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Intuitionistic Fuzzy Graph of Third Type

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Abstract- Intuitionistic Fuzzy Graph of second type was studied by [1]. In this paper we introduce Intuitionistic Fuzzy graph of Third Type say (IFGTT), Join of two intuitionistic fuzzy graph of Third Type and three theorems are discussed.

***Keywords*:** Intuitionistic Fuzzy Graph Of Third Type, Strong Intuitionistic Fuzzy Graph Of Third Type, Join of Intuitionistic Fuzzy Graph Of Third Type.

I. INTRODUCTION

In 1965 Lotfi. A. Zadeh introduced fuzzy sets as a generalization of crisp set and later in 1983 Krassimir T. Atanassov introduced the notion on intuitionistic fuzzy set.

The concept of fuzzy graph was introduced by Rosenfeld in 1975 and then K. T. Atanassov extended it to intuitionistic fuzzy graph in 1999. Research in intuitionistic fuzy graph and its application have been increased considerably in recent years. We have studied Intuitionistc Fuzzy Graph of Second Type [1]. In this paper we develop the concept of intuitionistic fuzzy graph of third type (IFGTT) and some of its properties.

This paper is structured as follows: Section 2, contains basic definition that are necessary for the development of the following section. In Section3, we introduced Intuitionistic fuzzy graph of Third Type, Strong and complete IFGTT and Join of two IFGTT and some theorems and results, and section 4 concludes this paper.

II. PRELIMINARIES

Definition 2.1 Intuitionistic Fuzzy sets [3]

Let X be a given set. An Intuitionistic fuzzy set A in X is given by $A = \{(x, \mu_A(x), \vartheta_A(x))/x \in X\}$. Where $\mu_A: X \to [0,1]$, $\vartheta_A: X \to [0,1]$ and $0 \le \mu_A(x) + \vartheta_A(x) \le 1$. Where $\mu_A(x)$ is the degree of membership of the element x in A and $\vartheta_A(x)$ is the degree of non membership of the element x in A. Also for each $x \in X$, $\pi_A(x) = 1 - \mu_A(x) - \vartheta_A(x)$ is called the degree of hesitation.

Definition 2.2 Intuitionistic Fuzzy set Of Third Type[1]

Let X be a non empty set. An Intuitionistic fuzzy set third type in X is defined as an object of the form $A = \{(x, \mu_A(x), \vartheta_A(x))/x \in X\}$ where $\mu_A: X \to [0,1]$, $\vartheta_A: X \to [0,1]$ and $0 \le {\mu_A}^3(x) + {\vartheta_A}^3(x) \le 1$. Where $\mu_A(x)$ is the degree of membership of the element x in A and, $\vartheta_A(x)$ is the degree of non membership of x in A.

Definition 2.3 Intuitionistic Fuzzy Graph (IFG)

An Intuitionistic fuzzy graph is of the form G = [V, E], where

(i) V = $\{v_1, v_2 \cdots v_n\}$ such that $\mu_V: V \to [0,1]$ and $\vartheta_V: V \to [0,1]$ denote the degree of membership and degree of non membership of the element $v_i \in V$, respectively and $0 \le \mu_V(x) + \vartheta_V(x) \le 1, \forall\, v_i \in V, (i = 1,2, \cdots n)$

(ii) $E \subseteq V \times V$ where $\mu_E: V \times V \to [0,1]$ and $\vartheta_E: V \times V \to [0,1]$ such that
$\mu_E(v_i\, v_j) \le \min[\mu_V(v_i), \mu_V(v_j)]$,
$\vartheta_E(v_i\, v_j) \le \max[\vartheta_V(v_i), \vartheta_V(v_j)]$, and $0 \le \mu_E(v_i\, v_j) + \vartheta_E(v_i\, v_j) \le 1, \forall\, (v_i v_j) \in E, (i,j = 1,2, \cdots n)$,

III. INTUITIONISTIC FUZZY GRAPH OF THIRD TYPE (IFGTT)

In this section, we define Intuitionistic fuzzy graph of third type and some other related definitions, theorems and we deduced some results.

