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On Almost πGβ closed mapping and Contra πGβ closed mapping in Intuitionistic Fuzzy Topological Spaces

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Abstract— In this paper we introduce intuitionistic fuzzy almost pi generalized beta closed mappings intuitionistic fuzzy contra pi generalized beta continuous mappings and intuitionistic fuzzy almost contra pi generalized beta continuous mappings.

Keywords—Intuitionistic fuzzy almost πGβ continuous mappings, intuitionistic fuzzy contra πGβ continuous mappings, intuitionistic fuzzy almost contra πGβ continuous mappings and intuitionistic fuzzy contra πGβ closed mappings.

1. INTRODUCTION

Zadeh [13] introduced fuzzy sets in 1965, and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, the notion of intuitionistic fuzzy sets was introduced by
Atanassov [1] as a generalization of fuzzy sets. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In 2005, Young Bae Jun and Seok Zun Song [12] introduced Intuitionistic fuzzy beta continuous mappings in intuitionistic fuzzy topological spaces. S.Jothimani and T.Jenitha premalatha [7] introduced the notion of intuitionistic fuzzy πgeneralized beta closed mappings and intuitionistic fuzzy π generalized beta open mappings. In this paper we introduce intuitionistic fuzzy almost π generalized beta closed mappings, intuitionistic fuzzy contra π generalized β continuous mappings, and intuitionistic fuzzy almost contra πgeneralized β continuous mappings. We investigate some of their properties.

2. PRELIMINARIES

Definition 2.1: [1] An intuitionistic fuzzy set(IFS in short) A in X is an object having the form $A=\{<x, \mu_A(x), \nu_A(x)>/x\in X\}$ where the functions $\mu_A:X\rightarrow[0,1]$ and $\nu_A: X \rightarrow[0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x\in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{<x, \mu_A(x), \nu_A(x)>/ x\in X\}$ and $B = \{<x, \mu_B (x), \nu_B(x)> / x\in X\}$. Then

(a)$A \subseteq B$ if and only if $\mu_A (x) \leq \mu_B (x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x\in X$

(b)A = B if and only if $A \subseteq B$ and $B \subseteq A$

(c)$A^c = \{< x, \nu_A(x), \mu_A(x)>/ x \in X\}$

(d)$A \cap B = \{<x, \mu_A(x) \cap \mu_B(x), \nu_A(x) \cup \nu_B(x)> / x\in X\}$

(e)$A \cup B = \{<x, \mu_A(x) \cup \mu_B(x), \nu_A(x) \cap \nu_B(x)> / x \in X\}$

We shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [11] The IFS $p (\alpha, \beta) = < x, p_\alpha, p_{1-\beta} >$ where $\alpha \in(0, 1]$, $\beta \in[0, 1)$ and $\alpha + \beta \leq 1$ is called an intuitionistic fuzzy point (IFP for short) in X.

Definition 2.4: [6] Let p (α, β) be an IFP of an IFTS (X, τ). An IFS A of X is called an intuitionistic fuzzy neighborhood of p (α, β) if there exists an IFOS B in X such that p $(\alpha, \beta) \in B \subseteq A$.

Definition 2.5: [3] An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \in \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ are known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^C of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.6: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by $int(A) = \cup\{ G/G$ is an IFOS in X and $G \subseteq A\}$ and $cl(A) = \cap\{ K/K$ is an IFCS in X and $A \subseteq K \}$. Note that for any IFS A in (X, τ), we have $cl(A^C) = [int(A)]^C$ and $int(A^C) = [cl(A)]^C$ [11].

(i)

(ii)

(iii)

Definition 2.7: [9] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) Intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$

(ii) Intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A)) \subseteq A$

(iii) intuitionistic fuzzy α closed set (IFαCS in short) if $cl(int(cl(A)) \subseteq A$. These respective complements of the above IFCS s are called their respective IFOSs. The family of all IFSCSs, IFPCSs, and IFαCSs (respectively IFSOSs, IFPOSs and IFαOSs) of an IFTS(X, τ) are respectively denoted by IFSC(X), IFPC(X) and IFαC(X) (respectively IFSO(X), IFPO(X) and IFαO(X)).

Definition 2.8: [6] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an intuitionistic fuzzy beta closed set (IFβCS in short) if $int(cl(int(A))) \subseteq A$.

Definition 2.9:[10] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized beta closed set (IFGβCS for short) if $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Definition 2.10: [7] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy π generalized beta closed set (IFGβCS for short) if $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFπOS in (X, τ).The family of all IFπGβCSs of an IFTS (X, τ) is denoted by IFπGβC(X).

Definition 2.11: [6] Let A be an IFS in an IFTS (X, τ). Then $\beta int(A) = \cup\{G / G$ is an IFβOS in X and $G \subseteq A\}$. $\beta cl(A) = \cap \{K / K$ is an IFβCS in X and $A \subseteq K\}$. Note that for any IFS A in (X, τ), we have $\beta cl(A^C) = (\beta int(A))^C$ and $\beta int(A^C) = (\beta cl(A))^C$ [7].

Definition 2.12: [7] The complement A^C of IFπGβCS in an IFTS (X, τ) is called an IFπGβOS in X.

Definition 2.13: [6] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ).Then f is said to be an intuitionistic fuzzy closed mapping ((IFCM) for short) if f (A) is IFC(X) in Y, for each ICS B in X.

Definition 2.14: [9] Let a mapping f: $(X, \tau) \to (Y, \sigma)$ Then f is said to be an

(i) Intuitionistic fuzzy semi continuous mapping if $f^{-1}(B) \in$ IFSO(X) for every $B \in \sigma$.

(ii) Intuitionistic fuzzy α-continuous mapping if $f^{-1}(B) \in$ IFαO(X) for every $B \in \sigma$.

(iii) Intuitionistic fuzzy pre continuous mapping if $f^{-1}(B) \in$ IFPO(X) for every $B \in \sigma$.

(iv) Intuitionistic fuzzy β continuous mapping if $f^{-1}(B) \in$ IFβO(X) for every $B \in \sigma$.

Definition 2.15: [10] Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping. Then f is said to be an intuitionistic fuzzy generalized βcontinuous mapping (IFGβCM) if $f^{-1}(B) \in$ IFGβC in X for every IFCS B in Y.

Definition 2.16: [7] Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping. Then f is said to be an intuitionistic fuzzy π generalized β continuous mapping (IFπGβCM) if $f^{-1}(B) \in$ IFπGβC in X for every IFCS B in Y.

Definition 2.17: [12] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy almost continuous (IFA continuous in short) if $f^{-1}(B) \in$ IFC(X) for every IFRCS B in Y.

Definition 2.18: [11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy almost πGβ continuous (IFAπGβ continuous in short) if $f^{-1}(B) \in$ IFπGβC(X) for every IFRCS B in Y.

