

## Vertex Antimagic Edge Labeling of the Mongolian Tent Graph

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**Abstract-** In graph theory, the word 'label' to the vertices and the edges refer to the weights associated with the labels and edges. The concept of magic and antimagic labeling in graphs has been one of the much sought after topics in Graph Theory. Researchers have been using this concept effectively to different types of graphs. The impetus to apply the concept of vertex antimagic edge labeling is the definition of the Mongolian Tent Graph  $M_n$ . In this paper, we have shown that the Mongolian tent graph admits vertex antimagic edge labeling.

**Keywords-** Antimagic Labeling, Mongolian Tent Graph, Vertex Antimagic Edge Labeling

**AMS Subject Classification:** 05C78

### I. INTRODUCTION

Grid graphs are one of the most prominent types of graphs with rich hassle free applications in interconnected networks. They are easy to construct with minimum congestion. Grid graphs are used to construct different graphs such as extended grids, Mongolian Tent etc. Labeling in graph theory has been one of the most fascinating and happening topics with a lot of variety and applications. The acuity of labeling the vertices and edges in graphs has thrived with types of labeling being applied to different graphs by the research scholars. One among the prominent types of labeling is the magic and antimagic labeling. The motivation behind the development of this paper is to apply the concept of antimagic labeling to diverse types of graphs namely the Mongolian Tent graph.

In section I, we consider a brief introduction of the vertex antimagic edge labeling and the graphs where it was applied. In section II, a literature survey, in brief, is presented with the concept of vertex antimagic edge labeling is being applied to different graphs. We also consider a special case of the grid graph, the Mongolian Tent graph  $M_n$ , its definition and the construction. In section III, we have proved that the Mongolian Tent graph admits vertex antimagic edge labeling. The proof is constructed with multiple claims in two cases, one where  $n$

$\neq 2^k$  and when  $n = 2^k$ . In section IV, a specific labeling function is given for the Mongolian Tent graph formed from a  $3 \times 3$  grid.

We have concluded the paper in section V with the notion that the vertex antimagic edge labeling can also be applied for the Mongolian Tent graph that can be formed from a grid  $P_m \times P_n$ ,  $m \neq n$  and extended grid graphs.

#### Definition 1.1:

Let  $G$  be a graph and let  $V(G)$  and  $E(G)$  be the set of vertices and edges of  $G$  respectively. The weight  $w(v)$  of a vertex  $v$  in  $V(G)$  under an edge labeling is given by  $g: E(G) \rightarrow \{1, 2, \dots, |E(G)|\}$ , is the sum of values  $g(e)$  assigned to the edges that are incident with the vertex  $v$ .

#### Definition 1.2:

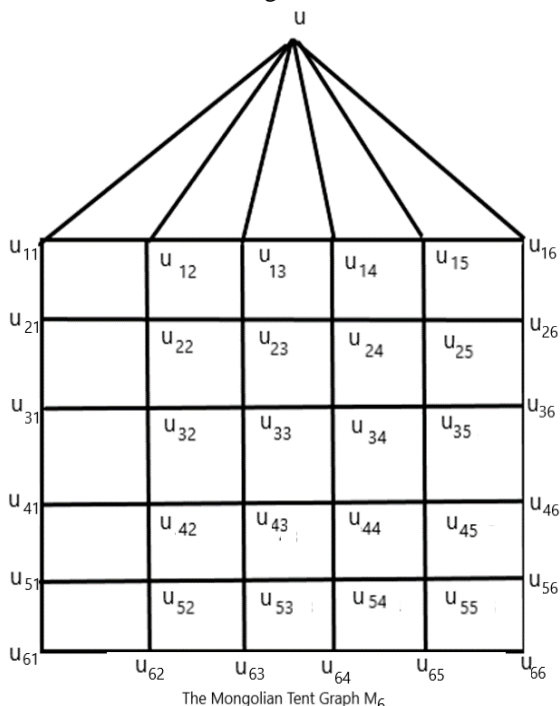
A connected graph  $(V(G), E(G))$  is said to be  $(a, d)$  antimagic labelled graph if there exist integers  $a > 0$ ,  $d \geq 0$  and a bijective map  $g: E(G) \rightarrow \{1, 2, \dots, |E(G)|\}$  such that the induced mapping  $f_g: V(G) \rightarrow W$  where  $W = \{a, a+d, a+2d, \dots, a+(|V(G)|-1)d\}$  is also a bijection.