Definition 3.1 *An Intuitionistic Fuzzy graph of Third Type (IFGTT)* is of the form G= [V,E] where

(i) The vertex set V = $\{v_1, v_2 \cdots v_n\}$ such that $\mu_V: V \to [0,1]$ and $\vartheta_V: V \to [0,1]$ denote the degree of membership and non membership of the element $v_i \in V$,respectively and
$0 \le \mu_V{}^3(x) + \vartheta_V{}^3(x) \le 1 \quad, \forall\, v_i \in V, (i = 1,2, \cdots n)$ … (1)

(ii) $E \subseteq V \times V$ where $\mu_E: V \times V \to [0,1]$ and $\vartheta_E: V \times V \to [0,1]$ such that

$$\left.\begin{array}{l}\mu_E(v_i v_j) \le \min[\mu_V{}^3(v_i), \mu_V{}^3(v_j)],\\ \vartheta_E(v_i v_j) \le \max[\vartheta_V{}^3(v_i), \vartheta_V{}^3(v_j)],\end{array}\right\} \quad (2)$$

and
$0 \le \mu_E{}^3(v_i v_j) + \vartheta_E{}^3(v_i v_j) \le 1$,
$\forall\, (v_i, v_j) \in E, (i,j = 1,2, \cdots n)$, (3)

Example 3.1

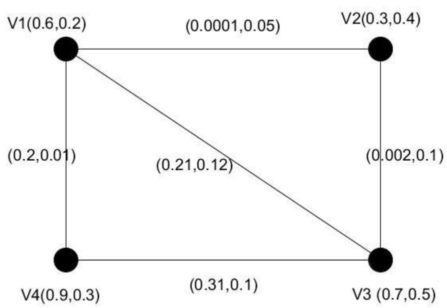


Figure1.IFGTT

Remark 3.1 IFGTT is an IFG, because
For an IFGTT

$$\mu_E(v_i v_j) \le \min[\mu_V{}^3(v_i), \mu_V{}^3(v_j)] < \min[\mu_V(v_i), \mu_V(v_j)]$$

$$\vartheta_E(v_i v_j) \le \max[\vartheta_V{}^3(v_i), \vartheta_V{}^3(v_j)] < max[\vartheta_V(v_i), \vartheta_V(v_j)]$$

Definition 3.2 Strong IFGTT An IFGTT G = [V, E] is said to be *Strong* if
$\mu_E(v_i\, v_j) = \min[\mu_V{}^3(v_i), \mu_V{}^3(v_j)]$, &
$\vartheta_E(v_i\, v_j) = \max[\vartheta_V{}^3(v_i), \vartheta_V{}^3(v_j)]$, $\forall\, v_i\, v_j \in E$ (4)

Definition 3.3 Complete IFGTT An IFGTT G= [V,E] is said to be *complete* if
$\mu_E(v_i\, v_j) = \min[\mu_V{}^3(v_i), \mu_V{}^3(v_j)]$, &
$\vartheta_E(v_i\, v_j) = \max[\vartheta_V{}^3(v_i), \vartheta_V{}^3(v_j)]$, $\forall\, v_i, v_j \in V$ (5)

Example 3.2

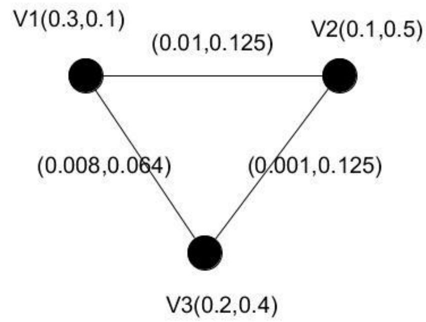


Figure 3.2 Complete IFGTT

(2)

