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Research Article

Analysis of the Cholera Disease and Control Using the SIRD Model

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Abstract—This Study is centered on modifying SIRD mathematical model for cholera dynamics incorporating four control strategies namely: therapeutic treatment, Surveillance, vaccination and water sanitation. This study is limited to only cholera disease. Data were collected from the Gombe state Hospital for infectious disease and leprosy control in Zambuk. According to this study, total human population has been classified into four sub population such as susceptible humans, recovered humans, Infected Humans and the mortality rate of humans(Death). The susceptible class,Infected class and the recovered and the death category is being related with the homotopy perturbation method and the runge kutta method to observe the most efficient method to use to control the disease better.

Keywords—SIRD model, Susceptible, Infected, Recovered, Death, Runge kutta(order 4and order 5) method, Homotopy perturbation method, Vibrio Cholerae

1. Introduction

The disease called Cholera has been a challenge in Africa for centuries, This disease has been occurring yearly due to poor drainage systems ,underdevelopment in Africa,poor drainage systems,Illitracy and lack of awareness. This study intends to focus on Nigeria and Gombe state to be precise. Cholera is a disease that is caused by a Bacteria called the ‘Vibrio Cholerae’ which if present in the available drinking water, at a particular place and at a particular time. Cholera can be contacted directly and indirectly which means Cholera can be contacted directly if the drinking water source of a particular environment has the Bacteria called ‘Vibrio Cholerae’. And again it can be contacted indirectly if the infected person gets in contact with any individual especially if the individual has poor immune system (susceptible individual). This indirect infection can also occur through Migration, poor toilet systems, untreated drinking water, lack of awareness of such disease and how it is caused, Lack of proper hygiene of environment and poor hygiene of foods that we eat.

In this study we intend to categorize the population into Susceptible, Infected, Recovered and Death category. Data was collected from the Zambuk Hospital for infectious diseases Gombe state. The four control strategies are the Vaccination, Washing of food items, Therapeutic treatment and surveillance. After the consideration of the four control strategy, The treatment and prevention process will categorise the population into different sect to make observation of which control is faster and can best curb the disease appropriately.

2. Related Work

MacElroy & Townsend(2009) outlined Cholera as an intestinal disease caused by ‘*Vibrio cholerae’* characterized by loosed stool and vomiting which can cause dehydration.. Tien et al outlined that, mathematical modeling gives an approach to gain basic knowledge in cholera dynamics. Mwasa and Tchuenche(2011) stated that prevention, and control strategies are based on knowledge and assumptions, variables and parameters to formulate a mathematical model. Lemos et al (2017), studied the spread of Cholera disease model by deciding that treatment and control can be achieved isolation of infected individuals. Nyaberi & Malonza (2019), worked on vaccination as a means to study the dynamics of cholera disease. (Hethcote, 2000) brought the idea That dynamics of cholera involve several co existence between the human host, the pathogen, and the environment. (Mukandavire et al, 2011),studied The outbreak of Cholera, which brings a huge burden on public health which lead to large spending of funds which can affect the economy. (Andrews et al 2011), worked on an epidemic model which forcussed on dynamics and control of cholera. (Chao et al 2011), studied the Vaccination as a control for Cholera disease and hence cheking the implication it might cause to a developing country. (Bertuzzo et al 2011), suggested that control strategies plays a vital role on Cholera Epidemic.

Cappaso and Paveri(1979) developed the cholera model consisting of two ordinary differential equations .Codeco(1959), Developed a mathematical model using the Paveri and Fontana(1979), model which used two systems of ordinary differential equation and tried to explain the complex nature and dynamics of cholera disease.

3. Experimental Method/Procedure/Design

On dividing the population into the SIRD which are the susceptible category, infected category, recovered category and the dead category at a given time t. In this study we tend to study how each control will have an effect on the cholera disease. However there are other parameters that should be considered in order to formulate the model. The S(t) which stands for the susceptible individuals at a given time(t) and the I(t) which is the infected individuals at time(t).The recovered individuals at a given time (t) and the dead category at a given time (t). The β(t) refers to the concentration of vibrio cholera present in the available drinking water at a given time (t) and at a particular location.The ᴧ(t) which is the loss rate of the immunity at a given time (t). The μ(t) is the therapeutic treatment being given to the infected and susceptible humans at time (t). γ(t) is the 50% chances of contacting cholera per day at a given time (t). $^{\delta}$(t) is the washing of food at time (t).
This lead to the formulation of a modified model.

