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On Friendly Index Set of Graphs

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***Abstract*—**A function f from V(G) to {0,1} where for each edge xy ,f*(xy) = (f(x) +f(y))(mod2), let $v_i(f)$ is the number of vertices v with f(v) = i and $e_i(f)$ is the number of edges e with f*(e) = i is called friendly if $|v_0(f) - v_1(f)| \leq 1$. The friendly index set of a graph G is FI(G) = {$|e_0(f) - e_1(f)|$, where f runs over all friendly labelings f of G}. In this paper we find the friendly index set of the umbrella graph, Spl($K_{1,n}$), Globe graph ,P_2+mk_1 and union of a path and a star sharing a vertex in common.

Keywords— *Friendly labeling, Friendly index set, Umbrella graph, Spl($K_{1,n}$), Globe graph*

I. INTRODUCTION

For all terminology and notations in graph theory we follow harary[3]. Unless mentioned or otherwise a graph in this paper shall mean a simple finite graph without isolated vertices. Let G be a graph with vertex set V(G) and edge set E(G). A labeling $f:V(G)\to Z_2$ induces an edge labeling f*: $E(G) \to Z_2$ defined by f*(xy) = f(x) + f(y) for each edge xy∈E(G). For $i\in Z_2$, let $v_f(i)$ = card{v∈V(G):f(v)=i}and $e_f(i)$ = card {e∈E(G) : f*(e) = i}. A labeling f of a graph G is said to be friendly if $|v_f(0) - v_f(1)| \leq 1$. The friendly index set of the graph G, FI(G) is defined as {$|e_f(0) - e_f(1)|$: the vertex labeling f is friendly}. Lee and Ng [3] define the friendly index set of graphs. This is a generalization of graph cordiality.

This paper consists of four sections. Section I is the introduction part of friendly index set. Section II contains preliminaries and notations.Section III contains main results and illustrations and section IV contains conclusion part .

II. PRELIMINARIES AND NOTATIONS

Definition 2.1[4] : A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

Definition 2.2 [10] : A fan graph f_n obtained by joining all vertices of a path P_n to further vertex, called the centre. Thus f_n contains n+1 vertices c,v_1,v_2, ..., v_n and (2n-1) edges say cv_i, $1 \leq i \leq n$ and $v_i v_{i+1}$, $1 \leq i \leq n-1$

Definition 2.3 [2] : Star $K_{1,n}$ is the graph with one vertex of degree n called apex and n vertices of degree one.

Definition 2.4[7] : A Umbrella graph U(m,n) is the graph obtained by joining a path P_n with the central vertex of a fan f_m.

Definition 2.5 [5] : For each vertex v of a graph G take a new vertex v. Join v_1 to all the vertices of G adjacent to v. The graph spl(G) thus obtained is called splitting graph of G.

Definition 2.6 [6] : A globe is a graph obtained from two isolated vertex are joined by n paths of length two. It is denoted by Gl(n)

III. MAIN RESULTS

Theorem 3.1 : The Umbrella graph U(m,n) where n = 2 has the friendly index set {0,2,4,...,m+1} if m is odd, $m \geq 9$ and {0,2,4,...,m} if m is even, $m \geq 4$

Proof:

Let G = U(m,2)

The vertex set of U(m,2) is V(G) = {x_1,x_2,...,x_m,y_1,y_2}
The edge set of U(m,2) is
E(G) = {$(x_i,x_{i+1})|1\leq i \leq m-1$} ∪{($y_1$,$y_2$)}∪{($x_i$,$y_1$), $1\leq i\leq m$}
Then |V(G)| = m+2, |E(G)| = 2m

Case (i) : m is odd

First label alternatively with 0's and 1's starting with 0 in x_i's and label y_1 by 1 and y_2 by 0. Then v(0) - v(1) = 1 and |e(0)-e(1)| = m+1

Next interchange only y_1 and y_2. Then v(0) - v(1) =1 and |e(0) - e(1)| = m-1.

Next label x_1 by '0' and next two vertices by '1' and following next two vertices by '0' and then alternatively by 0's and 1's starting with '1' and label y_1 by 1 and y_2 by 0. Then |v(0) - v(1)| = 1 and |e(0) - e(1)| = m-3.

Then interchange only y_1 and y_2. Again |v(0) - v(1)| = 1 and |e(0) - e(1)| = m-5.