Definition 2.19: [6] Let c (α, β) be an IFP in (X, τ). An IFSA of X is called an intuitionistic fuzzy beta neighborhood (IFβN for short) of c (α, β) if there is an IFβOS B in X such that c $(\alpha, \beta) \in B \subseteq A$.

Definition 2.20: [7] A mapping f: X→Y is said to be an intuitionistic fuzzy π generalized beta closed mapping (IFπGβCM, for short) if f (A) is an IFπGβCS in Y for every IFCS A in X.

Definition 2.21: [7] A mapping f: X →Y is said to be an intuitionistic fuzzy M π generalized beta closed mapping (IFMπGβCM, for short) if f (A) is an IFπGβCS in Y for every IFπGβCS A in X.

Definition 2.22: [8] A mapping f: X →Y is said to be an intuitionistic fuzzy almost πgeneralized beta continuous mapping (IFAπGβCM, for short) if $f^{-1}(A)$ is an IFπGβCS in X for every IFRCS A in Y.

Definition 2.23: [5] Two IFSs A and B are said to be q coincident (AqB in short) if and only if there exists and element x ∈X such that μA(x) > νB(x) or νA(x) < μB(x).

Definition 2.24: [4] A mapping f: (X, τ) → (Y, σ) is called an

i) intuitionistic fuzzy contra continuous if $f^{-1}(B)$ is an IFCS in X for every IFOS B in Y [4]

ii) intuitionistic fuzzy contra beta continuous if $f^{-1}(B)$ is an IFβCS in X for every IFOS B in Y.[4]

iii) intuitionistic fuzzy contra πGβ continuous if $f^{-1}(B)$ is an IFπGβCS in X for every IFOS B in Y.[8]

3. INTUITIONISTIC FUZZY ALMOST πGβ CLOSED MAPPINGS

In this section we have introduced intuitionistic fuzzy almost πGβ open mappings. We have investigated some of its properties.

Definition 3.1: A map f: X→Y is called an intuitionistic fuzzy almost πgeneralized beta closed mapping (IFAπGβCM for short) if f (A) is an IFπGβCS in Y for each IFRCS A in X.

Example 3.2: Let X ={a,b},Y={u,v} and G_1 = ⟨x,(0.4a,0.3b),(0.5a, 0.6b)⟩, G_2 = ⟨y,(0.2u,0.3v),(0.8u, 0.7v)⟩.Then τ ={0~, G_1, 1~}and σ = { 0~, G_2, 1~} are IFTs on X and Y respectively. Define a mapping f: (X, τ) →(Y,σ) by f (a) = u and f (b) = v. Then f is an IFAπGβCM.

Definition 3.3: A map f: X→Y is called an intuitionistic fuzzy almost π generalized β open mapping (IFAπGβ OM for short) if f (A) is an IFπGβOS in Y for each IFROS A in X.

Theorem 3.4: Every IFCM is an IFAπGβCM but not conversely.

Proof: Let f: X→Y be an IFCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. Then f (A) is an IFCS in Y. Since every IFCS is an IFπGβCS, f (A) is an IFπGβCS in Y. Hence f is an IFAπGβ CM.

Example 3.5: In Example 3.2 f is an IFAπGβ CM but not an IFCM since G_1^c = ⟨x,(0.5a, 0.6b), (0.4a, 0.3b)⟩ is an IFCS in X but $f(G_1^c)$ = ⟨y,(0.5u,0.6v), (0.4u,0.3v)⟩ is not an IFCS in Y, since $cl(f(G_1^c)) = G_2^c \not\subset f(G_1^c)$.

Theorem 3.6: Every IFSCM is an IFAπGβCM but not conversely.

Proof: Let f: X→Y be an IFSCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. Then f (A) is an IFSCS in Y. Since every IFSCS is an IFπGβCS, f (A) is an IFπGβCS in Y. Hence f is an IFAπGβCM.

Example 3.7: Let X ={a,b},Y={u,v} and G_1 = ⟨x,(0.4a,0.3b), (0.5a, 0.6b)⟩, G_2 = ⟨y,(0.5u,0.4v), (0.2u, 0.3v)⟩. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are IFTs on X and Y respectively. Define a mapping f: (X,τ)→(Y,σ) by f(a) = u and f(b) = v. Then f is an IFAπGβCM but not an IFSCM, since G_1^c = ⟨x,(0.5a,0.6b), (0.4u,0.3b)⟩ is an IFCS in X but $f(G_1^c)$ = ⟨y,(0.5u, 0.6v), (0.4u,0.3v)⟩ is not an IFSCS in Y, since $int(cl(f(G_1^c))) = 1\sim \not\subset f(G_1^c)$.

Theorem 3.8: Every IFαCM is an IFAπGβCM but not conversely.

Proof: Let f: X→Y be an IFαCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. Then f (A) is an IFαCS in Y. Since every IFαCS is an IFπGβCS, f (A) is an IFπGβCS in Y. Hence f is an IFAπGβCM.

Example 3.9: In Example 3.2, f is an IFAπGβCM but not an IFαCM since G_1^c = ⟨x,(0.5a,0.6b), (0.4u,0.3b)⟩ is an

IFCS in X, but f($G1^C$) = ⟨y,(0.5u,0.6v), (0.4u, 0.3v) ⟩ is not an IFαCS in Y, since cl(int(f($G1^C$))) = 1 ⊄ f($G1^C$).

Theorem 3.10: Every IFPCM is an IFAπGβCM but not conversely.

Proof: Let f: X →Y be an IFPCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X Then f (A) is an IFPCS in Y. Since every IFPCS is an IFπGβCS, f (A) is an IFπGβCS in Y. Hence f is an IFAπGβCM.

Example 3.11: In Example 3.2 f is an IFAπGβCM but not an IFPCM, since $G1^c$ = ⟨x, (0.5a, 0.6b), (0.4a, 0.3b)⟩ is an IFCS in X but f($G1^c$) = ⟨y, (0.5u, 0.6v), (0.4u, 0.3v)⟩is not an IFPCS in Y, since cl(int(f($G1^c$))) = $G2^c$ ⊄f($G1^c$) .

Theorem 3.12: Every IFGβCM is an IFAπGβCM but not conversely.

Proof: Let f: X→Y be an IFGβCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X Then f (A) is an IFπGβCS in Y. Hence f is an IFAπGβCM.