$$\frac{dS}{dt} = dD(t) - \beta SI(t) - \mu S(t) \quad (1)$$

$$\frac{dI}{dt} = \beta S(t)I(t) - (\lambda+\mu)I(t) \quad (2)$$

$$\frac{dR}{dt} = \gamma I(t) - \mu R(t) \quad (3)$$

$$\frac{dD}{dt} = \Lambda(t) - (^{\delta}+\mu)\, D(t) \quad (4)$$

With initial conditions
$S(t)= S_0$, $I(t)= I_0$, $R(t)= R_0$, $D(t)= D_0$ (5)

Table 1. Description of the parameters

Parameters	Parameter description	Values
S	susc. Category	200,250,300,…1050.
I	infected category	100, 150, 200, 250…500.
R	Recovered category	150, 200, 300, 400…500
D	Dead category	50,400,30, 20,…0
Λ	Surveillance	0.01
μ	Therapeutic treatment	0.1
γ	50% chances of con. cholera	0.15
r	loss rate of immunity	0.0001
β	conc.of vibrio cholera	0.5
$^{\delta}$	washing of food items	0.018
λ	Vaccination	0.25
t	period of time	0.1,0.2,0.3,..1

4. Results and Discussion

4.1Results

Table 2. Numerical solution for susceptible humans

Time	Runge- Kutta	HPM	Error
0.0	10000	10000	0
0.1	16153.0850	16153.0850	4.93074E-05
0.2	22145.2435	22145.2426	0.000827317
0.3	27980.6848	27980.6786	0.006220119
0.4	336633.5081	33663.4822	0.02587213
0.5	39197.7056	3197.6268	0.07867993
0.6	44587.1643	44586.9697	0.194616374
0.7	49835.6706	49835.2519	0.418711643
0.8	54946.9112	54946.0984	0.812878598
0.9	59924.4760	59923.0180	1.457988928
1.0	64771.8618	64769.4036	2.4582722091

Table 3. Numerical solution for infected humans

Time	Runge- kutta	HPM	Error
0.0	500.0000	500.0000	0
0.1	474.9230	474.9230	-3.35343E.06
0.2	428.5419	428.5153	-534592E-05
0.3	428.5149	451.1162	-0.000391696
0.4	407.0581	407.0397	-0.001606786
0.5	386.3478	367.3597	-0.004841169
0.6	367.3478	367.3597	-0.011888652
0.7	348.9867	349.0121	-0.025420482
0.8	331.5544	331.6036	-0.049079107
0.9	315.0040	315.0915	-0.087586605
1.0	299.2903	299.4373	-0.146971

Table 4. Numerical solution for Recovered humans

Time	Runge-Kutta	HPM	Error
0.0	300.0000	300.0000	0
0.1	651.5348	651.5348	-5.062E.05
0.2	1154.1676	1154.1684	-0.0008617
0.3	1803.6770	1803.6835	-0.006512
0.4	2595.9540	2595.9811	-0.0271016
0.5	3526.9986	3527.0811	-0.0824508
0.6	4592.9177	4593.1217	-0.2039295
0.7	5789.9211	57903599	-0.4387205
0.8	7114.3195	7115.1712	-0.8516212

0.9	**8562.5224**	**8564.0497**	**-1.5272787**
1.0	**10131.0334**	**10133.6081**	**-2.5747044**
Time	**Runge- kutta**	**HPM**	**Error**
0.0	**1000.000**	**1000.000**	**0**
0.1	**997.8831**	**997.8831**	**5.54961E-06**
0.2	**994.6159**	**994.6158**	**9.13095E-05**
0.3	**990.2896**	**990.2889**	**0.000686257**
0.4	**984.9899**	**984.9870**	**0.002842679**
0.5	**978.7972**	**978.7886**	**0.008610256**
0.6	**971.7872**	**971.7660**	**0.021209245**
0.7	**964.0308**	**963.9853**	**0.0454438487**
0.8	**955.5945**	**955.5066**	**0.087842946**
0.9	**946.5408**	**946.3858**	**0.15690524**
1.0	**936.9282**	**936.6647**	**0.263453118**

