International Journal of Scientific Research in Mathematical and Statistical Sciences **Research Paper**
Vol.6, Issue.2, pp.164-167, April (2019) **E-ISSN:** 2348-4519
DOI: https://doi.org/10.26438/ijsrmss/v6i2.164167

Composition of Functions with Separation Axioms

R. Buvaneswari

Dept. of Mathematics, Cauvery College for Women, Trichirappalli – 620 018, Tamil Nadu, India.

Corresponding Author: rambuviabi@yahoo.com, Tel.: +91-9443996054

Available online at: www.isroset.org

Received: 07/Apr/2019, Accepted: 14/Apr/2019, Online: 30/Apr/2019

Abstract— The aim of this paper is to present some new mappings like feebly regular clopen mapping, totally feebly regular continuous, strongly feebly regular continuous, slightly feebly regular continuous functions, feebly regular irresolute, feebly regular clopen irresolute and super feebly regular clopen continuous functions. The notion of feebly regular clopen separation space is introduced. Among several results we prove the composition of functions and characterizations of totally feebly regular continuous functions. Some fundamental properties are investigated in the separation axioms by using the concepts of feebly regular clopen-T_1, feebly regular clopen-T_2 .

Keywords— *feebly regular clopen set, feebly regular clopen mapping, feebly regular clopen -T_1 space, feebly regular clopen-T_2*

I. INTRODUCTION

In General topological space, the concept of feebly open and feebly closed sets are introduced by S.N. Maheswari and P.C.Jain [5]. In this concept, further works are developed by many researchers. Recently A.P. Dhana Balan and R. Buvaneswari analyzed some concepts like feebly open sets (resp. feebly closed sets) and its mappings and also initiated some sets feebly regular open (resp. feebly regular closed) and its characterizations [3].

Section I contains the introduction of General Topological Space, Section II contain the basic necessarily needed concepts, Section III contain the main results, Section IV contain the conclusion with further work. Finally, the alphabetic arrangements of references are listed.

II. PRELIMINARIES

Definition 2.1 [6] : Let X be a topological space and A be a subset of X. It is said to be semi-regular open if A=s int(s cl(A)) and also defined on other hand, it is said to be semi-regular open if both semi open (if A⊂cl(int(A))**[4]**) and semi closed (if int(cl(A))⊂A).

Definition 2.2 [5]: A subset A of a topological space X is said to be feebly open (resp. feebly closed) if A⊂s cl(int(A)) (resp. s int(cl(A))⊂A).

Definition 2.3 [7]: A map f:X→Y is said to be feebly closed (resp. feebly open) if the image of each closed set (resp. open set) in X is feebly closed (resp. feebly open) set in Y.

Remark 2.4 [1]: (i) Every open set is feebly open **(ii)** Every closed set is feebly closed.

Definition 2.5 [3]: A subset A of X is said to be feebly regular open (briefly F.reg.open) if A=f.int(f.cl(A)).

Definition 2.6 [3]: A subset A of X is said to be feebly regular closed if A=f.cl(f.int(A)) (briefly F.reg.closed).

Remark 2.7 [3]: The feebly regular open set is analyzed in the way if A is both feebly open and feebly closed.

Definition 2.8 [3]: A subset A of X is said to be feebly regular clopen if A=f.int(f.cl(f.int(A))). On the other hand, if A is F.reg.open and F.reg.closed.

Definition 2.9 [3]: Let A be subset of X. The regular closure of A (briefly F.reg.cl(A)) is the intersection of all feebly regular closed set containing A and F.reg.int(A) is the union of all feebly regular open set contained in A.

Definition 2.10 [2]: A space X is said to be feebly regular-T_1 (briefly F.reg.-T_1) if for each pair of disjoint points x and y of X, there exist F.reg.open sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

Definition 2.11 [2]: A space X is said to be feebly regular-T_2 (briefly F.reg.-T_2 or F.reg.Hausdorff) if for each pair of distinct points x and y in X, there exist disjoint F.reg.open sets U and V in X such that $x \in U$ and $y \in V$.

III. Composition of Functions with Separation Axioms

Definition 3.1: A function $f : X \to Y$ is said to be feebly regular clopen if the image of every open and closed set in X is respectively feebly regular open and feebly regular closed in Y.