Example3.13: Let X ={a ,b},Y={u, v} and G_1 = ⟨x,(0.1a,0.1b), (0.4a,0.4b)⟩, G_2 = ⟨x,(0.2a,0b), (0.5a,0.5b)⟩, G_3 = ⟨y,(0.5u,0.6v), (0.2u,0v)⟩ and G_4 = ⟨y,(0.4u,0.1v), (0.2u,0.1v)⟩. Then τ = {0~,G_1,G_2,1~}and σ = {0~,G_3,G_4,1~}are IFTs on X and Y respectively. Define a mapping f: (X, τ) →(Y,σ) by f (a) = u and f (b) = v. Then f is an IFAπGβM but not an IFGβCM , since G_2^C = ⟨x,(0.5a,0.5b), (0.2a, 0b)⟩is an IFCS in X but f(G_2^C) = ⟨y,(0.5u,0.5v), (0.2u, 0v)⟩ is not an IFπGβCS in Y, since f(G_2^C) ⊆ G_3 but βcl(f(G_2^C) = 1~⊄G_3.

Theorem 3.14: Every IFACM is an IFAπGβCM but not conversely.

Proof: Let f: (X, τ) →(Y,σ) be an IFACM. Let A be an IFRCS in X. Since f is IFACM, f (A) is an IFCS in Y. Since every IFCS is an IFπGβCS, f (A) is an IFπGβCS in Y .Hence f is an IFAπGβCM.

Theorem 3.15: Let f: X →Y be a mapping. Then the following are equivalent
(i) f is an IFAπGβOM
(ii) f is an IFAπGβCM.

Proof: Straightforward

Theorem3.16 A bijective mapping f: X →Y is an IFAπGβ closed mapping if and only if the image of each IFROS in X is an IFπGβOS in Y.

Proof Necessity: Let A be an IFROS in X. This implies A^C is IFRCS in X. Since f is an IFAπGβ closed mapping, f (A^C) is an IFπGβCS in Y .Since f (A^C) = $(f (A))^C$, f (A) is an IFπGβOS in Y.

Sufficiency: Let A be an IFRCS in X. This implies A^C is an IFROS in X. By hypothesis, f (A^C) is an IFπGβOS in Y. Since f (A^C) = $(f (A))^C$, f (A) is an IFπGβCS in Y. Hence f is an IFAπGβ closed mapping.

Theorem3.17 Let f :(X, τ) →(Y,σ) be an IFAπGβ closed mapping. Then f is an IFA closed mapping if Y is an IFπβ$T_{1/2}$space.

Proof: Let A be an IFRCS in X. Then f (A) is an IFπGβCS in Y, by hypothesis. Since Y is an IFπβ$T_{1/2}$ space, f (A) is an IFCS in Y. Hence f is an IFA closed mapping.

Theorem3.18: Let f: X→Y be a mapping where Y is an IFπβ$T_{1/2}$ space .Then the following are equivalent:
(i) f is an IFAπGβCM
(ii) βcl(f(A))⊆f(cl(A))for every IFβOS A in X
(iii)βcl(f(A))⊆f(cl(A))for every IFSOS A in X.
(iv) f (A)⊆βint(f(int(cl(A)))) for every IFPOS A in X.

Proof: (i)⇒(ii) Let A be an IFβOS in X .Then cl(A) is an IFRCS in X. By hypothesis f (A) is an IFπGβCS in Y and hence is an IFβCS in Y, since Y is an IFπβT1/2space.This implies βcl(f(cl(A))) = f(cl(A)).
Now βcl(f(A)) ⊆βcl(f(cl(A)))= f(cl(A)).Thus βcl(f(A))⊆f(cl(A)).
(ii)⇒(iii) Since every IFSOS is an IFβOS ,the proof directly follows.
(iii)⇒(i) Let A be an IFRCS in X. Then A = cl(int(A)). Therefore A is an IFSOS in X. By hypothesis, βcl(f(A)) ⊆ f(cl(A)) = f(A) ⊆ βcl(f(A)). Hence f (A) is an IFβCS and

hence is an IFπGβCS in Y. Thus f is an IFAπGβCM.

(i)⇒(iv)Let A be an IFPOS in X. Then A⊆int(cl(A)).Since int(cl(A)) is an IFROS in X, by hypothesis, f(int(cl(A))) is an IFπGβOS in Y. Since Y is an IFπβT1/2 space , f(int(cl(A))) is an IFβOS in Y. Therefore f(A)⊆f(int(cl(A)))⊆βint(f(int(cl(A)))).

(iv)⇒(i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis, f(A)⊆βint(f(int(cl(A)))) =βint(f(A))⊆f(A). This implies f(A) is an IFβOS in Y and hence is an IFπGβOS in Y. Therefore f is an IFAπGβCM.

Theorem 3.19: Let f: X →Y be a mapping. Then f is an IFAπGβCM if for each IFP c(α,β)∈Y and for each IFβOS B in X such that f^{-1}(c(α,β)∈B, βcl(f(B)) is an IFβN of c(α,β)∈Y.

Proof: Let c(α,β)∈Y and let A be an IFROS in X. Then A is an IFβOS in X. By hypothesis f^{-1}(c(α,β))∈A ,that is c(α,β)∈f(A) in Y and βcl((f(A)) is an IFβN of c(α, β) in Y. Therefore there exists an IFβOSB in Y such that c(α,β))∈B ⊆ βcl(f(A)).We have c(α,β)∈f(A)⊆βcl(f(A)).Now B=∪{c(α,β)/c(α,β)∈B}= f(A).

Therefore f(A) is an IFβOS in Y and hence an IFπGβOS in Y Thus f is an IFAπGβOM.

Hence by Theorem 3.15 f is an IFAπGβCM.

Theorem 3.20: Let f :X→Y be a mapping. If f is an IFAπGβCM then πGβcl(f(A)⊆f(cl(A)) for every IFβOS A in X.

Proof: Let A be an IFβOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(cl(A)) is an IFπGβCS in Y. Then πGβcl(f(cl(A)) = f(cl(A)). Now πGβcl(f(A))⊆πgβcl(f(cl(A)))⊆f(cl(A)).That is πGβcl(f(A)⊆f(cl(A)).

Corollary 3.21: Let f : X→Y be a mapping. If f is an IFAπGβCM, then πGβcl(f(A)⊆f(cl(A))for every IFSOS A in X.

Proof: Since every IFSOS is an IFβOS, the proof directly follows from the Theorem 3.20

Corollary3.22: Let f : X→Y be a mapping. If f is an IFAπGβCM, then πGβcl(f(A)⊆fcl(A)) for every IFPOS A in X.

Proof: Since every IFPOS is an IFβOS, and hence πGβOS, the proof directly follows from the Theorem 3.20.

Theorem 3.23: Let f: X →Y be a mapping. If f is an IFAπGβCM, then πGβcl(f(A))⊆f(cl(βint(A))) for every IFβOS A in X.