Next label x_1 by 0 and next three vertices by 1 and following next three vertices by 0 and then alternatively by 0's and 1's starting with 1. Then label y_1 by 1 and y_2 by 0. Then |v(0) - v(1) | = 1 and |e(0) - e(1) | = m-9.

Continuously proceeding like this ie, label x_1 by 0 and next continuously label up to $\frac{m-3}{2}$ vertices by 1 and $\frac{m-3}{2}$ vertices by 0 and then alternatively 0's and 1's starting with 1.

At the last step we get the vertex is labeled friendly and | e(0) - e(1)| = 0.

∴The friendly index set is {0,2,4,...,m+1}

Case (ii) : m is even

Fix y_1 by 1 and y_2 by 0.

Label the vertices x_1, x_2,..., x_m alternatively by 0's and 1's starting with '0'. Therefore the vertex labeling is friendly and | e(0) - e(1)| = m.

Keep x_1 and x_2 remain unchanged and then label all the other vertices ie, x_3,x_4,...,x_m by their complement. Then the vertex labeling is friendly and |e(0) - e(1)| = m-2.

Then keep x_1, x_2, x_3, x_4 remain unchanged and then label all the other vertices ie, x_5, x_6,...,x_m by its complement. Again the vertex labeling is friendly and |e(0) - e(1)| = m-4.

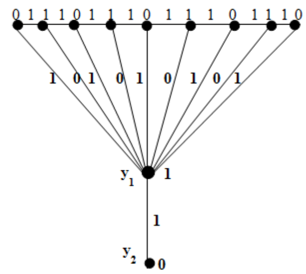
Continue this process to get the friendly index set {2,4,...,m}

Atlast change the labels x_m and y_1 to its complement and we get the vertex is friendly and | e(0) - e(1)| = 0.

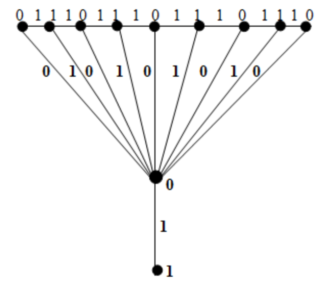
∴The friendly index set is {0,2,4,...,m}.

Illustration 3.1 :