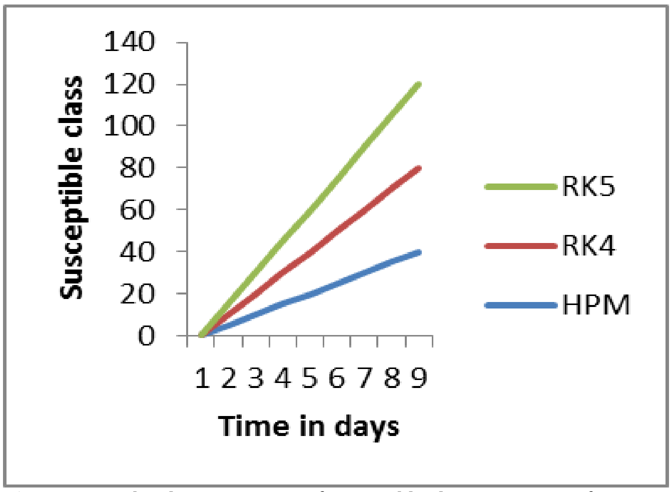

Figure 1. Graphical representation of susceptible class against time for HPM, Runge kutta

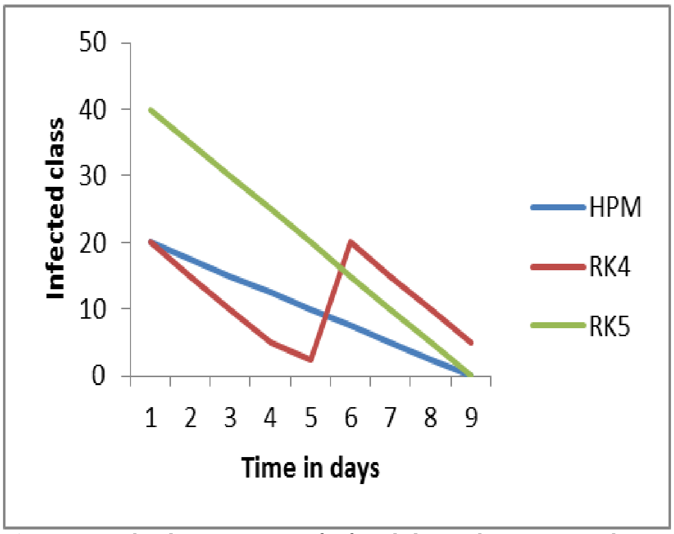

Figure 2. Graphical representation of Infected class with HPM ,Runge kutta

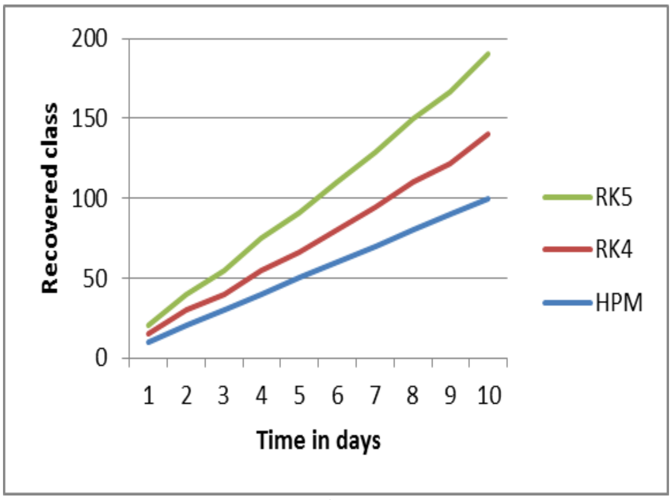

Figure 3. Graphical representation of the recovered class with HPM ,Runge kutta.

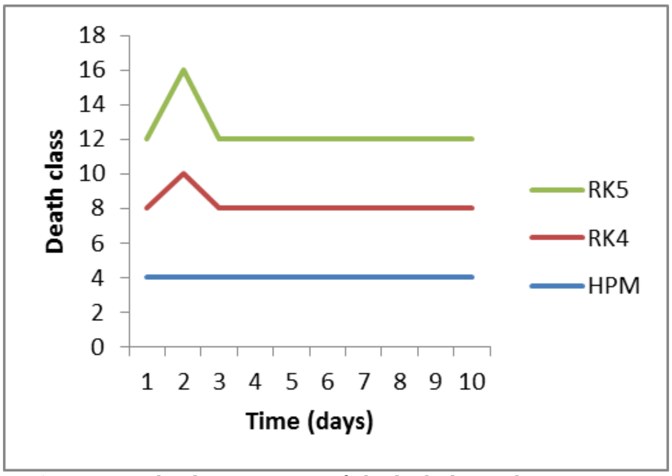

Figure 4. Graphical representation of the death class with HPM, Runge kutta.