### II. RELATED WORK:

The concept of magic labeling was first introduced by *J. Sedlacek* [8][9] in 1963. As an extension of magic labeling, vertex labeling was first suggested by *J.A. Macdougall*,

M. Miller and W.D. Wallis [7] in 2002. The concept of antimagic labeling was first introduced by N. Hartsfield and G. Ringel [4] in 1990 and they proved that the paths  $P_n$  ( $n \geq 3$ ), cycles  $C_n$ , wheels  $W_n$  and complete graphs  $K_n$  ( $n \geq 3$ ) are all antimagic. They also conjectured that every connected graph except  $K_2$  is antimagic. The concept of  $(a, d)$  antimagic labeling was first introduced by R. Bodendiek and G. Walther [1][2] in 1993. An extensive work on magic and antimagic labeling was done by J.A. Gallian [3].

The Mongolian Tent Graph  $M_n$  as defined by Sharmila Mary Arul, K. Subashini [10], is obtained by joining a vertex on top of  $P_n \times P_n$  grid with the top row vertices of the grid. The number of vertices in the graph  $M_n$  is  $n^2 + 1$  and the number of edges is  $2n^2 - n$ .



### III. MAIN RESULT

**THEOREM.** The Mongolian Tent graph  $M_n$  ( $n > 3$ ) admits vertex antimagic edge labeling.

**Proof:** Consider the Mongolian Tent graph  $M_n$  containing  $n$  vertices,  $n \neq 2^k$ . The graph  $M_n$  is constructed from a grid  $P_n \times P_n$  formed from the vertices  $\{u_{ij} / 1 \leq i \leq n, 1 \leq j \leq n\}$  and let  $u$  be the vertex above the grid that is adjacent with every vertex of the top row. We prove the theorem in two cases.

**Case (1).** Let the vertex set be defined by  $V(M_n) = V_1 \cup V_2$ , where  $V_1 = \{u_{ij} / 1 \leq i \leq n, 1 \leq j \leq n\}$  and  $V_2 = \{u\}$  and the edge set be given by  $E(M_n) = E_i \cup E'_i \cup E''$ , where

$$E_i = \{u_{ij}u_{i(j+1)} / 1 \leq j \leq n-1, 1 \leq i \leq n,$$

$$E'_i = \{u_{ji}u_{(j+1)i} / 1 \leq j \leq n-1, 1 \leq i \leq n \text{ and}$$

$$E'' = \{u u_{1j} / 1 \leq j \leq n\}$$

Define a function  $f: E \rightarrow R$  as

$$\text{For } e \in E'', f(u u_{1j}) = 1 + 2(j - 1)n, 1 \leq j \leq n,$$

For  $e \in E'_i$ ,

$$f(u_{ji} u_{(j+1)i}) = \begin{cases} 2((i - 1)n + j) + 1, & 1 \leq j \leq n, 1 \leq i \leq n - 2 \\ 2i(n - 1) + j + 1, & 1 \leq j \leq n, i = n - 1 \end{cases}$$

$$\text{For } e \in E_i, f(u_{ij} u_{i(j+1)}) = 2((j - 1)n + i), 1 \leq i \leq n, 1 \leq j \leq n - 1,$$

We hereby prove the uniqueness of edge labels using the following claims;

**Claim (1).** For some  $j \neq k$ , assume that  $f(u u_{1j}) = f(u u_{1k})$

$$\Rightarrow 1 + 2(j - 1)n = 1 + 2(k - 1)n$$

$$\Rightarrow j = k, \text{ a contradiction to the assumption.}$$

$$\Rightarrow \text{The edge labels are unique.}$$

**Claim (2).** For some  $i$  and  $j$ , it is evident that  $f(u u_{1j}) \neq f(u_{ij} u_{i(j+1)})$  as  $1 + 2(j - 1)n$  is an odd value and

$2[(j - 1)n + i]$  is even. Hence the edge labels are unique.

