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Magicness of Circular Human Chain Graph

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Abstract - In this paper, we introduce a new graph called circular human chain graph and investigate the existence of Z_3-vertex magic total, Z_3- edge magic total, Z_4-bi magic and n-edge magic labeling of circular human chain graph.

Key words:Circular human chain,Magic labeling,Vertex –magic labeling,Edge –magic labeling

I. INTRODUCTION

A labeling of a graph G(V,E) is a mapping from the set of vertices, edges or both vertices and edges to the set of labels. Based on the domain we distinguish vertex labeling, edge labeling and total labeling. The concept of Human chain graph was introduced by K.Anitha and B.Selvam[1] and they have proved many results of human chain graph[2,3]. For a summary on various labeling see the dynamic survey of graph labeling by Gallian [5]. We have referred Z_3- vertex magic total labeling and Z_3- edge magic total labeling which has been extracted from various articles [6]. In this paper, we introduce a new graph called circular human chain graph and investigate Z_3- vertex magic total, Z_3- edge magic total, Z_4-bi magic and n-edge magic labeling of circular human chain graph.

II. PRELIMINARIES

In this section, we provide some basic definitions relevant to this paper.

Definition 2.1 Z_3-Vertex magic total labeling : A graph G(V,E) is said to admit Z_3- vertex magic total labeling if $f: V \cup E \rightarrow A^*$ where $A^*=Z_3-[0]$ such that the induced map f* on V defined by $f^*(v_i)= \{f(v_i)+\sum f(e)\}(mod3) = k$, a constant where e is the edge incident at v_i. A graph which admits Z_3-vertex magic total labeling is called Z_3- vertex magic total graph.

Definition 2.2 Z_3-edge magic total labelling: A graph G(V,E) is said to admit Z_3- edge magic total labeling if $f: V \cup E \rightarrow A^*$ where $A^*=Z_3-[0]$ such that the induced map f* on E defined by $f^*(v_iv_j)= \{f(v_i)+f(v_j)+f(v_i v_j)\}(mod3) = k$, a constant for all edges $v_i v_j \in E$. A graph which admits Z_3-edge magic total labeling is called Z_3- edge magic total graph.

Definition 2.3 Z_4-bi magic labeling : A graph G(V,E) is said to admit Z_4 - bi magic labeling if there exists a function $f: E\rightarrow \{1,2,3\}$ such that the induced map f* on V defined by $f^*(v_i)= \sum f(e)(mod\ 4) = k_1$ or k_2 , a constant $e=v_iv_j \in E$.

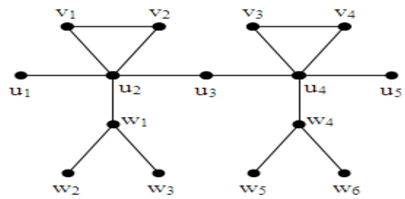
Definition 2.4 n-edge magic labeling : Let G(V,E) be a graph. Let $f: V\rightarrow \{-1, n+1\}$ and $f^*:E\rightarrow\{n\}$ such that for all $uv \in E$, $f^*(uv)= f(u)+f(v)=n$, then the labeling is called n-edge magic labeling.

Definition 2.5 Human chain graph

A human chain graph $HC_{n,m}(p,q)$ is obtained from a path $u_1 ,u_2 ,\ldots,u_{2n+1}$, $n\in N$ by joining a cycle of length m (C_m) and Y-tree Y_{m+1}, $m\geq3$ to each u_{2i} for $1\leq i\leq n$. The vertices of C_m and Y-tree Y_{m+1} are $v_1 ,v_2 ,\ldots,v_{(m-1)n}$ and $w_1 ,w_2 ,\ldots,w_{mn}$ respectively and the vertices of path is $u_1 ,u_2 ,\ldots,u_{2n+1}$.

