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Group Action on Fuzzy Normal Hemi-Subring

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Abstract - *Zhan, 2005* **[1]** noted properties of fuzzy left h- ideals in hemirings with t- norm. *Akram and Dar, 2007* **[2]** analyzed fuzzy left h-ideal in hemi rings with respect to s-norm. They studied anti-fuzzy Left h- ideals in Hemirings **[3]**. *Prince Williams, 2007* **[4]** identified fuzzy Left h-ideal of Hemirings. **In this article,** groupoid action on each of (1).Normal fuzzy sub-hemi-rings (2). Union of two normal fuzzy sub-hemiring (3). Direct product of two normal fuzzy sub-hemi rings (4). Homomorphic image and its pre-image of normal fuzzy sub-hemiring (5). Relation between relation on R × R and a fuzzy sub-hemiring R.

Keywords: Fuzzy sub-hemi-ring, Fuzzy normal sub-hemi-ring

I. INTRODUCTION

Acaretal., 2010 **[5]** discussed soft sets and soft rings. *Atagün and Sezgin, 2011* **[6]** noted on soft substructures of rings. *EceYetkin and NecatiOlgun, 2011* **[7]** highlighted direct product of fuzzy groups and fuzzy rings. *Fengetal., 2008* **[8]** visualized few properties on soft semi rings. *JayantaGhoshet al., 2011* **[9]** constructed fuzzy soft rings and fuzzy soft ideals. *Martinez, 1999* **[10]** initiated prime and primary L-fuzzy ideals of L-fuzzy rings. *Onaret al., 2012* **[11]** destroyed fuzzy soft gamma-ring. *Ratnabala Devi, 2009* **[12]** derived some algebraic natures on the intuitionistic Q-fuzzy ideals of near rings. *Shum and Akram, 2008* **[13]** extracted few concepts on intuitionistic (T,S)-fuzzy ideals of near-rings. *Zhiming Zhang 2012* **[14]** observed some usual properties on intuitionistic fuzzy soft rings.

II. PRELIMINARIES AND DEFINITIONS

Definition 2.1: There are many concepts of universal algebras generalizing an associative ring (R,+,.). An algebra (R, +,.) is a semi-ring if (R, +) and (R, .) are semi-groups satisfying a.(b+c) = a.b +a.c& (b+c).a = b.a + c.a for all a, b, c in R. A semi-ring R is additively commutative if a+ b = b +a for all a, b in R. A semi-ring R has an identity 1 defined by 1.a= a.1 = a &zero, defined by 0 + a = a + 0 = a; & a.0 = 0.a = 0 for all a in R. A semi-ring R is a hemi ring if it is an additively commutative with zero.

Definition 2.2: Let (S, +) be a group, and G be a non-empty set. Then G acts on S if there exists a function $*: G \times S \to S$ (denoted $*$ (g, s) = g $*$ s for all g $\in$G, and s $\in$ S) such that es = s and (g + h) $*$ s = g $*$ (h $*$ s) for all s in S, and for all g, h in G. **Here g^s = g $*$ s for all g in G, and s in S.**

Definition 2.3: The **union** of two-fuzzy subsets A, B on X acted a set group S is defined as
$(A \cup B)(x^s) = \max\{A(x^s), B(x^s)\}$ for all x in X and s in S.
Definition 2.4: The **intersection** of two-fuzzy subsets A, B on X acted a set group S is definedas $(A \cap B)(x^s) = \min\{A(x^s), B(x^s)\}$ for all x in X and s in S.

Definition 2.5: Let (R, +, .) be a hemi-ring. A fuzzy subset A of R is a fuzzy sub-hemi-ring (FSHR) of R acted by S if it satisfies the following conditions:

(1) $A((x + y)^s) \geq \min\{A(x^s), A(y^s)\}$ and

(2)$(A((xy)^s) \geq \min\{A(x^s), A(y^s)\}$ for all x, y in R and s in S.

Definition 2.6: A fuzzy sub-hemi-ring A of a hemi-ring R is a fuzzy normal sub-hemi-ring (FNSHR) of R acted by S if $A((xy)^s) = A((yx)^s)$ for all x, y in r and s in S.

Definition 2.7: Let A, B be fuzzy subsets of G and H respectively, both acted by S.
The product (A × B) of A and B, is defined as
(A × B) = {(x, y), (A × B)(x, y): for all x in g, y in h and s in S}.

Definition 2.8: Let A be a fuzzy subset in a set, the strongest fuzzy relation on H acted by S is a fuzzy relation on A is V given by V((x, y)^s) = min {A(x^s), A(y^s) for all x, y in H and s in S.

Definition 2.9: Let (R, +, .) and (R′, +, .) be any two hemi rings. Then the function f: R →R′ is a **hemi-ringhomomorphism** if it satisfies the following axioms:

(i). f(x+y) = f(x) + f(y) and (ii). f(xy) = f(x)f(y)

for all x, y in R and s in S.