Proof: Let A be an IFβOS in X. Then cl(A) is an IFRCS in X. By hypothesis , f(cl(A)) is an IFπGβCS in Y. Then πGβcl(f(A)) ⊆πgβcl(f(cl(A)))= f(cl(A)) ⊆f(cl(βint(A))), since βint(A) = A.

Corollary3.24: Let f :X→Y be a mapping. If f is an IFAπGβCM, then πGβcl(f(A))⊆f(cl(βint(A))) for every IFSOS A in X.

Proof: Since every IFSOS is an IFβOS, the proof directly follows from the Theorem 3.23.

Corollary3.25: Let f :X→Y be a mapping. If f is an IFAπGβCM, then πGβcl(f(cl(A))) ⊆ f(cl(βint(A))) for every IFPOS A in X.

Proof: Since every IFPOS is an IFβOS, the proof directly follows from the Theorem 3.23.

Theorem 3.26: Let f: X →Y be a mapping. If f(βint(B))⊆βint(f(B)) for every IFSB in X, then f is an IFAπGβOM.

Proof: Let B ⊆ X be an IFROS. By hypothesis, f(βint(B)) ⊆ βint(f(B)).Since B is an IFROS, it is an IFβOS in X. Therefore βint(B)=B. Hence f(B)=f(βint(B)) ⊆βint(f(B)) ⊆f(B). This implies f(B) is an IFβOS and hence an IFπGβOS in Y. Thus f is an IFAπGβOM.

Theorem 3.27: Let f : X →Y be a mapping. If βcl(f(B))⊆f(βcl(B)) for every IFSB in X, then f is an IFAπGβCM.

Proof: Let B ⊆ X be an IFRCS. By hypothesis, βcl(f(B)) ⊆ f(βcl(B)). Since B is an IFRCS ,it is an IFβCS in X. Therefore βcl(B)=B. Hence f(B) = f(βcl(B)) ⊇βcl(f(B)) ⊇f(B). This implies f(B) is an IFβCS and hence an IFπGβCS in Y. Thus f is an IFAπGβCM.

Theorem 3.28: Let f :X →Y be a mapping where Y is an IFπβ$T_{1/2}$ space Then the following are equivalent.

(i) f is an IFAπGβOM

(ii) for each IFPc(α,β) in Y and each IFROSB in X such that f^{-1}(c(α,β))∈B, cl(f(cl(B))) is an IFβN of c(α,β) in Y.

Proof: (i)⇒(ii) Let c(α,β)∈Y and let B be an IFROS in X such that f^{-1}(c(α, β))∈B. That is c(α,β) ∈f(B).
By hypothesis f(B) is an IFπGβOS in Y. Since Y is an IFπβ$T_{1/2}$ space, f(B) is an IFβOS in Y.
Now c(α,β)∈f(B) ⊆f(cl(B)) ⊆cl(f(cl(B))). Hence cl(f(cl(B))) is an IFβN of c(α,β) in Y.
(ii)⇒(i) Let B be an IFROS in X . Then f^{-1}(c(α,β))∈B. This implies c(α,β)∈f(B). By hypothesis, cl(f(cl(B))) is an IFβN of c(α,β). Therefore there exists an IFβOS A in Y such that c(α,β)∈A ⊆ cl(f(cl(B))).
Now A= ∪{c(α ,β)/c(α, β)∈A} =f(B). Therefore f(B) is an IFβOS and hence an IFπGβOS in Y .
Thus f is an IFAπGβOM.

Theorem 3.29: The following are equivalent for a mapping f : X → Y where Y is an IFπβ$T_{1/2}$ space

(i) f is an IFAπGβCM

(ii) βcl(f(A)) ⊆ f(αcl(A)) for every IFβOS A in X

(iii)βcl(f(A)) ⊆ f(αcl(A))for every IFSOS A

(iv) f(A) ⊆ βint(f(scl(A))) for every IFPOS A in X.

Proof:(i)⇒(ii) Let A be an IFβOS in X .Then cl(A) is an IFRCS in X. By hypothesis f(A) is an IFπGβCS in Y and hence is an IFβCS in Y, since Y is an IFπβ$T_{1/2}$ space .This implies βcl(f(cl(A)))=f(cl(A)).
Now βcl(f(A)) ⊆ βcl(f(cl(A))) = f(cl(A)). Since cl(A) is an IFRCS, cl(int(cl(A)))= cl(A).
Therefore βcl(f(A)) ⊆ f(cl(A))=(cl(int(cl(A)))) ⊆ f(A∪cl(int(cl(A)))) ⊆ f(αcl(A)).
Hence βcl(f(A)) ⊆ f(αcl(A)).
(ii)⇒(iii) Let A be an IFSOS in X. Since every IFSOS is an IF□OS, the proof is obvious.
(iii)⇒(i)Let A be an IFRCS in X. Then A =cl(int(A)). Therefore A is an IFSOS in X. By hypothesis,
βcl(f(A)) ⊆ f(αcl(A)) ⊆ f(cl(A))=f(A)⊆ βcl(f(A)).That is βcl(f(A))=f(A).
Hence f(A) is an IFβCS and hence is an IFπGβCS in Y. Thus f is an IFAπGβCM.
(i)⇒(iv) Let A be an IFPOS in X. Then A ⊆ int(cl(A)).Since int(cl(A)) is an IFROS in X, by hypothesis f(int(cl(A))) is an IFπGβOS in Y. Since Y is an IFπβ$T_{1/2}$ space, f(int(cl(A))) is an IFβOS inY.Therefore f(A) ⊆ f(int(cl(A))) ⊆ βint(f(int(cl(A)))) =βint(f(A∪int(cl(A))))
=βint(f(scl(A))). That is f(A)⊆βint(f(scl(A))).
(iv)⇒(i)Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis, f(A) ⊆ βint(f(scl(A))). This implies f(A) ⊆ βint (f(A∪int(cl(A)))) ⊆ βint(f(A∪A))=βint(f(A)) ⊆ f(A).Therefore f(A) is an IFβOS in Y and hence an IFπGβOS in Y .Thus f is an IFAπGβCM by Theorem 3.13

Theorem 3.30: Let f : X→Y be a mapping where Y is an IFπβ$T_{1/2}$ space. If f is an IFAπGβCM, then int(cl(int(f(B))))⊆ f(βcl(B)) for every IFRCS B in X.

Proof: Let B⊆X be an IFRCS. By hypothesis, f(B) is an IFπGβCS in Y. Since Y is an IFπβ$T_{1/2}$ space, f(B) is an IFβCS in Y. Therefore βcl(f(B))=f(B). Now int(cl(int(f(B)))) ⊆f(B)∪int(cl(int(f(B)))) =βcl(f(B))=f(B)= f(βcl(B)).
Hence int(cl(int(f(B))))⊆f(βcl(B)).