The vertex set and edge set of $HC_{n,m}$ are $\{u_i , v_j , w_k /1\leq i\leq 2n+1,\ 1\leq j\leq(m-1)n,\ 1\leq k\leq mn\}$and $\{u_i\ u_{i+1}/ 1\leq i\leq 2n\}$U $\{u_{2i}\ w_{m(i-1)+1};\ u_{2i}\ v_{(m-1)i}\ ;\ u_{2i}\ v_{(m-1)(i-1)+1};\ w_{mi}\ w_{mi-2}/1\leq i\leq n\}$ U $\{w_{mi+j}\ w_{mi+j+1};\ v_{(m-1)i+j}\ v_{(m-i)+j+1}/0\leq i\leq n-1,1\leq j\leq m-2\}$.

Example1:$HC_{2,3}$



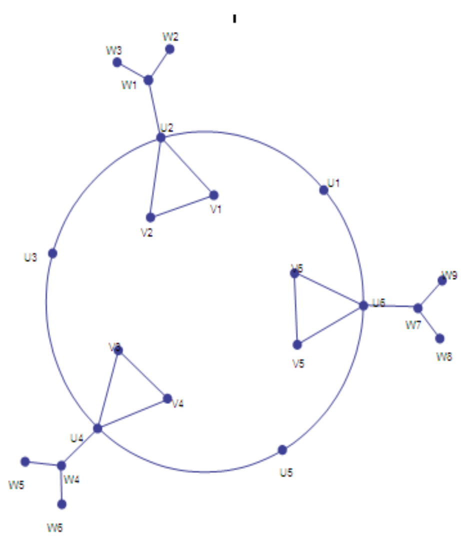
III. MAIN RESULTS

In this section, we introduce a new definition of circular human chain graph and investigate Z_3- vertex magic total, Z_3- edge magic total, Z_4-bi magic and n-edge magic labeling of circular human chain graph.

Definition 3.1 Circular Human chain graph

A circular human chain graph $CHC_{n,m}(p,q)$ is obtained from a circle $u_1, u_2, \ldots, u_{2n}, u_1$, $n \geq 3$ by joining a cycle of length m (C_m) and Y-tree Y_{m+1}, $m \geq 3$ to each u_{2i} for $1 \leq i \leq n$. The vertices of C_m and Y-tree Y_{m+1} are $v_1, v_2, \ldots, v_{(m-1)n}$ and $w_1, w_2, \ldots, w_{mn}$ respectively.
The vertex set and edge set of $CHC_{n,m}$ are $\{u_i, v_j, w_k / 1 \leq i \leq 2n, 1 \leq j \leq (m-1)n, 1 \leq k \leq mn\}$ and $\{u_i u_{i+1} / 1 \leq i \leq 2n-1\} \cup \{u_{2i} w_{m(i-1)+1}; u_{2i} v_{(m-1)i}; u_{2i} v_{(m-1)(i-1)+1}; w_{mi} w_{mi-2} / 1 \leq i \leq n\} \cup \{w_{mi+j} w_{mi+j+1}; v_{(m-1)i+j} v_{(m-i)+j+1} / 0 \leq i \leq n-1, 1 \leq j \leq m-2\} \cup \{u_1 u_{2n}\}$.

Example 2: $CHC_{3,3}$



Algorithm 3.1

We use the following algorithm to prove that the existence of Z_3-vertex magic total labeling for the circular human chain graph.

Procedure: (Z_3-Vertex magic total labeling of $CHC_{n,m}$)
Input:

$V \leftarrow \{u_1, u_2, \ldots, u_{2n}, v_1, v_2, \ldots, v_{(m-1)n}, w_1, w_2, \ldots, w_{mn}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{2mn+2n}\}$