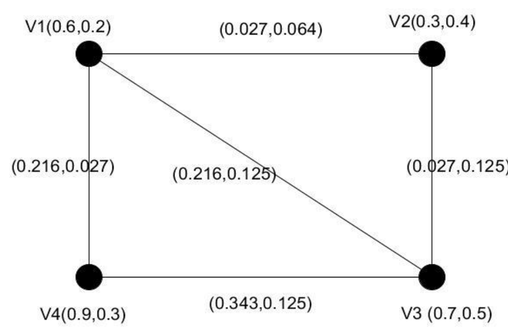


Figure 3.4 Strong IFGTT

Remark 3.2 If G is an IFGTT Then $\bar{\bar{G}} = G$

Definition 3.4 The Join Of two IFGTT

The *Join* of two IFGTT $G_1 = [V_1, E_1] and\ G_2 = [V_2, E_2]$, denoted by $G_1 \triangle G_2 = [V_1 \cup V_2, E_1 \cup E_2 \cup E']$, where E' is the new edge joining $V_1 \& V_2$, its membership and non membership are defined as follows.

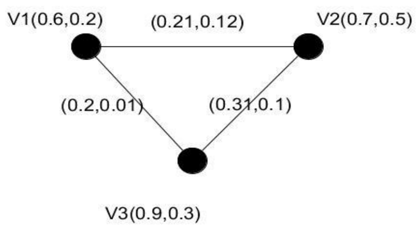
$$(\mu_{V_1} + \mu_{V_2})(u) = \begin{cases} \mu_{V_1}(u)\ if\ u \in V_1 \\ \mu_{V_2}(u)\ if\ u \in V_2 \end{cases}$$

$$(\vartheta_{V_1} + \vartheta_{V_2})(u) = \begin{cases} \vartheta_{V_1}(u)\ if\ u \in V_1 \\ \vartheta_{V_2}(u)\ if\ u \in V_2 \end{cases}$$

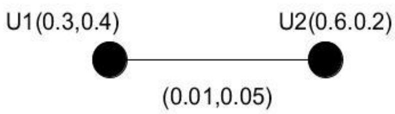
$$(\mu_{E_1} + \mu_{E_2})(u\, v) = \begin{cases} \mu_{E_1}(uv) & if u\, v \in E_1 \\ \mu_{E_2}(uv) & if\ u\, v \in E_2 \\ \min(\mu_{V_1}{}^3(u), \mu_{V_2}{}^3(v) & if\ u\, v \in E' \end{cases}$$

$$(\vartheta_{E_1}+\vartheta_{E_2})(u\,v)=\begin{cases}\vartheta_{E_1}(uv) & if\,u\,v\in E_1\\ \vartheta_{E_2}(uv) & if\;u\,v\in E_2\\ \max({\vartheta_{V_1}}^3(u),{\vartheta_{V_2}}^3(v) & if\;u\,v\in E'\end{cases}$$

Example 3.3



G_1



G_2

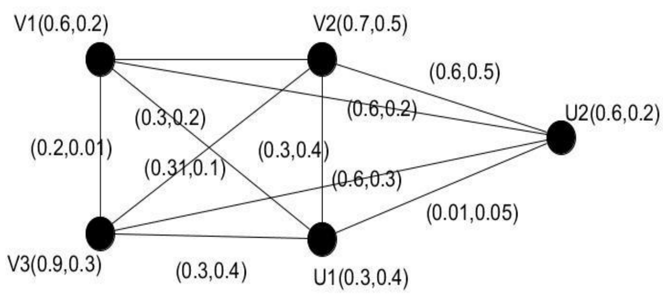


Figure 3.5 G_1 ⨺ G_2

Theorem 3.1

The Join of two IFGTT is again an IFGTT

Proof

Let $G_1=[V_1,E_1]$ and $G_2=[V_2,E_2]$ be IFGTT ,We have to prove that the Join of $G_1\,\&\,G_2$ denoted by G_1 ⨺ $G_2=[V_1\cup V_2,E_1\cup E_2\cup E']$, where E' is the new edge joining $V_1\&\,V_2$ is an IFGTT.