Theorem 3.31 Let f: XπY be a mapping where Y is an IFπβ$T_{1/2}$ space. If f is an IFAπGβCM, then f(βint(B))⊆cl(int(cl(f(B)))) for every IFROS B in X.

Proof: This theorem can be easily proved by taking complement in Theorem 3.30

Theorem3.32: Let f : (X ,τ) →(Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an IFπβ$T_{1/2}$ space.

(i) f is an IFAπGβCM

(ii) f is an IFAπGβOM

(iii) f(int(A)) ⊆ int(cl(int(f(A)))) for every IFROS A in X.

Proof : (i) ⇒ (ii) It is obviously true.
(ii) ⇒ (iii) Let A be any IFROS in X. This implies A is an IFOS in X. Then int(A) is an IFOS in X. Then f(int(A)) is an IFπGβOS in Y. Since Y is an IFπβ$T_{1/2}$ space , f(int(A)) is an IFOS in Y.
Therefore f(int(A)) = int (f(int(A)) ⊆ int (cl(int (f(A)))).
(iii) ⇒ (i) Let A be an IFRCS in X. Then its complement A^c is an IFROS in X. By hypothesis
f(int(A^c))⊆ int(cl(int(f(A^c)))).This implies f(A^c) ⊆ int(cl(int(f(A^c)))). Hence f(A^c) is an IFαOS in Y.
Since every IFαOS is an IFπGβOS ,f(A^c) is an IFπGβOS in Y. Therefore f(A) is an IFπGβCS in Y. Hence f is an IFAπGβCM.

Theorem 3.33 Let f : (X, τ) → (Y, σ) be an IFA closed mapping and g : (Y, σ) → (Z, δ) is IFAπGβ closed mapping, then $g_o f$: (X, τ) →(Z, δ) is an IFA closed mapping. if Z is an IFπβT1/2 space.

Proof: Let A be an IFRCS in X. Then f(A) is an IFCS in Y. Since g is an IFπGβ closed mapping, g(f(A)) is an IFπGβCS in Z. Therefore g(f(A)) is an IFCS in Z, by hypothesis. Hence g□f is an IFA closed mapping.

Theorem 3.34 Let f : (X, τ) → (Y, σ) be an IFA closed mapping and g : (Y, σ) → (Z, η) be an IFπGβ closed mapping. Then gof : (X, τ) → (Z, η) is an IFAπGβ closed mapping.

Proof: Let A be an IFRCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since g is an IFπGβ closed mapping, g(f(A)) is an IFπGβCS in Z. Hence gof is an IFAπGβ closed mapping.

Theorem 3.35 If f : (X, τ) → (Y, σ) is an IFAπGβ closed mapping and Y is an IFπβT1/2 space, then f(A) is an IFGCS in Y for every IFRCS A in X .

Proof: Let f : (X, τ) → (Y, σ) be a mapping and let A be an IFRCS in X. Then by hypothesis f(A) is an IFπGβCS in Y. Since Y is an IFπβT1/2 space, f(A) is an IFGCS in Y.

Theorem 3.36 Let f: X→ Y be a bijective mapping. Then the following are equivalent.
(i) f is an IFAπGβOM. (ii) f is an IFAπGβCM.
(iii) f^{-1} is an IFAπGβ continuous mapping.

Proof : (i) ⇒ (ii) is obvious from the Theorem 3.15.
(ii) ⇒(iii) Let A ⊆ X be an IFRCS. Then by hypothesis, f(A) is an IFπGβCS in Y. That is $(f^{-1})^{-1}(A)$ is an IFπGβCS in Y. This implies f^{-1} is an IFAπGβ continuous mapping.
(iii) ⇒ (ii) Let A ⊆ X be an IFRCS. Then by hypothesis $(f^{-1})^{-1}(A)$ is an IFπGβCS in Y. That is f(A) is an IFπGβCS in Y. Hence f is an IFAπGβCM.

Theorem 3.37 Let f : X → Y be an IFAπGβOM , where Y is an IFπβT1/2 space. Then for each IFP c(α, β) in Y and each IFROS B in X such that
f^{-1}(c(α, β)) ∈B, cl(f(cl(B))) is an IFβN of c(α, β) in Y.

Proof: Let c(α, β) ∈ Y and let B be an IFROS in X such that f^{-1}(c(α, β)) ∈ B. That is c(α, β) ∈ f(B). By hypothesis, f(B) is an IFπGβOS in Y. Since Y is an IFπβT1/2 space, f(B) is an IFβOS in Y.
Now c(α, β) ∈ f(B) ⊆ f(cl(B)) ⊆ cl(f(cl(B))). Hence cl(f(cl(B))) is an IFβN of c(α, β) in Y.

Remark 3.38 If an IFS A in an IFTS (X, τ) is an IFπGβCS in X, then πgβcl(A) = A. But the converse may not be true in general, since the intersection does not exist in IFπGβCSs.

Remark 3.39 If an IFS A in an IFTS (X, τ) is an IFπGβOS in X, then πgβint(A) = A. But the converse may not be true in general, since the union does not exist in IFπGβOSs.

Theorem 3.40 Let f: X → Y be a mapping. If f is an IFAπGβCM, then πgβcl(f(A)) ⊆ f(cl(A)) for every IFβOS A in X.

Proof: Let A be an IFβOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(cl(A)) is an IFπGβCS in Y. Then πgβcl(f(cl(A)) = f(cl(A)). Now πgβcl(f(A)) ⊆ gβcl(f(cl(A)))=f(cl(A)).
That is πgβcl(f(A)) ⊆ f(cl(A)).

Corollary 3.41 Let f: X → Y be a mapping. If f is an IFAπGβCM, then πgβcl(f(A) ⊆ f(cl(A)) for every IFπGβOS A in X.

Proof: Since every IFSOS is an IFGβOS, the proof is obvious from the Theorem 3.40.

Corollary 3.42 Let f: X → Y be a mapping. If f is an IFAπGβCM, then πgβcl(f(A) ⊆f(cl(A)) for every IFGOS A in X.

Proof: Since every IFGOS is an IFπGβOS, the proof is obvious from the Theorem 3.38.

Theorem 3.43 Let f: X → Y be a mapping. If f is an IFAπGβCM, then πgβcl(f(A)) ⊆f(cl(βint(A))) for every IFβOS A in X.

Proof: Let A be an IFβOS in X. Then cl(A) is an IFRCS in X. By hypothesis, f(cl(A)) is an IFπGβCS in Y.
Then πgβcl(f(A)) ⊆ πgβcl(f(cl(A))) = f(cl(A)) ⊆ f(cl(βint(A))), since βint(A) = A.

4: INTUITIONISTIC FUZZY CONTRA πGβ OPEN MAPPINGS

In this section we have introduced intuitionistic fuzzy contra πGβ open mappings. We have investigated some of its properties.