Theorem 3.2: Every feebly regular clopen mapping is feebly regular closed map and feebly regular open map.
proof: Let $f: X \to Y$ be a feebly regular clopen mapping. Let H be any clopen subset of X.
Since f is feebly regular clopen mapping , f(H) is feebly regular closed and feebly regular open in Y. Hence f is feebly regular clopen mapping.
The converse of this theorem is trivial.

Definition 3.3: A function f from the topological spaces (X,τ) to (Y,σ) is said to be totally feebly regular continuous if the inverse image of every open subset of (Y,σ) is a feebly regular clopen subset of (X,τ).

Definition 3.4: A function f from the topological space (X,τ) to (Y,σ) is said to be strongly feebly regular continuous if the inverse image of every subset of Y is feebly regular clopen subset of (X,τ).

Definition 3.5: A function f from the topological space (X,τ) to (Y,σ) is said to be slightly feebly regular continuous if the inverse image of every feebly regular clopen set in Y is feebly regular open in X.

Theroem 3.6: Every totally feebly regular continuous mapping is feebly regular continuous.
Proof: Let a function f from the topological space (X,τ) to (Y,σ) be the totally feebly reglar continuous. Let H be any open set in Y. Since f is totally feebly regular continuous, $f^{-1}(H)$ is feebly regular clopen in X. By definition 3.3, $f^{-1}(H)$ is feebly regular open and feebly regular closed set in X. This implies that $f^{-1}(H)$ is feebly regular open in X. Hence f is feebly regular continuous.

Theorem 3.7: A function $f : X \to Y$ is totally feebly regular continuous if and only if the inverse image of every closed subset of Y is feebly regular clopen in X.

Proof: Let B be any closed set in Y. Then $Y - B$ is open set in Y. By definition 3.3, $f^{-1}(Y - B)$ is feebly regular clopen in X. That is, $X - f^{-1}(B)$ is feebly regular clopen in X.
This implies that the $f^{-1}(B)$ is feebly regular clopen in X. On the other hand if V is open in Y then Y-V is closed in Y. By hypothesis $f^{-1}(Y - V) = X - f^{-1}(V)$ is feebly regular clopen in X,
which implies $f^{-1}(V)$ is feebly regular clopen in X. Thus the inverse image of every open set in Y is feebly reglar clopen in X. Therefore, f is totally feebly regular continuous function.

Theorem 3.8: Every strongly feebly regular continuous function is totally feebly regular continuous function.
Proof: Suppose a function f from the topological space (X,τ) to (Y,σ) is strongly feebly regular continuous function. Let E be open in Y. By definition, $f^{-1}(E)$ is feebly regular clopen in X.
Therefore f is totally feebly regular continuous.

Remark 3.9:
The following implication gives the inter-relationship.

Strongly feebly regular continuous → Totally feebly regular continuous

Strongly feebly regular continuous → Feebly regular continuous

Totally feebly regular continuous → Feebly regular continuous

Theorem 3.10: Let a function f from the topological space (X,τ) to (Y,σ). The following are equivalent

(i) f is totally feebly regular continuous

(ii) for each point $p \in X$ and each open set M in Y with $f(p) \in M$, there is a feebly regular clopen set E in X such that $p \in E$ and $f(E) \subset M$.

Proof : (i)⇒(ii) Suppose a function f from the topological spaces (X,τ) to (Y,σ) is totally feebly regular continuous and M be any open set in Y containing f(x), so that $p \in f^{-1}(M)$.
Since f is totally feebly regular continuous, $f^{-1}(M)$ is feebly regular clopen in X. Let $E = f^{-1}(M)$. Then E is feebly regular clopen set in X and $p \in E$. Also $f(E) = f(f^{-1}(M)) \subset M$. This implies that $f(E) \subset M$.
(ii)⇒(i) Let M be open in Y. Let $p \in f^{-1}(M)$ be any arbitrary point. This implies that $f(p) \in M$. Therefore by (ii) there is a feebly regular clopen set $f(J_p) \subset X$ containing p such that $f(J_p) \subset M$, which implies $J_p \subset f^{-1}(M)$, we have $p \in J_p \subset f^{-1}(M)$. This implies that $f^{-1}(M)$ is feebly regular clopen of each of its points. Hence it is feebly reglar clopen set in X. Therefore, f is totally feebly regular continuous.