By definition we have, If $u\in V_1$ then $(\mu_{V_1}+\mu_{V_2})(u)=\mu_{V_1}(u)$ & $(\vartheta_{V_1}+\vartheta_{V_2})(u)=\vartheta_{V_1}(u)$

$\therefore 0\le (\mu_{V_1}+\mu_{V_2})^3(u)+(\vartheta_{V_1}+\vartheta_{V_2})^3(u)\le 1$

Similarly when $u\in V_2$ and if $u\,v\in E_1$then $(\mu_{E_1}+\mu_{E_2})(u\,v)=\mu_{E_1}(uv)=>$

$$(\mu_{E_1}+\mu_{E_2})(u\,v)\le\min[{\mu_{V_1}}^3(u),{\mu_{V_1}}^3(v)],$$

$$\le\min\{(\mu_{V_1}+\mu_{V_2})^3(u),(\mu_{V_1}+\mu_{V_2})^3(v)\}$$

And $\quad(\vartheta_{E_1}+\vartheta_{E_2})(u\,v)=\vartheta_{E_1}(uv)$

$$\le\max[{\vartheta_{V_1}}^3(u),{\vartheta_{V_1}}^3(v)]$$

$$\le max\left\{(\vartheta_{V_1}+\vartheta_{V_2})^3(u),(\vartheta_{V_1}+\vartheta_{V_2})^3(v)\right\}$$

Similarly if $u\,v\in E_2$, $(\mu_{E_1}+\mu_{E_2})(u\,v)\le\min\{(\mu_{V_1}+\mu_{V_2})^3(u),(\mu_{V_1}+\mu_{V_2})^3(v)\}$

And$(\vartheta_{E_1}+\vartheta_{E_2})(u\,v)\le max\left\{(\vartheta_{V_1}+\vartheta_{V_2})^3(u),(\vartheta_{V_1}+\vartheta_{V_2})^3(v)\right\}$

If

$u\,v\in E'$,$(\mu_{E_1}+\mu_{E_2})(u\,v)=$
$\min\left({\mu_{V_1}}^3(u),{\mu_{V_2}}^3(v)\right)=min\left((\mu_{V_1}+\mu_{V_2})^3(u),(\mu_{V_1}+\mu_{V_2})^3(v)\right)$

$$(\vartheta_{E_1}+\vartheta_{E_2})(u\,v)=\max\left({\vartheta_{V_1}}^3(u),{\vartheta_{V_2}}^3(v)\right)=max\left((\mu_{V_1}+\mu_{V_2})^3(u),(\vartheta_{V_1}+\vartheta_{V_2})^3(v)\right)$$

$\therefore G_1$ ⨺ G_2 is also an IFGTT

Theorem 3.2 If $G_1\&\,G_2$ are strong IFGTT, then their Join denoted by G_1 ⨺ G_2 is again strong IFGTT.

Proof

Since $G_1\&\,G_2$ are strong

$\mu_{E_1}(v_i\,v_j)=\min[{\mu_{V_1}}^3(v_i),{\mu_{V_1}}^3(v_j)]$ & $\vartheta_{E_1}(v_i\,v_j)=\max[{\vartheta_{V_1}}^3(v_i),{\vartheta_{V_1}}^3(v_j)]$, $\forall\, v_i\,v_j\in E_1$

And

$\mu_{E_2}(v_i\,v_j)=\min[{\mu_{V_2}}^3(v_i),{\mu_{V_2}}^3(v_j)]$ & $\vartheta_{E_2}(v_i\,v_j)=\max[{\vartheta_{V_2}}^3(v_i),{\vartheta_{V_2}}^3(v_j)]$, $\forall\, v_i\,v_j\in E_2$

$$\therefore(\mu_{E_1}+\mu_{E_2})(v_i\,v_j)=\begin{cases}\mu_{E_1}(v_i\,v_j)\;if\;v_i\,v_j\in E_1\\ \mu_{E_2}(v_i\,v_j)\;if\;v_i\,v_j\in E_2\end{cases}$$

$$=\begin{cases}\min[{\mu_{V_1}}^3(v_i),{\mu_{V_1}}^3(v_j)]\\ \min[{\mu_{V_2}}^3(v_i),{\mu_{V_2}}^3(v_j)]\end{cases}$$

$$(\vartheta_{E_1}+\vartheta_{E_2})(v_i\,v_j)=\begin{cases}\vartheta_{E_1}(v_i\,v_j)\;if\;v_i\,v_j\in E_1\\ \vartheta_{E_2}(v_i\,v_j)\;if\;v_i\,v_j\in E_2\end{cases}$$

$$=\begin{cases}\max[{\vartheta_{V_1}}^3(v_i),{\vartheta_{V_1}}^3(v_j)]\\ \max[{\vartheta_{V_2}}^3(v_i),{\vartheta_{V_2}}^3(v_j)].\end{cases}$$