Definition 4.1: A mapping f : (X, τ) → (Y, σ) is said to be an intuitionistic fuzzy contra π generalized beta open mapping (IFCπGβOM for short) if f(A) is an IFπGβCS in Y for every IFOS A in X.

Example 4.2: Let X = {a, b}, Y = {u, v} and G1 = ⟨x, (0.3, 0.1), (0.6, 0.7) ⟩,G2 = ⟨y, (0.5, 0.4), (0.5, 0.6) ⟩. Then τ = {0~, G1, 1~} and σ = {0~, G2, 1~} are IFTs on X and Y respectively. Define a mapping f : (X, τ) →(Y, σ) by f(a) = u and f(b) = v. Then f is an IFCπGβOM.

Definition 4.3: A mapping f : (X, τ) → (Y, σ) is called an intuitionistic fuzzy contra π generalized beta closed mapping (IFCπGβ closed in short) if for every IFCS A of (X, τ), f(A) is an IFπGβOS in (Y,σ).

Theorem 4.4: For a bijective mapping f : (X, τ) → (Y, σ), where Y is an IFπβT1/2 space, the following statements are equivalent:

(i) f is an IFCπgβOM.

(ii) for every IFCS A in X, f(A) is an IFπGβOS in Y

(iii) for every IFOS B in X, f(B) is an IFπGβCS in Y .

(iv) for any IFCS A in X and for any IFP p(α, β)∈ Y, if f^{-1}(p(α, β)) q A, then p(α, β) q βint(f(A))

(v) For any IFCS A in X and for any p(α, β) ∈ Y, if f^{-1}(p(α, β)) qA, then there exists an IFπGβOS B such that p(α, β)qB and f^{-1}(B) ⊆ A.

Proof: (i) ⇒ (ii) Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis, f(A^c) is an IFπGβCS in Y. That is $f(A)^c$ is an IFπGβCS in Y. Hence f(A) is an IFπgβOS in Y.

(ii) ⇒ (i) Let A be an IFOS in X. Then A^c is an IFCS in X. By hypothesis, f(A^c) = $(f(A))^c$ is an IFπGβOS in Y. Hence f(A) is an IFπGβCS in Y. Thus f is an IFCπGβOM.

(ii) ⇒ (iii) is obvious.

(ii) ⇒ (iv) Let A ⊆ X be an IFCS and let p(α, β)∈Y. Assume that f^{-1}(p(α, β)) q A. Then p(α, β) q f(A). By hypothesis, f(A) is an IFπGβOS in Y. Since Y is an IFβT1/2 space, f(A) is an IFβOS in Y. This implies βint(f(A)) = f(A).Hence p(α, β) qβint(f(A)).

(iv) ⇒ (ii) Let A ⊆ X be an IFCS and let p(α, β) ∈ Y. Assume that f^{-1}(p(α, β)) q A. Then p(α, β) q f(A). By hypothesis p(α, β) qβint(f(A)). That is f(A) ⊆βint(f(A)) ⊆f(A).Therefore f(A) = βint(f(A)) is an IFβOS in Y and hence an IFπGβOS in Y.

(iv) ⇒ (v) Let A ⊆ X be an IFCS and let p(α, β) ∈ Y. Assume that f^{-1}(p(α, β)) q A. Then p(α, β) q f(A). This implies p(α, β) q βint(f(A)). Thus f(A) is an IFβOS in Y and hence an IFπGβOS in Y. Let f(A) = B.

Therefore p(α, β)q B and f^{-1}(B) = f^{-1}(f(A)) ⊆ A.

(v) ⇒ (iv) Let A ⊆X be an IFCS and let p(α, β) ∈Y. Assume that f^{-1}(p(α, β)) q A. Then p(α, β) q f(A). By hypothesis there exists an IFπGβOS B in Y such that p(α, β) q B and f^{-1}(B) ⊆ A. Let B = f(A).

Then p(α, β) q f(A). Since Y is an IFβT1/2 space, f(A) is an IFβOS in Y. Therefore p(α, β) qβint(f(A)).

Theorem 4.5: Let f : (X, τ) → (Y, σ) be a bijective mapping. Suppose that one of the following properties hold:

(i) f(cl(B)) ⊆ int(βcl(f(B))) for each IFS B in X

(ii) cl(βint(f(B))) ⊆ f(int(B)) for each IFS B in X

(iii) f^{-1}(cl(βint(A))) ⊆ int(f^{-1}(A)) for each IFS A in Y

(iv)f^{-1}(cl(A)) ⊆ int(f^{-1}(A)) for each IFβOS A in Y

Then f is an IFCπGβOM.

Proof: (i) ⇒ (ii) is obvious by taking the complement in (i).

(ii) ⇒ (iii) Let A ⊆ Y. Put B = f^{-1}(A) in X. This implies A = f(B) in Y.

Now cl(βint(A)) = cl(βint(f(B))) ⊆ f(int(B)) by (ii).

Therefore f^{-1}(cl(βint(A))) ⊆ f^{-1}(f(int(B))) = int(B) = int(f^{-1}(A)).

(iii) ⇒ (iv) Let A ⊆ Y be an IFβOS . Then βint(A) = A. By hypothesis, f^{-1}(cl(βint(A))) ⊆ int(f^{-1}(A)).

Therefore f^{-1}(cl(A)) ⊆ int(f^{-1}(A)).

Suppose (iv) holds: Let A be an IFOS in X. Then f(A) is an IFS in Y and βint(f(A)) is an IFβOS in Y. Hence by hypothesis, we have f^{-1}(cl(βint(f(A)))) ⊆ int(f^{-1}(βint(f(A)))) ⊆ int(f^{-1}(f(A))) = int(A) ⊆ A.

Therefore cl(βint(f(A))) = f(f^{-1}(cl(βint(f(A))))) ⊆f(A). Now cl(int(f(A))) ⊆ cl(βint(f(A))) ⊆ f(A).

This implies f(A) is an IFPCS in Y and hence an IFπGβCS in Y. Thus f is an IFCπGβOM.

Theorem 4.6: Let f : (X, τ) → (Y, σ) be a bijective mapping. Suppose that one of the following properties hold:

(i) $f^{-1}(\beta cl(A)) \subseteq int(f^{-1}(A))$ for each IFS A in Y
(ii) $\beta cl(f(B)) \subseteq f(int(B))$ for each IFS B in X
(iii) $f(cl(B)) \subseteq \beta int(f(B))$ for each IFS B in X Then f is an IFCπGβOM.