**Claim (3).** For some  $i$  and  $j$ , in interval  $1 \leq i \leq n, 1 \leq j \leq n - 2$ , assume that  $f(u u_{1j}) = f(u_{ij} u_{i(j+1)})$

$$\Rightarrow 1 + 2(j - 1)n = 1 + 2[(i - 1)n + j]$$

$$\Rightarrow n(j - i) = j$$

$$\Rightarrow n < j, \text{ a contradiction}$$

$$\Rightarrow \text{The edge labels are unique.}$$

**Claim (4).** In the interval  $1 \leq i \leq n, j = n - 1$ , assume that  $f(u u_{1j}) = f(u_{ji} u_{(j+1)i})$

$$\Rightarrow 1 + 2(j - 1)n = 2i(n - 1) + j + 1$$

$$\Rightarrow n = \frac{1}{2} \left( \frac{j - 2i}{j - i - 1} \right). \text{ Choose } i \text{ so that } i < \left\lfloor \frac{j}{2} \right\rfloor \text{ and hence}$$

$$n < 0, \text{ a contradiction}$$

$$\Rightarrow \text{The edge labels are unique.}$$

**Claim (5)** For some  $j \neq l$ , assume that

$$f(u_{ij} u_{i(j+1)}) = f(u_{il} u_{i(l+1)})$$

$$\Rightarrow 2[(j - 1)n + i] = 2[(l - 1)n + i]$$

$$\Rightarrow l = j, \text{ a contradiction}$$

$$\Rightarrow \text{The edge labels are unique.}$$

**Claim (6).** For some  $j \neq m$  in  $1 \leq i \leq n, 1 \leq j \leq n - 2$ , assume that  $f(u_{ji} u_{(j+1)i}) = f(u_{mi} u_{(m+1)i})$

$$\Rightarrow 2[(i - 1)n + j] + 1 = 2[(i - 1)n + m] + 1$$

$\Rightarrow j = m$ , a contradiction

$\Rightarrow$  The edge labels are unique.

**Claim (7).** In the interval  $1 \leq i \leq n$ ,  $j = n - 1$ , assume that  $f(u_{ji} u_{(j+1)i}) = f(u_{mi} u_{(m+1)i})$

$$\Rightarrow 2i(n - 1) + j + 1 = 2i(n - 1) + m + 1$$

$\Rightarrow j = m$ , a contradiction

$\Rightarrow$  The edge labels are unique.

**Claim (8).** For some  $i$  and  $j$ , in  $1 \leq i \leq n$ ,  $1 \leq j \leq n - 2$ , it is evident that  $f(u_{ij}, u_{ij+1}) \neq f(u_{ji}, u_{j+1i})$  as

$$\Rightarrow 2[(j - 1)n + i] \text{ is even and } 2[(i - 1)n + j] + 1 \text{ is odd.}$$

$\Rightarrow$  The edge labels are unique.

**Claim (9).** In the interval  $1 \leq i \leq n$ ,  $j = n - 1$ , it is clear that  $f(u_{ij}, u_{ij+1}) \neq f(u_{ji}, u_{j+1i})$  as

$$2((j - 1)n + i) \text{ is even and } 2i(n - 1) + j + 1 \text{ is odd.}$$

$\Rightarrow$  The edge labels are unique.

The label of a vertex is the sum of the labels of edges that are incident with that vertex. From the claims discussed above, it follows that the labels of the edges are all distinct and so are the labels of the vertices.