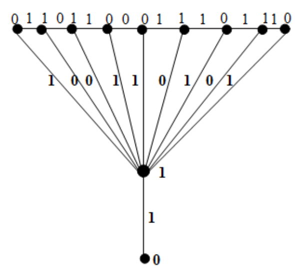
FI(U(9,2)) = {0,2,4,6,8,10}

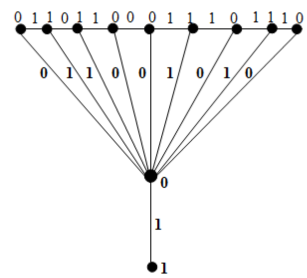


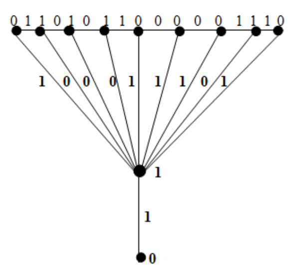
|e(0) - e(1)| = 10



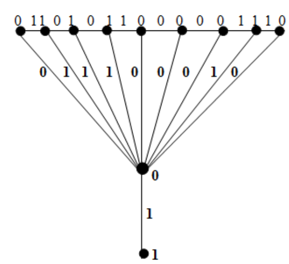
|e(0) - e(1)| = 8





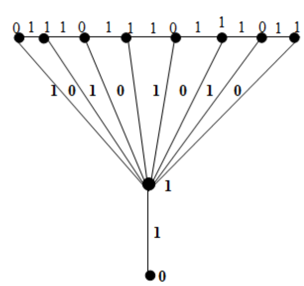


|e(0) - e(1)| = 2

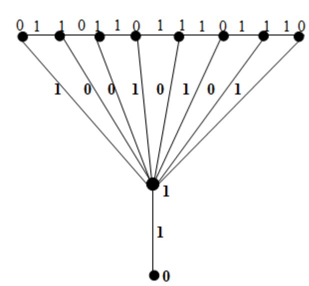


|e(0) - e(1)| = 0

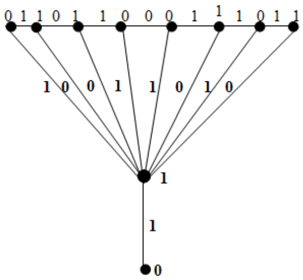
FI(U(8,2)) = {0,2,4,6,8}



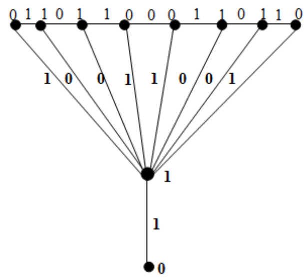
|e(0) - e(1)| = 8



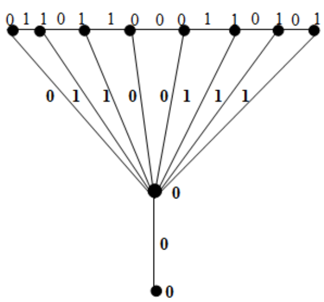
|e(0) - e(1)| = 6



|e(0) - e(1)| = 4



|e(0) - e(1)| = 2



|e(0) - e(1)| = 0

Theorem 3.2 :

The friendly index set of spl($k_{1,n}$) is {0,2,4,...,n}, if n is even and {1,3,...,n}, if n is odd.

Proof :

Let $v_1,v_2,..., v_n$ be the pendant vertices, v be the apex vertex of $K_{1,n}$ and $u,u_1,u_2,...u_n$ are the added vertices corresponding to $v,v_1, v_2,..., v_n$ in spl ($k_{1,n}$)

Let V(G) = { $v, v_1, v_2, ..., v_n, u, u_1,u_2,...,u_m$}

E(G) = {$vv_i/1\le i \le n$} ∪ {$uu_i/1 \le i \le n$} ∪ {$uv_i/1 \le i \le n$}

Thus spl ($k_{1,n}$) has 2n+2 vertices and 3n edges.

Case (i)

When n is even

Fix u = 0 and v = 1

First label the vertices v_i's as (0,0,...,0) and u_i's as (1,1,...,1) in spl (k1,n). Then the vertex labeling is friendly and |e(0) - e(1)| = n

In next step decrease one vertex labeling ie, '0' from v_i's and label with its complement ie, '1. Also decrease one vertex labeling, ie '1' from u_i's and label with its complement ie, '0'. Again the vertex labeling is friendly and |e(0) - e(1)| = n-2

The vertices remain unchanged and continue the above step upto $\left(\frac{n}{2}\right)$ vertices having the label 0 in v_i's and $\left(\frac{n}{2}\right)$ vertices having the label 1 in u_i's.

∴The friendly index set is {0,2,4,...,n}

Case (ii)

When n is odd

Fix u = 0 and v= 1

First label the vertices v_i's as (0,0,...,0) and u_i's as (1,1,...,1) in spl($k_{1,n}$). Then the vertex labeling is friendly and |e(0) - e(1)| = n

In the next step decrease one vertex labeling ie '0' from v_i's and label with its complement ie, '1'. Also decrease one vertex labeling ie '1' from u_i's and label with its complement ie, '0'. Again the vertex labeling is friendly and |e(0) - e(1)| = n-2

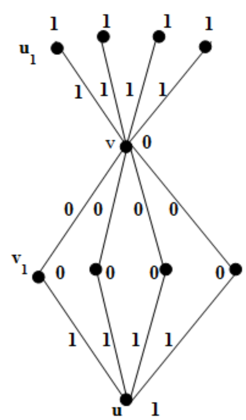
All changed vertex remain unchanged and continue this process upto $\left[\frac{n}{2}\right]$ vertices having the label 1 in v_i's and $\left[\frac{n}{2}\right]$ vertices having the label 0 in u_i's.