Proof: (i) ⇒ (ii) Let $B \subseteq X$. Then f(B) is an IFS in Y. By hypothesis, $f^{-1}(\beta cl(f(B))) \subseteq int(f^{-1}(f(B))) = int(B)$.
Now $\beta cl(f(B)) = f(f^{-1}(\beta cl(f(B)))) \subseteq f(int(B))$.
(ii) ⇒ (iii) is obvious by taking complement in (ii).
Suppose (iii) holds. Let B be an IFCS in X. Then cl(B) = B and f(B) is an IFS in Y.
Now $f(B) = f(cl(B)) \subseteq \beta int(f(B)) \subseteq f(B)$, by hypothesis. This implies f(B) is an IFβOS in Y and
Hence an IFπGβOS in Y. Thus f is an IFCπGβOM by Theorem 4.4.

Theorem 4.7: Let f : (X, τ) → (Y, σ) be a bijective mapping. Then f is an IFCπGβOM if
$cl(f^{-1}(A)) \subseteq f^{-1}(\beta int(A))$ for every IFS A in Y.

Proof: Let A be an IFCS in X. Then cl(A) = A and f(A) is an IFS in Y. By hypothesis
$cl(f^{-1}(f(A))) \subseteq f^{-1}(\beta int(f(A)))$. Therefore $A = cl(A) = cl(f^{-1}(f(A))) \subseteq f^{-1}(\beta int(f(A)))$.
Now $f(A) \subseteq f(f^{-1}(\beta int(f(A)))) = \beta int(f(A)) \subseteq f(A)$. Hence f(A) is an IFβOS in Y and hence an IFπGβOS in Y. Thus f is an IFCπGβOM by Theorem 4.4.

Theorem 4.8: If f : (X, τ) → (Y, σ) is an IFCπGβOM, where Y is an IFβT1/2 space, then the following conditions are hold:
(i) $\beta cl(f(B)) \subseteq f(int(\beta cl(B)))$ for every IFOS B in X
(ii) $f(cl(\beta int(B))) \subseteq \beta int(f(B))$ for every IFCS B in X

Proof: (i) Let $B \subseteq X$ be an IFOS. Then int(B) = B. By hypothesis f(B) is an IFπGβCS in Y. Since Y is an IFπβT1/2 space, f(B) is an IFβCS in Y. This implies $\beta cl(f(B)) = f(B) = f(int(B)) \subseteq f(int(\beta cl(B)))$.
(ii) can be proved easily by taking complement in (i).

Theorem 4.9: A mapping f : (X, τ) → (Y, σ) is an IFCπGβOM if $f(\beta cl(B)) \subseteq int(f(B))$ for every IFS B in X.

Proof: Let $B \subseteq X$ be an IFCS. Then cl(B) = B. Since every IFCS is an IFβCS , βcl(B) = B. Now by hypothesis, $f(B) = f(\beta cl(B)) \subseteq int(f(B)) \subseteq f(B)$. This implies f(B) is an IFOS in Y. Therefore f(B) is an IFπGβOS in Y. Hence f is an IFCπGβOM.

Theorem 4.10: A mapping f : (X, τ) → (Y, σ) is an IFCπGβOM, where Y is an IFπβT1/2 space if and only if $f(\beta cl(B)) \subseteq \beta int(f(cl(B)))$ for every IFS B in X.

Proof: Necessity: Let $B \subseteq X$ be an IFS. Then cl(B) is an IFCS in X. By hypothesis f(cl(B)) is an IFπGβOS in Y. Since Y is an IFβT1/2 space, f(cl(B)) is an IFβOS in Y. Therefore $f(\beta cl(B)) \subseteq f(cl(B)) = \beta int(f(cl(B)))$.
Sufficiency: Let $B \subseteq X$ be an IFCS. Then cl(B) = B. By hypothesis, $f(\beta cl(B)) \subseteq \beta int(f(cl(B))) = \beta int(f(B))$. But βcl(B) = B. Therefore $f(B) = f(\beta cl(B)) \subseteq \beta int(f(B) \subseteq f(B)$. This implies f(B) is an IFβOS in Y and hence an IFπGβOS in Y. Hence f is an IFCπGβOM.

Theorem 4.11: An IFOM f : (X, τ) → (Y, σ) is an IFCπGβOM if IFπGβO(Y) = IFπGβC(Y).
Proof: Let $A \subseteq X$ be an IFOS. By hypothesis, f(A) is an IFOS in Y and hence is an IFπGβOS in Y. Thus f(A) is an IFπGβCS in Y, since IFπGβO(Y) = IFπGβC(Y). Therefore f is an IFCπGβOM.

Definition 4.12: A mapping f : (X, τ) → (Y, σ) is said to be an intuitionistic fuzzy almost contra π generalized β open mapping (IFACπGβOM for short) if f(A) is an IFπGβCS in Y for every IFROS A in X.

Example 4.13: Let X = {a, b}, Y = {u, v} and G1 = ⟨x, (0.4 0.2), (0.5, 0.4⟩,G2 = ⟨y, (0.5, 0.3), (0.5, 0.4) ⟩. Then τ = {0~, G1, 1~} and σ = {0~, G2, 1~} are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and
f(b) = v. Then f is an IFACπGβOM.

Theorem 4.14: If f : (X, τ) → (Y, σ) is a bijective mapping, where Y is an IFπβT1/2 space, then the following conditions are equivalent:
(i) f is an IFACπGβOM.
(ii) $f(A) \subseteq$ IFπGβO(Y) for every A ∈ IFRC(X) .
(iii) f(int(cl(A))) βIFπGβC(Y) for every IFOS A ∈ X .
(iv) $f(cl(int(A))) \subseteq$ IFπGβO(Y) for every IFCS A ∈ X .

Proof: (i) ⇒ (ii) is obvious.
(i) ⇒ (iii) Let A be any IFOS in X. Then int(cl(A)) is an IFROS in X. By hypothesis, f(int(cl(A))) is an IFπGβCS in

Y. Hence f(int(cl(A))) ∈ IFπGβC(Y).
(iii) ⇒ (i) Let A be any IFROS in X. Then A is an IFOS in X. By hypothesis, f(int(cl(A))) ∈ IFπGβC(Y).
That is f(A) ∈ IFπGβ (Y), since int(cl(A)) = A. Hence f is an IFACπGβOM.
(ii) ⇒ (iv) is similar as (i) ⇒ (iii).

Theorem 4.15: If f : (X, τ) → (Y, σ) is a mapping, where X is an IFβT1/2 space, then the following are equivalent:

(i) f is an IFACπGβ continuous mapping .

(ii) f^{-1}(A) ∈ IFπGβO(X) for every A ∈ IFRC(Y)

(iii) f^{-1}(int(cl(G)))∈IFπGβC(X) for every IFOS G ⊆ Y

(iv) f^{-1}(cl(int(H)))∈IFπGβO(X) for every IFCS H ⊆ Y

Proof: (i) ⇒ (ii) Let A be an IFRCS in Y. Then A^c is an IFROS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFπGβCS in X. Therefore f^{-1}(A) is an IFπGβOS in X. Therefore f^{-1}(A) is an IFπGβOS in X.