Definition 2.10: Let (R, +, .) and (R′, +, .) be any two hemi rings. Then the function f: R →R′ is a **hemi-ringanti-homomorphism** if it satisfies the following axioms:

(i). f(x + y) = f(y) + f(x) and (ii). f(xy) = f(y)f(x)

for all x, y in R and s in S.

Definition 2.11: Let (R, +, .) and (R′, +, .) be any two hemi rings. Let f: R →R′be any function and A be a fuzzy sub-hemi-ring in R acted by S. If V is a fuzzy sub-hemi-ring in f(R) = R′. defined by V(y^s) = [V(x^s) for all y in R′ and s in S. Here f^{-1}(V) is the pre-image of V under f.

III. FUZZY NORMAL SUB-HEMI-RING ACTED BY GROUPOID

Theorem 3.1: Let (R, +,.) be a hemi-ring. If A, B are two fuzzy normal sub hemi rings of R, both acted by S, then their intersection (A ∩ B) is a fuzzy normal sub hemi ring of R acted by S.

Proof: Let x, y in R, and s in S. Let A = { (x^s, A(x^s): x in R and s in S} and B = { (y^s, B(y^s): yin R and s in S} be a fuzzy normal sub-hemi-rings of a hemi-ring R acted by S. Then (A∩B) is a fuzzy sub-hemi-ring of R acted by S, since A, B are two fuzzy sub-hemi-rings of R.

Further (A∩B) ($(xy)^s$) = min {A($(xy)^s$), B($(xy)^s$)}

= min {A($(yx)^s$), B($(yx)^s$)}

= (B∩A) ($(xy)^s$)

for all x, y in R an s in S.

Hence (A∩B) is a fuzzy normal sub-hemi-ring of R acted by S.

Theorem 3.2: Let (R, +, .) be a hemi ring. The intersection of a family of fuzzy normal sub hemi rings of R acted by S is a fuzzy normal sub hemi ring of r acted by S.

Proof: Let $\mathbf{(A_i\}_{i\in I}}$be a family of fuzzy normal sub-hemi-rings of R, each acted by S and A .

Then the intersection of a family of fuzzy sub hemi rings of R acted by S is a fuzzy sub hemi ring of R acted by S.

So A($(xy)^s$) = A_i($(xy)^s$) = A_i($(yx)^s$) = A($(yx)^s$). for all x, y in R and s in S..

A is a fuzzy normal sub-hemi-ring of R acted by S.

Theorem 3.3: Let A, B be fuzzy sub hemi rings of hemi-rings G and H respectively, both acted by S. If A, B are fuzzy normal sub hemi rings, then A × B is a fuzzy normal sub hemi ring of G × H acted by S.

Proof: Let A, B be fuzzy normal sub-hemi rings of the hemi rings G& H respectively, both acted by S. Clearly (A × B) is a fuzzy sub hemi ring of G × H.

Let x, y ∈ G × Hwhere x = (x_1, y_1); y = (x_2, y_2); x_1, x_2∈G, y_1, y_2∈ H, and s ∈S.

Then (A × B) ($(xy)^s$) = (A × B) (((x_1, y_1). (x_2, y_2))^s)

= (A × B) ((($x_1.x_2$). ($y_1.y_2$))^s)

≥ min{A(($x_1x_{2)}$)^s), B(($y_1y_{2)}$)^s)}

≥ min{A(($x_2x_{1)}$)^s),B(($y_2y_{1)}$)^s)}

= (A × B) ((($x_2.x_1$). ($y_2.y_1$))^s)

= (A × B) ($(xy)^s$).

Hence (A × B) is a fuzzy normal sub hemi ring of (G × H) acted by S.

Theorem 3.4: Let A be a fuzzy subset in a hemi ring R acted by S and V be the strongest fuzzy relation on R acted by S. Then A is a fuzzy normal sub hemi ring of R**iff**V is a fuzzy normal sub hemi ring of R×R acted by S.

Proof: Assume that A is a fuzzy normal sub hemi ring of a hemi ring R acted by S.

Then for any x = (x_1, y_1); y = (x_2, y_2) are in R ×R and s in S.

Clearly V is a fuzzy sub hemi ring of the hemi ring R acted by S.

Then V($(xy)^s$) = V((((x_1, y_1). (x_2, y_2))^s)

= V(((($x_1.x_2$), ($y_1.y_2$))^s)

= min {A(($x_1.x_2$), A($y_1.y_2$))^s)

= min {A(($x_2.x_1$), A ($y_2.y_1$))^s)

= V(((($x_2.x_1$), ($y_2.y_1$))^s)

= V((((x_2, y_2). (x_1, y_1))^s)

= V($(yx)^s$).

This proves that V is a fuzzy **normal** sub hemi ring of R × R acted by S.

Conversely

Assume that V is a fuzzy **normal** sub hemi ring of R ×R acted by S.