Hence the Mongolian Tent graph  $M_n$  admits vertex antimagic edge labeling for  $n \neq 2^k$ .

**Case (2):**

Let  $M_n$  be the Mongolian Tent graph formed from  $n$  vertices  $n = 2^k$ .

Let the edge set partitioned into the following sets

$$E_1 = \{uu_{lj} / 1 \leq j \leq \frac{n}{2}\}$$

$$E_2 = \{uu_{lj} / \frac{n}{2} + 1 \leq j \leq n - 1\}$$

$$E_3 = \{u_{ji}u_{(j+1)i} / 1 \leq i \leq \frac{n}{2}, 1 \leq j \leq n - 1\}$$

$$E_4 = \{u_{ji}u_{(j+1)i} / \frac{n}{2} + 1 \leq i \leq n - 1, 1 \leq j \leq n - 1\}$$

$$E_5 = \{u_{ij}u_{i(j+1)} / 1 \leq i \leq n, 1 \leq j \leq \frac{n}{2}\}$$

$$E_6 = \{u_{ij}u_{i(j+1)} / 1 \leq i \leq n, \frac{n}{2} + 1 \leq j \leq n - 1\}$$

Define a function  $f: E \rightarrow R$  as

$$f(u u_{1j}) = \begin{cases} 2(j - 1)(n + 1) + 1, & 1 \leq j \leq \frac{n}{2} \\ (2j - 3)n + 2, & \frac{n}{2} + 1 \leq j \leq n \end{cases}$$

$$f(u_{ji} u_{(j+1)i}) =$$

$$\begin{cases} 2(j + (i - 1)(n - 1)), & 1 \leq j \leq n - 1, 1 \leq i \leq \frac{n}{2} \\ 2(j + 1) + (2i - 3)n, & 1 \leq j \leq n - 1, \frac{n}{2} + 1 \leq i \leq n \end{cases}$$

$$f(u_{ij} u_{i(j+1)}) =$$

$$\begin{cases} 2i + 2(j - 1)(n + 1) + 1, & 1 \leq i \leq n, 1 \leq j \leq \frac{n}{2} \\ (2i - 1) + (2j - 1)n, & 1 \leq i \leq n, \frac{n}{2} + 1 \leq j \leq n - 1 \end{cases}$$

As similar to Case (1), we prove that the edge labels are distinct using the following claims;

**Claim i.** For some  $j \neq k$  in the interval  $1 \leq j \leq \frac{n}{2}$ , assume that  $f(u u_{1j}) = f(u u_{1k})$

$$\Rightarrow 2(j - 1)(n + 1) + 1 = 2(k - 1)(n + 1) + 1$$

$\Rightarrow j = k$ , a contradiction to the assumption.

$\Rightarrow$  The edge labels of  $E_1$  are unique.

**Claim ii.** For  $j \neq k$  in the interval  $\frac{n}{2} + 1 \leq j \leq n$ , assume that  $f(u u_{1j}) = f(u u_{1k})$

$$\Rightarrow (2j - 3)n + 2 = (2k - 3)n + 2$$

$\Rightarrow j = k$ , a contradiction to the assumption.

$\Rightarrow$  The edge labels of  $E_2$  are unique.

**Claim iii.** For some  $j_1$  in the interval  $1 \leq j_1 \leq \frac{n}{2}$  and for some  $j_2$  in  $\frac{n}{2} + 1 \leq j_2 \leq n$ , it is evident that the function

$$f(uu_{j_1}) \neq f(uu_{j_2})$$

$$\Rightarrow 2(j_1 - 1)(n + 1) + 1 \neq (2j_2 - 3)n + 2 \text{ since } 2(j_1 - 1)(n + 1) + 1 \text{ is odd and } (2j_2 - 3)n + 2 \text{ is even.}$$

$\Rightarrow$  The edge labels of  $E_1$  and  $E_2$  are distinct.