(i) ⇒ (iii) Let G be any IFOS in Y. Then int(cl(G)) is an IFROS in Y. By hypothesis, f^{-1}(int(cl(G))) is an IFπGβCS in X. Hence f^{-1}(int(cl(G))) ∈ IFπGβC(X).

(iii) ⇒(i) Let A be any IFROS in Y. Then A is an IFOS in Y. By hypothesis, we have f^{-1}(int(cl(A))) ⊆IFπGβC(X). That is f^{-1}(A) ∈ IFπGβC(X), since int(cl(A)) = A. Hence f is an IFACπGβ continuous mapping.

(ii) ⇒ (iv) is similar to (i) ⇒ (iii).

Definition 4.16: A mapping f : (X, τ) → (Y, σ) is said to be an intuitionistic fuzzy contra MπGβ open mapping (IFCMπGβOM) if f(A) is an IFπGβCS in Y for every IFπGβOS A in X.

Example 4.17: Let X = {a, b}, Y = {u, v} and G1 = 〈x, (0.5, 0.6), (0.4, 0.3)〉 , G2 = 〈y, (0.2, 0.3), (0.8, 0.7)〉. Then τ = {0~, G1, 1~} and σ = {0~, G2, 1~} are IFTs on X and Y respectively. Define a mapping
f : (X, τ) →(Y, σ) by f(a) = u and f(b) = v. Then f is an IFCMπGβOM.

Theorem 4.18: Let f : (X, τ) → (Y, σ) be a bijective mapping. Then the following statements are equivalent:
(i) f is an IFCMπGβOM,
(ii) f(A) is an IFπGβOS in Y for every IFπGβCS A in X.

Proof: (i) ⇒ (ii) Let A be an IFπGβCS in X. Then A^c is an IFπGβOS in X. By hypothesis, $f(A^c)$ is an IFπGβCS in Y. That is $f(A)^c$ is an IFπGβCS in Y. Hence f(A) is an IFπGβOS in Y.
(ii) ⇒ (i) Let A be an IFπGβOS in X. Then A^c is an IFπGβCS in X. By hypothesis, $f(A^c)$ is an IFπGβOS in Y.
Hence f(A) is an IFπGβCS in Y. Thus f is an IFCMπGβOM.

Theorem 4.19: Every IFCMπGβOM is an IFCπGβOM but not conversely.

Proof: let f : (X, τ) → (Y, σ) be an IFCMπGβOM , and A⊆ X be an IFOS. Then A is an IFπGβOS in X. By hypothesis, f(A) is an IFπGβCS in Y. Hence f is an IFCπGβOM.

Example 4.20 Let X = {a, b}, Y = {u, v} and G1 = 〈x, (0a, 0.3b), (0.5a, 0.4b)〉 ,G2 =〈 y, (0.2u, 0.4v), (0.5u, 0.4v)〉 G3 =〈 y, (0.1u, 0.3v), (0.3u, 0.4v)〉 ,G4 =〈 y, (0.1u, 0.3v), (0.5u, 0.4v)〉 , G5 =〈 y, (0.2u, 0.4v), (0.3u, 0.4v) 〉and G6 = 〈y, (0.4u, 0.4v), (0.3u, 0.4v) 〉. Then τ = {0~, G1, 1~} and σ = {0~, G2, G3 ,G4 ,G5, G6, 1~} are IFTs on X and Y respectively. Define a mapping f: (X, τ) (Y,) by f(a) = u and f(b) = v. Then f is an IFCπGβOM but not an IFCMπGβOM , since A = x, (0a, 0.3b), (0.5a, 0.4b) is an IFCMπGβOS in X but f(A) = y, (0u,0.3v), (0.5u,0.4v) is not an IFπGβCS in Y.

Theorem 4.21 (i) If f : (X, τ) → (Y, σ) is an IFOM and g : (Y, σ) → (Z, η) be an IFCπGβOM, then $g_o f$ is an IFCπGβOM.
(ii) If f : (X, τ) → (Y, σ) is an IFCπGβOM and g : (Y, σ) → (Z, η) is an IFMπGβCM, then $g_o f$ is an IFCπGβOM.
(iii) If f : (X, τ) → (Y, σ) is an IFπGβOM and g : (Y, σ) →(Z, η) is an IFCMπGβOM, then $g_o f$ is an IFCπGβOM.
(iv) If f : (X, τ) → (Y, σ) is an IFCπGβOM and g : (Y, σ) → (Z, η) is an IFCMπGβOM, then $g_o f$: (X, τ) → (Z, η) is an IFπGβOM.

Proof: (i) Let A be an IFOS in X. Then f(A) is an IFOS in Y. Therefore g(f(A)) is an IFπGβCS in Z. Hence g□f is an IFCπGβOM.
(ii) Let A be an IFOS in X. Then f(A) is an IFπGβCS in Y. Therefore g(f(A)) is an IFπGβCS in Z. Hence g□f is an IFCπGβOM.

(iii) Let A be an IFOS in X. Then f(A) is an IFπgβOS in Y. Therefore g(f(A)) is an IFπGβCS in Z. Hence g∘f is an IFCπGβGOM.

(iv) Let A be an IFOS in X. Then f(A) is an IFπGβCS in Y, since f is an IFCπGβOM. Since g is an IFCMπGβOM, g(f(A)) is an IFπGβOS in Z. Therefore g∘f is an IFπGβOM.

Theorem 4.22: If f : (X, τ) → (Y, σ) is an IFCMπGβOM, then for any IFπGβCS A in X and for any IFP p(α, β) ∈ Y, if f^{-1}(p(α, β)) q A, then p(α, β)$_q$πGβ int(f(A)).

Proof: Let A ⊆ X be an IFπGβCS and let p(α, β) ∈ Y. Assume that f^{-1} (p(α, β)) $_q$ A. Then p(α, β) $_q$ f(A). By hypothesis, f(A) is an IFπGβOS in Y. This implies πGβ**i**nt(f(A)) = f(A). Hence p(α, β) $_q$πGβ**i**nt(f(A)).

Theorem 4.23: If f : (X, τ)→ (Y, σ) is an IFCπGβ closed mapping and Y is an IFπβT1/2 space, then f(A) is an IFGOS in Y for every IFCS A in X.

Proof: Let f : (X, τ) → (Y,σ) be an IFCπGβ closed mapping and let A be an IFCS in X. Then by hypothesis f(A) is an IFπGβOS in Y. Since Y is an IFπβT1/2 space, f(A) is an IFGOS in Y.

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