if m,n≥3

for i= 1 to2n do

$u_i \leftarrow 1$

end for

for i= 1 to(m-1)n do

$v_i \leftarrow 1$

end for

for i= 1 to mn do

$w_i \leftarrow 1$

end for

for i= 1 to 2n-1 do

$u_i u_{i+1} \leftarrow 1$

$u_1 u_{2n} \leftarrow 1$

end for

for i= 1 to n do

$u_{2i} v_{(m-1)i} \leftarrow 1$

$u_{2i} v_{(m-1)(i-1)+1} \leftarrow 1$

$u_{2i} w_{m(i-1)+1} \leftarrow 1$

$w_{mi} w_{mi-2} \leftarrow 2$

end for

for i= 0 to n-1 do

for j= 1 to m-2 do

$v_{(m-1)i+j} v_{(m-1)i+j+1} \leftarrow 1$

$w_{(mi+j)} w_{(mi+j+1)} \leftarrow 2$

end for

end for

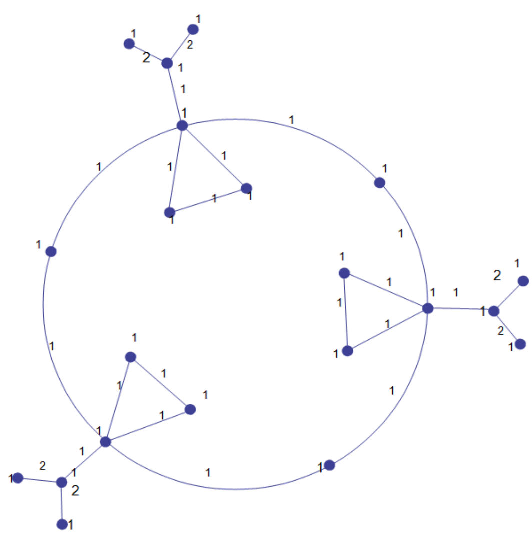
end if

end procedure

Theorem 3.1 For m,n ≥3, the circular human chain graph admits Z_3 – vertex magic total labeling.

Proof: Let $CHC_{n,m}(p,q)$ be the circular human chain graph with p= 2mn+n vertices and q= 2mn+2n edges. Using algorithm 3.1, the 2mn+n vertices and 2mn+2n edges are labeled by defining a function $f:V \cup E \rightarrow \{1,2\}$. The induced function is defined by $f^*:V \rightarrow N \cup \{0\}$, such that $f^*(v_i)= \{f(v_i)+\sum f(e)\}(mod3) = k$, a constant for all edges $v_iv_j \in E$. The total weight of each vertex is $f^*(v)= \{f(v)+\sum f(uv)\}(mod3) = 3$ or $6(mod\ 3)= 0$, a constant for all edges $uv \in E$. Thus the induced function yields the weight '0' to all the vertices. Therefore, for $m,n \geq 3$, the circular human chain graph admits Z_3 – vertex magic total labeling.

Example 3: Z_3 – vertex magic total labelling of $CHC_{3,3}$



Algorithm 3.2

We use the following algorithm to prove that the existence of Z_3-edge magic total labeling for the circular human chain graph.

Procedure: (Z_3-edge magic total labeling of $CHC_{n,m}$)
Input:

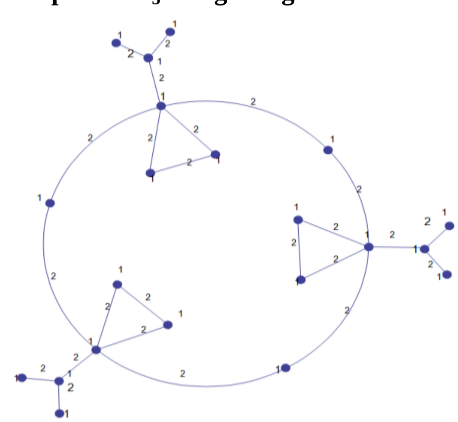
$V \leftarrow \{u_1,u_2,\ldots,u_{2n}, v_1,v_2,\ldots,v_{(m-1)n}, w_1,w_2,\ldots,w_{mn}\}$
$E \leftarrow \{ e_1,e_2,\ldots,e_{2mn+2n}\}$

if $m,n \geq 3$
for i= 1 to 2n do
  $u_i \leftarrow 1$
end for
for i= 1 to(m-1)n do
  $v_i \leftarrow 1$
end for
for i= 1 to mn do
  $w_i \leftarrow 1$
end for
for i= 1 to n do
  $u_{2i}\, v_{(m-1)i} \leftarrow 2$
  $u_{2i}\, v_{(m-1)(i-1)+1} \leftarrow 2$
  $u_{2i}\, w_{m(i-1)+1} \leftarrow 2$
  $w_{mi}\, w_{mi-2} \leftarrow 2$
end for
for i= 1 to 2n-1 do
  $u_iu_{i+1} \leftarrow 2$
end for
for i= 0 to n-1 do
  for j= 1 to m-2 do
    $v_{(m-1)i+j}\, v_{(m-1)i+j+1} \leftarrow 2$
    $w_{(mi+j)}\, w_{(mi+j+1)} \leftarrow 2$
    $u_1u_{2n} \leftarrow 2$
  end for
end for
end if
end procedure

Theorem 3.2 For $m,n \geq 3$, the circular human chain graph admits Z_3 – edge magic total labeling.