∴The friendly index set is {1,3,...,n}

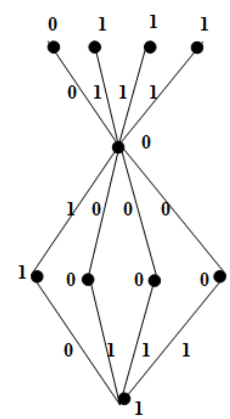
Illustration 3.2

When n is even

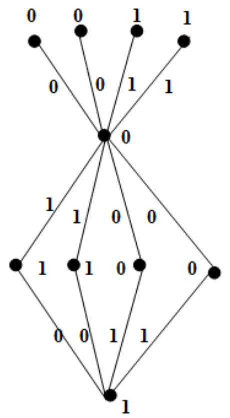
FI (Spl($k_{1,4}$)) = {0,2,4}



|e(0) - e(1)| = 4



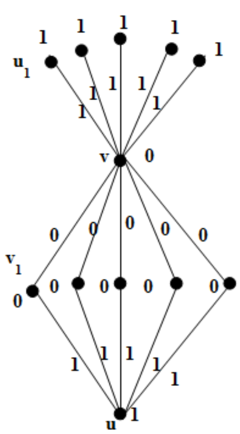
|e(0) - e(1)| = 2



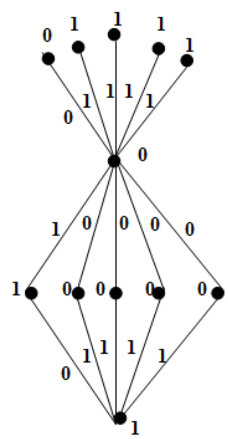
|e(0) - e(1)| = 0

when n is odd

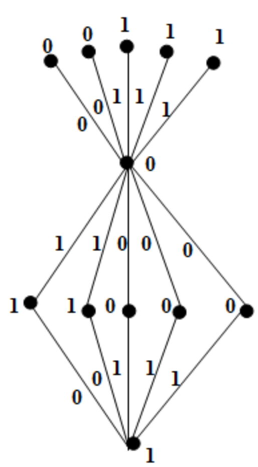
FI (Spl($k_{1,5}$)) = {1,3,5}



|e(0) - e(1)| = 5



|e(0) - e(1)| = 3



|e(0) - e(1)| = 1

Theorem 3. 3 :

FI(Gl(n)) = {0,4}, if n is even and FI(Gl(n)) = {0,2,6}, if n is odd

Proof :

Let (x_1,x_2,...,x_n) be the vertices of n paths of length two. Let u,v be the vertices which are joined by those n paths.

It has n+2 vertices and 2n edges.

Case(i) : When n is even

Label u by 0 and v by 1 and label $\left(\frac{n}{2}\right)$ vertices of x_i's by 0 and other $\left(\frac{n}{2}\right)$ vertices by 1. Then v(0) - v(1) = 0. Therefore |e(0) - e(1)| = 0

Next label u and v by 0 and label $\left(\frac{n}{2}\right) - 1$ vertices of x_i's by 0 and $\left(\frac{n}{2}\right) + 1$ vertices of x_i's by 1. Again v(0) - v(1) = 0. Then |e(0) - e(1)| = 4

Then the friendly index set is {0,4}

Case (ii) : When n is odd

Label u and v by 0 and label $\left\lceil\frac{n}{2}\right\rceil$ vertices of x_i's by 0 and the remaining vertices by 1. Then the vertex labeling is friendly with v(0)-v(1)=1. Therefore |e(0) - e(1)| = 2.

Next Label u by 0 and v by 1 and label $\left\lceil\frac{n}{2}\right\rceil + 1$ vertices of x_i's by 0 and the remaining vertices by 1.

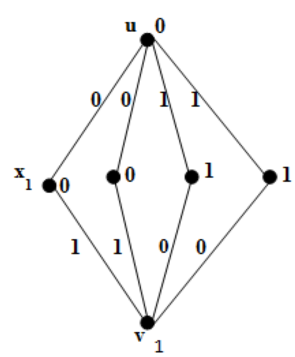
Then v(0)-v(1) = 1. Therefore |e(0) - e(1)| = 0.

Next label u and v by 1 and label $\left\lceil\frac{n}{2}\right\rceil + 2$ vertices of x_i's by 0 and the remaining vertices by 1. Again v(0) - v(1) = 1 and |e(0) - e(1)| = 6.