4.2 Discussions

In figure 1,the susceptible class increased in an alarming rate since the infection had spread which made reasonable number of people to be vulnerable. This shows that the susceptible class will move to the infected class the HPM,RK4,RK5 are linear on the graph as they move in the same direction only that the HPM method increases but slower than RK4 and RK5. In figure 2, which is the infected category shows the drop of the infection with respect to time. This due to the Control strategies incorporated to control the spread and severity of the disease. Except for the RK4 which is unstable at a certain time or subsequently. In figure 3, the three methods are linear on the graph which shows a reasonable amount of recovery with respect to time. In figure 4, the graph shows that the amount of death is not greater than 10 which signify that the control strategy has helped in the eradication of the disease .Furthermore, The Four control strategies are suitable the study as the methods which align with each other because the error is relatively small.

5. Conclusion and further work

Tables 3-5 presented shows that the two methods align with each other and the two methods have relatively small errors.

This method is faster as to curb a pandemic. The four control strategies incorporated are suitable for the control of the Cholera disease. Furthermore, the use of the four control strategies and the Homotopy perturbation method and Runge Kutta method can be use to curb the Cholera disease.

Data Availability
Data was collected from the Zambuk Hospital for infectious disease Gombe state and other parameter values were obtained from previous literature and studies

Conflict of Interest
There was no conflict of ideas and interest incurred during this study.

Funding Source
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Authors' Contributions
The mathematical model was adjusted by professor Atureta who suggested that the model should be modified and professor Lasisi suggested that the control strategy should be four to observe the best among the control strategies. Munira salisu perfomed the numerical analysis using the model parameters.

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References

[1] A. Macelroy and P. townsend, ‘’ Medical arthropology in ecological perspective’’ 2nd Edition, Boulder,co- westview, USA, **pp 375, 2009.**

[2] A.P. Lemos paiao, C.J. Silva, D.F. Torres, ‘’ An epidemic model for Cholera with optimal control treatment’’ A journal of computational and applied mathematics, **Vol.318, No1, pp 1-7, 2016.**

[3] A.J. Robert and S.Basu, ‘’ Transmission dynamics and control of Cholera in Haiti’’ A journal on epidemic model Lancet. **Vol.377, No 3, pp 1248- 1255, 2011.**

[4] C. Virginia and S.L. Paveri- Fontana, ‘’ A mathematical model for the Cholera epidemic in the European Mediterrenean region’’ proceedings of the Revue depimoligie et de Sante publique, France **Vol.27 pp 121-132, 1979.**

[5] C. T. Codeco ‘’ Epidemic dynamics of Cholera the role of acquatic reservoir’’ proceedings of the BMC infectious diseases, Berlin **pp1-9, 2001.**

[6] C. Denny, M.E. Halloran,I.M. Longini Jr, ‘’ Vaccination strategies for epidemic Cholera in Haiti with implications for the developing world’’ proceedings of the national acdemy of science USA,**pp 7081- 7085, 2011.**

[7] E. Berttuzo, R. Casagrandi, M. Gatto, C.R. Blokes, ‘’ prediction of the spatial evolution and effects of control measures for the unfolding Haiti Cholera outbreak’’ proceedings of the geophysical research letters, Haiti. **Vol. 38, pp 106403, 2011.**

[8] H.Hethcote,’’The mathematics of infectious disease’’ A journal of mathematical biology, **Vol.42, No2, pp599, 2000.**

[9] J. Tien, A.R. Tuite,D.N. Fisman ‘’Using transmission models to explain spatial spread of disease and identify optimal control interventions’’ An international journal of medicine, **Vol.154, No 1 pp593-601,2010.**

[10]S.L. Paveri-Fontana ‘’A mathematical model for the 1973 cholera epidemic in the European mediterenean region’’ proceedings of the Revue Depidemioligue et de Sante publique, France, **pp121-132.1973.**

[11] Z. Mukanvire,S. liao, J. Wang,H.Gaff, D.L. Smith, ‘’Estimating the reproductive numbers for the 2008-2009 cholera outbreaks’’ Proceeding of the National academy of science, Zimbabwe, **pp 8767-8772, 2011.**

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