Theorem 3.11: If f : X $\rightarrow$Y is totally feebly regular continuous and A is feebly regular clopen subset of X, then the restriction f/A : A $\rightarrow$Y is totally feebly regular continuous .

Proof: Consider the function f/A : A$\rightarrow$Y and let V be any open set in Y. Since f is totally feebly regular continuous, $f^{-1}(V)$ is feebly regular clopen subset of X. Since A is feebly regular clopen subset of X and $(f/A)^{-1}(V) = A\cap f^{-1}(V)$ is feebly regular clopen in A, it follows $(f/A)^{-1}$ (V) is feebly regular clopen in A. Hence f/A is totally feebly regular continuous.

Definition 3.12: A map f : $(X,\tau)\rightarrow(Y,\sigma)$ is said to be feebly regular irresolute if $f^{-1}(V)$ is feebly regular open in (X,τ) for each feebly regular open set V of (Y,σ).

Definition 3.13: A function f from the domain (X,τ) into the co-domain (Y,σ) , where X and Y are topological spaces is said to be feebly regular clopen irresolute if $f^{-1}(V)$ is feebly regular clopen in (X,τ) for each feebly regular clopen set V of (Y,σ).

Theorem 3.14: If the functions f and g from X to Y and from Y to Z are slightly feebly regular continuous and totally feebly regular continuous respectively, then the function g$\circ$f from X to Z is feebly regular continuous.

Proof : Let O be a open set in Z. Since g is totally feebly regular continuous, $g^{-1}(O)$ is feebly regular clopen in Y. Now since f is slightly feebly regular continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is feebly regular open in X. Hence g$\circ$ f : X $\rightarrow$Z is feebly regular continuous.

Theorem 3.15: If the functions f and g from X to Y and from Y to Z are feebly regular irresolute function and slightly feebly regular continuous respectively, then the function g $\circ$ f from X to Z is slightly feebly regular continuous.

Proof: Let O be feebly regular clopen in Z. Since g is slightly feebly regular continuous, $g^{-1}(O)$ is feebly regular open in Y. Now since f is feebly regular irresolute function, $f^{-1}(g^{-1}(O)) =(g \circ f)^{-1}$ (O) is feebly regular open in X. Hence g $\circ$f : X $\rightarrow$Z is slightly feebly regular continuous.

Theorem 3.16: If the functions f and g from X to Y and from Y to Z are feebly regular clopen irresolute function and totally feebly regular continuous respectively, then the function g $\circ$ f from X to Z is totally feebly regular continuous.

Proof: Let O be open in Z. Since g is totally feebly regular continuous, $g^{-1}(O)$ is feebly regular clopen in Y. Now since f is feebly clopen regular irresolute function, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}$ (O) is feebly regular clopen in X. Hence g $\circ$f : X $\rightarrow$Z is totally feebly regular continuous.

Theorem 3.17: If the functions f and g from X to Y and from Y to Z are is slightly feebly regular continuous and feebly regular clopen irresolute function respectively, then the function g $\circ$ f from X to Z is slightly feebly regular continuous.

Proof: Let O be feebly regular clopen in Z. Since g is feebly regular clopen irresolute function, $g^{-1}(O)$ is feebly regular clopen in Y. Now since f is slightly feebly regular continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}$ (O) is feebly regular open in X. Hence g $\circ$ f : X $\rightarrow$Z is slightly feebly regular continuous.

Theorem 3.18: If the functions f and g from X to Y and from Y to Z are feebly regular irresolute function and feebly regular continuous respectively, then the function g $\circ$ f from X to Z is feebly regular continuous.

Proof: Let O be open in Z. Since g is feebly regular continuous, $g^{-1}(O)$ is feebly regular open in Y. Now since f is feebly regular irresolute function, $f^{-1}(g^{-1}(O))=(g \circ f)^{-1}(O)$ is feebly regular open in X. Hence g $\circ$f : X $\rightarrow$Z is feebly regular continuous.

Definition 3.19: A space X is said to be feebly regular clopen- T_1 if for each pair of distinct points p and q of X, there exist feebly regular clopen sets E and M containing p and q respectively such that q and p do not belong to E and M.