Proof: Let $CHC_{n,m}(p,q)$ be the circular human chain graph with p= 2mn+n vertices and q= 2mn+2n edges. Using algorithm 3.2, the 2mn+n vertices and 2mn+2n edges are labeled by defining a function $f:V \cup E \rightarrow \{1,2\}$. The induced function is defined by $f^*:E \rightarrow N \cup \{0\}$, such that $f^*(uv)= \{f(u)+f(v)+f(uv)\}(mod3) = k$. The induced function yields the labels for as follows. $f^*(uv)= \{f(u)+f(v)+f(uv)\}= 1+1+2=4(mod3)=1$. Therefore, for $m,n \geq 3$, the circular human chain graph admits Z_3 – edge magic total labeling.

Example 4: Z_3 – edge magic total labelling of $CHC_{3,3}$



Algorithm 3.3

We use the following algorithm to prove that the existence of Z_4-bi magic total labeling for the circular human chain graph.

Procedure: (Z_4-bi magic lebeling of $CHC_{n,m}$)

Input:

$V \leftarrow \{u_1, u_2, \ldots, u_{2n}, v_1, v_2, \ldots, v_{(m-1)n}, w_1, w_2, \ldots, w_{mn}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{2mn+2n}\}$

if n,m≥3

$u_1 u_2 \leftarrow 1$

$u_1 u_{2n} \leftarrow 2$

for i= 1 to n do

$w_{mi}\, w_{mi-2} \leftarrow 3$

$w_{mi-2}\, w_{mi-1} \leftarrow 3$

$u_{2i}\, w_{m(i-1)+1} \leftarrow 2$

end for

for i= 1 to n-1 do

$u_{2i}\, u_{2i+1} \leftarrow 3$

end for

for i= 0 to n-1 do

for j=1 to m-2 do

$v_{(m-1)i+j}\, v_{(m-1)i+j+1} \leftarrow 2$

end for

end for

for i= 1 to n do

$u_{2i}\, v_{(m-1)i} \leftarrow 1$

$u_{2i}\, v_{(m-1)(i-1)+1} \leftarrow 1$

end for

for i= 1 to n-1 do

$u_{2i+1}\, u_{2i+2} \leftarrow 1$

end for

end if

if m>3

for i= 0 to n-1 do

for j= 1 to $\left\lfloor \frac{m-2}{2} \right\rfloor$ do

$w_{mi+2j}\, w_{mi+2j-1} \leftarrow 1$

end for

end for

end if

if m>4

for i= 0 to n-1 do

for j= 1 to $\left\lfloor \frac{m-3}{2} \right\rfloor$ do

$w_{mi+2j+1}\, w_{mi+2j} \leftarrow 2$

end for

end for

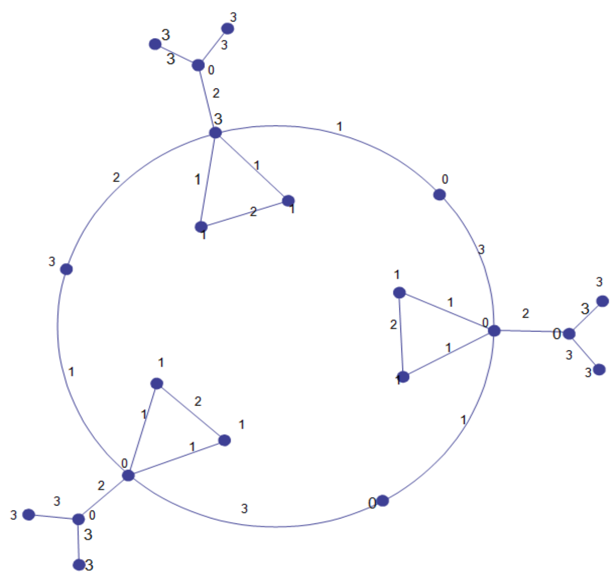
end if

end procedure

Theorem 3.3 For m,n≥3, the circular human chain graph admits Z_4 – bi magic labeling.