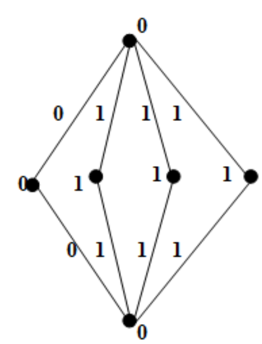
Then the friendly index set is {0,2,6}

Illustration 3.3

FI (Gl(4)) = {0,4}

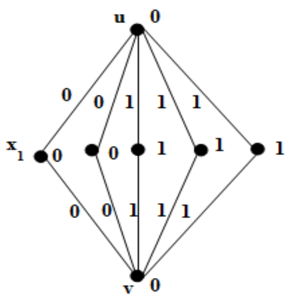


|e(0) - e(1)| = 0

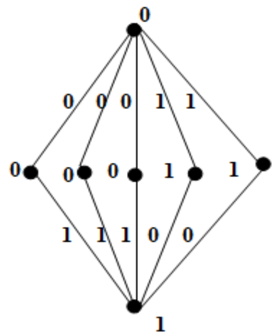


|e(0) - e(1)| = 4

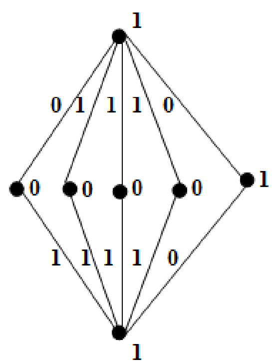
FI (Gl(5)) = {0,2,6}



|e(0) - e(1)| = 2



|e(0) - e(1)| = 0



|e(0) - e(1)| = 6

Theorem 3.4 :

FI (P_2+mk_1) = {1,3}, if m is even and FI(P_2+mk_1) = {1,5} if m is odd.

Proof :

Consider a path P_2 with two vertices v_1,v_2. Let $y_1,y_2,...,y_m$ be the m isolated vertices. Join v_1,v_2 with y_i, $1 \le i \le m$. The graph obtained is P_2+mk_1. The vertex set of G is V(G) = {$v_1,v_2,y_1,y_2,...,y_m$}. The edge set of G is

E(G) = { ($v_1 v_2$) } ∪ { ($v_1 y_i$) | $1 \le i \le m$} ∪ { ($v_2 y_i$) | $1 \le i \le m$}

Then | V (G) | = 2+m and | E (G) | = 2m+1

First consider an even m

Label v_1 and v_2 as 0 and $y_{\frac{m}{2}}, y_{\frac{m}{2}+1}$, ……$y_m$ as 1 and the remaining vertices as 0. Then the vertex labeling is friendly with v(1) – v(0) = 0 and | e(1) – e(0) |= 3.

Then label v_1 as 0 and v_2 as 1 and label the vertices of y_i as alternatively 0 and 1. This vertex labeling is also friendly with v(1) –v(0) = 0 Then | e(0) – e(1)| = 1.

∴The friendly index set is { 1,3}.

Next consider an odd m.

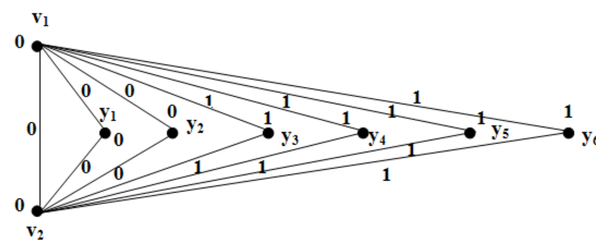
Label v_1 and v_2 as 0 and $y_{[\frac{m}{2}]+1}$, ….. y_m as 1 and all the remaining vertices as 0. This labeling is friendly with v(0) – v(1) = 1. Then | e(0) – e(1) |= 1.

Then label v_1 and v_2 as 0 and label $y_{[\frac{m}{2}]}$ ….. y_m as 1 and all the remaining vertices as 0. This labeling is also friendly with v(1) – v(0) = 1 Then |e(0) – e(1)| = 5

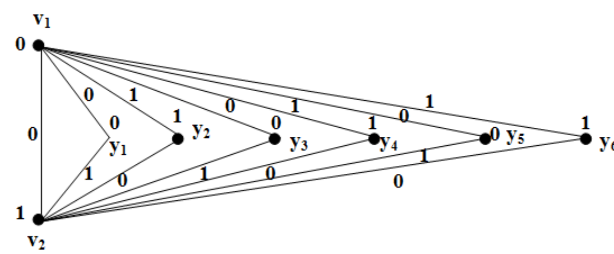
∴The friendly index set is {1,5}.