Definition 3.20: A space X is said to be feebly regular clopen -T_2 (feebly regular clopen Hausdorff) if for each pair of distinct points p and q in X, there exist disjoint feebly regular clopen sets E and M in X such that q and p belong to E and M.

Theorem 3.21: If a function f from X to Y is a slightly feebly regular continuous injection and Y is feebly regular clopen- T_1 then X is feebly regular - T_1.

Proof: Suppose that Y is feebly regular clopen- T_1. For any distinct points p and q in X., there exist feebly regular clopen set E and M such that f(p) ∈E, f(q) ∉E and f(p) ∉M, f(q)∈M. Since f is slightly feebly regular continuous, f^{-1}(E) and f^{-1}(M) are feebly regular open sets of X such that p∈f^{-1} (E), q∉f^{-1}(E) and p∉f^{-1}(M) , q∈f^{-1}(M). This shows that X is feebly regular - T_1.

Theorem 3.22: If a function f from X to Y is a slightly feebly regular continuous injection and Y is feebly regular clopen-T_2 then X is feebly regular-T_2.
Proof: For any pair of distinct points p and q in X, there exist disjoint feebly regular clopen sets E and M in Y such that f(p)∈E and f(q)∈M. Since f is slightly feebly regular continuous, f^{-1}(E) and f^{-1}(M) are feebly open in X containing p and q respectively. Therefore $f^{-1}(E)\cap f^{-1}(M)=\varphi$, because $E\cap M = \varphi$. This shows that X is feebly regular-T_2.

Definition 3.23: A map f : X→Y is said to be super feebly regular clopen continuous if for each x∈X and for each feebly regular clopen set V containing f(x) in Y, there exist a feebly regular open set U containing x such that f(U) ⊂ V.

Theorem 3.24 : Let (X,τ), (Y,σ) and (Z,η) be topological spaces. Then the composition g ∘f : (X,τ) → (Z,η) is super feebly regular clopen continuous function where f:(X,τ)→(Y,σ) is super feebly regular clopen continuous function and g:(Y,σ)→(Z,η) is feebly regular clopen irresolute function.
Proof : Let A be a feebly regular closed set of (X,τ). Since f is super feebly regular clopen continuous, f(A) is feebly regular clopen in (Y,σ). Then by hypothesis f(A) is feebly regular clopen set. Since g is feebly regular clopen irresolute, g(f(A)) = (g ∘f) (A). Therefore g ∘f is super feebly regular clopen continuous.

IV. CONCLUSION AND FUTURE WORK

Some characterizations are analyzed among these functions in General Topological Spaces. Further work will be done by applying these sets to Feebly Regular Compact Spaces and also in Soft Topological Space.

REFERENCES

[1] K. Ali, "*On semi separation axioms"*, M.Sc.Thesis, l.Mastaxiriyah Univ., **2003.**

[2] R.Buvaneswari and A.P.Dhana Balan, "*On Feebly Regular Set-Connected Function and Quasi Ultra Feebly Regular Open Map in Topological Space"*, Indian Streams Research Journal, Vol.**6**, Issue.**5**, **2016.**

[3] A.P.Dhana Balan and R.Buvaneswari, "*On almost feebly totally continuous functions in topological spaces"*, IJCSITR, Vol.**3**,pp. **274-279**, **2015**.

[4] N.Levine, "*Semi-open sets and semi-continuity in topological space"*, Amer Math.Monthly, Vol.**70**, pp.**36-41**, **1963**.

[5] S.N. Maheswari and P.C.Jain, "*Some new mappings"*, Mathematica,Vol.**24**, Issue.**47,** pp.**53-58**, **1982**.

[6] G.D.Maio and T. Noiri, "*On S-closed spaces"*, Indian J.Pure Appl. Math., Vol.**18**, Issue.**3**, pp.**226-233**, **1987**..

[7] V.Popa, Glasnik Math., Vol.**14**, Issue.**34**, pp. **359-3362**, **1979**.

AUTHOR PROFILE

Mrs. R. Buvaneswari is currently working as Assistant professor at Department of Mathematics in Cauvery College for Women, Trichirappalli ,Tamil Nadu, India. She has published 13 research papers in both reputed and international journals. Her areas of interest are General Topology and Soft Topology. She has 7 years of teaching experiences.