Proof: Let $CHC_{n,m}(p,q)$ be the circular human chain graph with p= 2mn+n vertices and q= 2mn+2n edges. Using algorithm 3.3, the 2mn+2n edges are labeled by defining a function $f\colon E \to \{1,2,3\}$ such that the induced function is defined by $f^*\colon V \to \{0,1,2,3\}$ defined by $f^*(v) = \{\sum f(uv) (mod\ 4) / u \in N(v)\} = k_1$ or k_2, constants. Thus all the weight of the vertices are either 0 or 3. Therefore, for m,n≥3 the circular human chain graph admits Z_4 – bi magic labeling.

Example 5: Z_4 – bi magic labelling of $CHC_{3,3}$



Algorithm 3.4

Procedure: (n-edge magic labeling of $CHC_{n,m}$, m is even, n≥3)

Input:

$V \leftarrow \{u_1, u_2, \ldots, u_{2n}, v_1, v_2, \ldots, v_{(m-1)n}, w_1, w_2, \ldots, w_{mn}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{2mn+2n}\}$

if n≥3, m≥4

for i= 1 to n do

$u_{2i-1} \leftarrow -1$

end for

for i= 1 to n do

$u_{2i} \leftarrow n+1$

end for

for i= 0 to n-1 do

for j= 1 to (m/2) do

$w_{mi+2j-1} \leftarrow -1$

end for

end for

for i= 0 to n-1 do

for j= 1 to (m-2)/2 do

$w_{mi+2j} \leftarrow n+1$

end for

end for

for i= 1 to n

$w_{mi} \leftarrow -1$

end for

for i= 1to n do

for j= 1 to (m/2) do

$v_{(m-1)i-m+2j} \leftarrow -1$

end for

end for

for i= 1 to n do

for j= 1 to (m-2)/2 do

$v_{(m-1)i-m+2j+1} \leftarrow n+1$

end for

end for

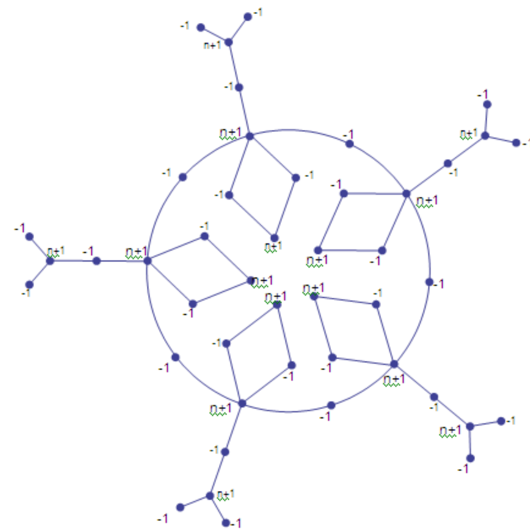
end if

end procedure

Theorem 3.4 For m≥4 and n≥3, the circular human chain graph admits n– edge magic labeling.

Proof: Let $CHC_{n,m}(p,q)$ be the circular human chain graph with p= 2mn+n vertices and q= 2mn+2n edges. Using algorithm 3.4, the 2mn+n vertices are labeled by defining a function f: V→{-1, n+1/n∈N}and 2mn+2n edges are labeled by defining a function f*:E→N**,**such that $f^*(uv)$= {f(u)+f(v)} = -1+n+1=n, a constant for all uv∈E. Therefore, for m≥4 and n≥3, the human chain graph admits n– edge magic labeling.

Example 6: n-edge magic labelling of $CHC_{5,4}$



IV. CONCLUSION

In this paper, we have constructed algorithms for labelling the vertices and edges and also proved the existence of Z_3- vertex magic total, Z_3- edge magic total, Z_4-bi magic and n-edge magic labeling of circular human chain graph.

V. ACKNOWLEDGEMENT

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