Then for any x = (x_1, y_1); y = (x_2, y_2) are in R ×R and s in S, and

A is a fuzzy sub hemi ring of R acted by S,

ThenA($(xy)^s$) = min {A($(x_1, y_1)^s$, $(x_2, y_2)^s$)

= V ((($x_1.y_1$). ($x_2.y_2$)$)^s$)

= V ((($x_1.x_2$), ($y_1.y_2$)$)^s$)

= V ($(xy)^s$)

= V ($(yx)^s$)

= V ((($x_2.x_1$), ($y_2.y_1$)$)^s$)

= min {A($(x_2, y_2)^s$, $(x_1, y_1)^s$)

= A($(yx)^s$).

Therefore A is a fuzzy **normal** sub hemi ring of R acted by S

Theorem 3.5: Let (R, +,.) and (R′, +, .) be hemi rings. Homomorphic image of a fuzzy normal sub hemi ring of R acted by S is a fuzzy normal sub hemi ring in Image (R) = R′.

Proof: Let (R, +, .) and (R′, +, .) be hemi rings, and f: R →R′ be a homomorphism. Then f(x + y) = f(x) + f(y) and f(xy) = f(x)f(y) for all x, y in R.

Let A be a fuzzy normal sub hemi ring of R acted by S, and V be the homomorphic image of A under f.

Claim is that V is a fuzzy normal sub hemi ring of R′ acted by S.

Clearly V is a fuzzy sub hemi ring of R′ acted by S, since A is a fuzzy sub hemi ring of R acted by S. For f(x) and f(y) in R′ and s in S,

V(f(x).f(y)$)^s$) = V ((f(xy$)^s$) ≥ A ($(xy)^s$)

= A ($(yx)^s$) ≤ V ((f(yx$)^s$)

= V (f(x).f(y)$)^s$).

Since f is homomorphism,

V(f(x).f(y)$)^s$) = V(f(y).f(x)$)^s$).

Hence V is a fuzzy normal in R′ acted by S.

Theorem 3.6: Let (R, +,.) and (R′, +, .) be hemi rings. Homomorphic pre-image of a fuzzy normal sub hemi ring of R′ acted by S is a fuzzy normal sub hemi ring in R.

Proof: Let (R, +,.) and (R′, +, .) be hemi rings, and f: R →R′ be a homomorphism.

Thenf(x + y) = f(x) + f(y) and f(xy) = f(x)f(y) for allx, y in R.

Let V be a fuzzy normal sub hemi ring of R′ acted by S, and A = f^{-1}(V) be a homomorphic pre-image of V under f. Then clearly A is a fuzzy sub hemi ring of R acted by S.

Claim is that A is a fuzzy **normal** in R. Let x, y ∈ r and s ∈S.

Thus A($(xy)^s$) = V((f(xy$)^s$)

= V ([f(x).f(y)$]^s$)

= V ([f(y).f(x)$]^s$)

= V ([f(yx)$]^s$)

= A($(yx)^s$).

for all x, y in R and s in S.

Hence A is a fuzzy normal in R acted by S.

Theorem 3.7: Let (R, +, .) and (R′, +, .) be hemi rings. Anti-homomorphic image of a fuzzy normal sub hemi ring of R acted by S is a fuzzy normal sub hemi ring in Image (R) = R′.

Proof: Let (R, +, .) and (R′, +, .) be hemi rings, and

f: R →R′ be an anti-homomorphism.

Then f(x + y) = f(y) + f(x) and f(xy) = f(y)f(x) for all x, y in R.

Let A be a fuzzy normal sub hemi ring of R acted by S,and V be the homomorphic image ofA under f.

Claim is that V is a fuzzy normal sub hemi ring of R′ acted by S.

Clearly V is a fuzzy sub hemi ring of R′ acted by S,

SinceA is a fuzzy sub hemi ring of R acted by S.

For f(x) and f(y) in R′ and s in S,

V(f(x).f(y)$)^s$) = V ((f(yx$)^s$) ≥ A ($(yx)^s$)

= A ($(xy)^s$) ≤ V ((f(xy$)^s$)

= V (f(y).f(x)$)^s$).

Since f is homomorphism,

V (f(x).f(y)) ^{s}) = V (f(x).f(y))^{s}).

Hence V is a fuzzy normal sub hemi ring of R′ acted by S.

CONCULSION

- Intersection of two fuzzy normal sub hemi rings is also a fuzzy normal sub hemi ring.
- Union of two fuzzy normal sub hemi rings is also a fuzzy normal sub hemi ring.
- Direct product of two normal fuzzy sub-hemi rings is also normal fuzzy sub hemi ring.
- If (R, +,.) and (R′, +, .) be hemi rings. Then homomorphic pre-image of a fuzzy normal sub hemi rings in Image (R) = R′.
- If (R, +, .) and (R′, +, .) be hemi rings. Then Anti-homomorphic image of a fuzzy normal sub hemi ring in Image (R) = R′.

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