**Claim iv.** For some  $j \neq l$  in the interval  $1 \leq j \leq n - 1$ ,  $1 \leq i \leq \frac{n}{2}$ , assume that  $f(u_{ji} u_{(j+1)i}) = f(u_{li} u_{(l+1)i})$

$$\Rightarrow 2(j + (i - 1)(n - 1)) = 2(l + (i - 1)(n - 1))$$

$\Rightarrow j = l$ , a contradiction.

$\Rightarrow$  The edge labels of  $E_3$  are unique.

**Claim v.** For some  $j \neq l$  in the interval  $1 \leq j \leq n - 1$ ,  $\frac{n}{2} + 1 \leq i \leq n$ , assume that  $f(u_{ji} u_{(j+1)i}) = f(u_{li} u_{(l+1)i})$

$$\Rightarrow 2(j + 1) + (2i - 3)n = 2(l + 1) + (2i - 3)n$$

$\Rightarrow j = l$ , a contradiction.

$\Rightarrow$  The edge labels of  $E_4$  are unique.

**Claim vi.** For some  $j_1$  and  $j_2$  in the interval  $1 \leq i \leq \frac{n}{2}$  and  $\frac{n}{2} + 1 \leq i \leq n - 1$  and  $1 \leq j \leq n - 1$ . Consider the

$$f(u_{j_1 i} u_{(j_1+1)i}) = 2(j_1 + (i - 1)(n - 1)) \text{ and } f(u_{j_2 i} u_{(j_2+1)i}) = 2(j_2 + 1) + (2i - 3)n$$

But  $2(j_1 + (i - 1)(n - 1)) \leq n^2 - n$  and

$$2(j_2 + 1) + (2i - 3)n \geq n^2 - n + 4. \text{ Since } n^2 - n + 4 > n^2 - n, \text{ it follows that the edge labels of } E_3 \text{ and } E_4 \text{ are unique.}$$

**Claim vii.** For some  $j \neq k$  in the interval  $1 \leq j \leq \frac{n}{2}$ ,  $1 \leq i \leq n$ , assume that  $f(u_{ij} u_{i(j+1)}) = f(u_{ik} u_{i(k+1)})$

$$\Rightarrow 2i + 2(j - 1)(n + 1) + 1 = 2i + 2(k - 1)(n + 1) + 1$$

$\Rightarrow j = k$ , a contradiction

$\Rightarrow$  The edge labels of  $E_5$  are unique.

**Claim viii.** For some  $j \neq k$  in the interval  $\frac{n}{2} + 1 \leq j \leq n - 1$ ,

$$\begin{aligned}
 & 1 \leq i \leq n, \text{ assume that } f(u_{ij} u_{i(j+1)}) = f(u_{ik} u_{i(k+1)}) \\
 & \Rightarrow (2i - 1) + (2j - 3)n = (2i - 1) + (2k - 3)n \\
 & \Rightarrow j = k, \text{ a contradiction} \\
 & \Rightarrow \text{The edge labels of } E_6 \text{ are unique.}
 \end{aligned}$$

**Claim ix** For some  $j_1$  and  $j_2$  in the intervals  $1 \leq i \leq n$ ,  $1 \leq j_1 \leq \frac{n}{2}$  and  $\frac{n}{2} + 1 \leq j_2 \leq n - 1$  respectively,

$$\begin{aligned}
 & \text{We have } f(u_{ij_1} u_{i(j_1+1)}) = 2i + 2(j_1 - 1)(n + 1) + \\
 & 1 \text{ and } f(u_{ij_2} u_{i(j_2+1)}) = (2i - 1) + (2j_2 - 1)n \\
 & \Rightarrow 2i + 2(j_1 - 1)(n + 1) + 1 \leq n^2 + n - 1 \text{ and} \\
 & (2i - 1) + (2j_2 - 1)n \geq n^2 + n + 1.
 \end{aligned}$$

Since  $n^2 + n - 1 < n^2 + n + 1$ , the edge labels of  $E_5$  and  $E_6$  are all distinct.