Illustration 3.4

FI (P_2+6K_1) = {1,3}

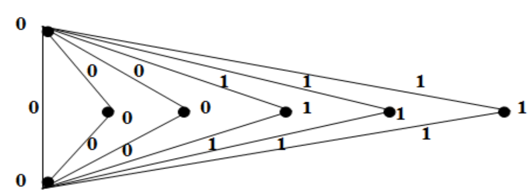


|e(0) - e(1)| = 3

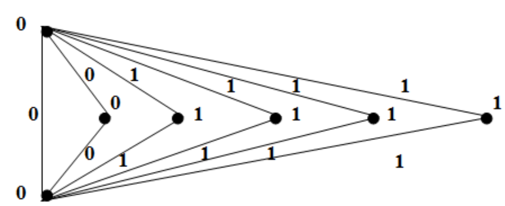


|e(0) - e(1)| = 1

FI (P_2+5K_1) = {1,5}



|e(0) - e(1)| = 1



|e(0) - e(1)| = 5

Theorem 3.5 :

Union of a path and a star sharing a vertex in common has the friendly index set { 1,3,5,…..2n-5}

Proof

Let the n vertices of path P_n be u_1, u_2, ….., u_n. Let the n spokes of star S_n be v_1, v_2, …. v_n and u_1 be the centre vertex of the star. Identify u_1 and v_1. Let the graph so obtained is G.

Clearly G has 2n vertices and 2n-1 edges.

First label (u_1, u_2, u_3… u_n) = (1,0,0,……0) and (v_1, v_2, …. v_n) = (0,1,1,…….1) The vertex labeling is friendly and | e(0) – e(1) | = 2n-5

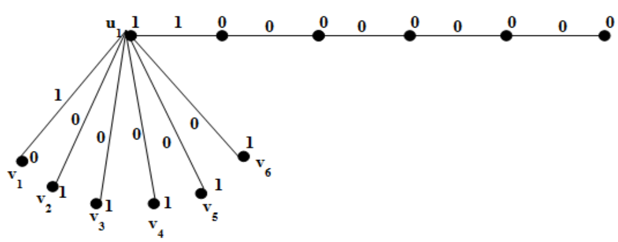
Now keep all the vertex remain unchanged. Next change the vertex u_2 and v_2 to its complement. This labeling is friendly. Then | e(0) – e(1) | = 2n-3

Now keep all the vertex remains unchanged and change the vertex u_3 and v_3 to its complement. This labeling is vertex friendly. Then | e(0) – e(1) | = 2n-1

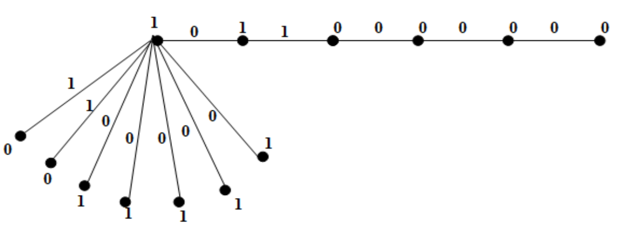
Continue this process upto changing the vertex u_{n-2} and v_{n-2} to its complement.

Then the friendly index set is { 1,3,5, ………, 2n-5}

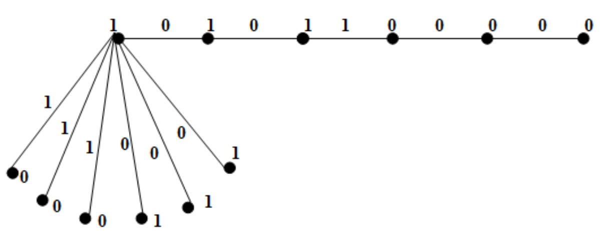
Illustration 3.5 :



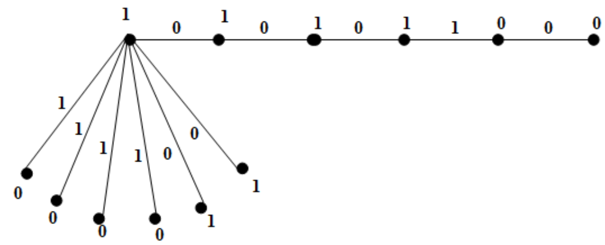
|e(0) - e(1)| = 7



|e(0) - e(1)| = 5



|e(0) - e(1)| = 3



|e(0) - e(1)| = 1

IV. CONCLUSION

Labeling of graphs in graph theory is one of the interesting research topic in the recent days. Here we have find new results of friendly index sets of five graphs related to labeling of graphs. Similar work can be carried out for other graphs also.

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