

Proof:

Let A be a αws-closed set is X. Since X is a door space, by Definition 2.3(iii), A is either open or closed. If A is closed, then A is α-closed. If A is open, then A is ws-open

Since A is αws-closed & A is ws-open, by Theorem 3.19, A is α-closed.

Theorem: 3.30

In an extremally disconnected space X, every αws-closed set is gs-closed.

Proof:

Let X be an extremally disconnected space and A be a αws -closed subset of X.

Let $A \subseteq U$ & U be open. Since every open set is ws-open in X and since A is αws-closed,

$\alpha cl A \subseteq U$. Since X is extremally disconnected space, by Lemma2.4(ii), $scl A = \alpha cl A \subseteq U$

$\Rightarrow scl A \subseteq U$. Hence A is gs-closed.

Theorem: 3.31

In an extremally disconnected sub maximal space X, every αws-closed set is w-closed.

Proof:

Let X be an extremally disconnected space and A be a αws-closed subset of X.

Let $A \subseteq U$ & U is semi-open. Since every semi open set is ws-open & since A is αws-closed,

$\alpha cl\, A \subseteq U$. Since X is extremally disconnected submaxinal space, by Lemma 2.5 $cl\, A \subseteq U$.

Hence A is w-closed.

Theorem: 3.32

In a T_b space X, Every gs-closed set is αws-closed

Proof:

Let A be a gs-closed set. Since X is T_b space, by Definition 2.3 (i), A is closed. By Preposition 3.2(i) A is αws-closed.

Theorem: 3.33 In a α-space X, every α-closed set is αws-closed.

Proof: Let A be a α-closed set in X. Since X is a α-space, by Definition 2.3(ii), A is closed. By Preposition 3.2(ii), A is αws-closed.

IV. CONCLUSION

In this paper, we have focused on Alpha Weakly Semi closed sets in topological spaces and found some important properties. In future this concept can be extended to bitopological and ideal topological spaces.

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