And if $v_i\,v_j\in E'$,$(\mu_{E_1}+\mu_{E_2})(v_i\,v_j)=min[{\mu_{V_1}}^3(v_i),{\mu_{V_2}}^3(v_j)]$,

$$(\vartheta_{E_1}+\vartheta_{E_2})(v_i\, v_j)=max[\vartheta_{V_1}{}^3(v_i),\vartheta_{V_2}{}^3(v_j)]$$

$\therefore G_1 \mathbin{\triangle\!\!\!\!\!+} G_2$ is strong.

Theorem 3.3 If $G_1 \& G_2$ are complete IFGTT, then their Join denoted by $G_1 \mathbin{\triangle\!\!\!\!\!+} G_2$ is again Complete

Proof

Since $G_1 \& G_2$ are complete

$\mu_{E_1}(v_i\, v_j)=\min[\mu_{V_1}{}^3(v_i),\mu_{V_1}{}^3(v_j)]$, & $\vartheta_{E_1}(v_i\, v_j)=\max[\vartheta_{V_1}{}^3(v_i),\vartheta_{V_1}{}^3(v_j)]$, $\forall\, v_i, v_j \in V_1$

And $\mu_{E_2}(v_i\, v_j)=\min[\mu_{V_2}{}^3(v_i),\mu_{V_2}{}^3(v_j)]$, & $\vartheta_{E_2}(v_i\, v_j)=\max[\vartheta_{V_2}{}^3(v_i),\vartheta_{V_2}{}^3(v_j)]$, $\forall\, v_i, v_j \in V_2$

$$\therefore (\mu_{E_1}+\mu_{E_2})(v_i\, v_j)=\begin{cases}\mu_{E_1}(v_i\, v_j)\ if\ v_i\, v_j\in E_1\\ \mu_{E_2}(v_i\, v_j)\ if\ v_i\, v_j\in E_2\end{cases}$$

$$=\begin{cases}\min[\mu_{V_1}{}^3(v_i),\mu_{V_1}{}^3(v_j)]\\ \min[\mu_{V_2}{}^3(v_i),\mu_{V_2}{}^3(v_j)]\end{cases}$$

$\forall\, v_i, v_j \in V_1$

$$(\vartheta_{E_1}+\vartheta_{E_2})(v_i\, v_j)=\begin{cases}\vartheta_{E_1}(v_i\, v_j)\ if\ v_i\, v_j\in E_1\\ \vartheta_{E_2}(v_i\, v_j)\ if\ v_i\, v_j\in E_2\end{cases}$$

$$=\begin{cases}\max[\vartheta_{V_1}{}^3(v_i),\vartheta_{V_1}{}^3(v_j)]\\ \max[\vartheta_{V_2}{}^3(v_i),\vartheta_{V_2}{}^3(v_j)]\end{cases}\quad \forall\, v_i, v_j \in V_2$$

And if $v_i\, v_j\in E'$, $(\mu_{E_1}+\mu_{E_2})(v_i\, v_j)=min[\mu_{V_1}{}^3(v_i),\mu_{V_2}{}^3(v_j)]$, $\forall\, v_i, v_j \in V_1$

$(\vartheta_{E_1}+\vartheta_{E_2})(v_i\, v_j)=max[\vartheta_{V_1}{}^3(v_i),\vartheta_{V_2}{}^3(v_j)]$ $\forall\, v_i, v_j \in V_2$

$\therefore G_1 \mathbin{\triangle\!\!\!\!\!+} G_2$ is complete.

IV. CONCLUSION

Here we have defined IFGTT, Complete and Strong IFGTT and Join of two IFGTT, also we established some of its properties. In upcoming papers we shall establish some more properties and application of IFGT.

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