**Claim x.** For some  $i$  and  $j$  in the interval  $1 \leq j \leq \frac{n}{2}$ ,  $1 \leq i \leq n$ , note that  $f(u_i u_{ij}) \neq f(u_{ji} u_{(j+1)i})$  as  $2(j - 1)(n + 1) + 1$  is odd and  $2[j + (i - 1)(n - I)]$  is even.

$\Rightarrow$  The edge labels of  $E_1$  and  $E_2$  are unique.

**Claim xi.** For some  $j_1$  in the interval  $1 \leq j_1 \leq \frac{n}{2}$  and for some  $j_2$  in  $\frac{n}{2} + 1 \leq j_2 \leq n - 1$ , it is evident that

$$\begin{aligned}
 & f(u_{ij_1} u_{i(j_1+1)}) \neq f(u_{j_2i} u_{(j_2+1)i}) \text{ since } 2i + 2(j_1 - 1)(n + \\
 & 1) + 1 \text{ is odd and} \\
 & 2(j_2 + 1) + (2i - 3)n, \text{ is even.}
 \end{aligned}$$

$\Rightarrow$  The edge labels of  $E_5$  and  $E_4$  are unique.

**Claim xii.** For some  $j_1$  and  $j_2$  in the intervals  $\frac{n}{2} + 1 \leq j_1 \leq n - 1$  and  $1 \leq j_2 \leq \frac{n}{2}$ , we have  $f(u_i u_{1j_1}) = (2j_1 - 3)n + 2$  and  $f(u_{j_2i} u_{(j_2+1)i}) = 2[j_2 + (i - 1)(n - I)]$

$$\begin{aligned}
 & \Rightarrow (2j_1 - 3)n + 2 \leq 2n^2 - 5n + 2 \text{ and} \\
 & 2[j_2 + (i - 1)(n - I)] \geq 2
 \end{aligned}$$

Since  $2 < 2n^2 - 5n + 2$ , the labels of the edges of  $E_2$  and  $E_3$  are all distinct.

**Claim xiii.** For some  $j_1$  and  $j_2$  in interval  $\frac{n}{2} + 1 \leq j_1 \leq n - 1$ ,

$$\begin{aligned}
 & 1 \leq j_2 \leq n - 1, \text{ assume that } f(u_i u_{1j_1}) = f(u_{j_2i} u_{(j_2+1)i}) \\
 & \text{But } (2j_1 - 3)n + 2 \leq 2n^2 - 5n + 2 \text{ and } 2(j_2 + 1) + \\
 & (2i - 3)n \geq n^2 - n + 4
 \end{aligned}$$

$$\Rightarrow n^2 - n + 4 < 2n^2 - 5n + 2, \text{ a contradiction to the assumption.}$$

$\Rightarrow$  The edge labels of  $E_2$  and  $E_4$  are all distinct.

**Claim xiv.** For some  $j_1$  and  $j_2$  in the interval  $1 \leq j \leq \frac{n}{2}$ , we have

$$\begin{aligned}
 & f(u_i u_{1j_1}) = 2(j_1 - 1)(n + 1) + 1 \text{ and} \\
 & f(u_{ij_2} u_{i(j_2+1)}) = 2i + 2(j_2 - 1)(n + 1) + 1
 \end{aligned}$$

$$\begin{aligned}
 & \text{Note that } 2(j_1 - 1)(n + 1) + 1 \leq n^2 - n - 1 \text{ and} \\
 & 2i + 2(j_2 - 1)(n + 1) + 1 \geq 3
 \end{aligned}$$

$$\Rightarrow n^2 - n - 1 > 3$$

$\Rightarrow$  The edge labels of  $E_1$  and  $E_5$  are all distinct.

**Claim xv.** For some  $j_1$  and  $j_2$  in the interval  $1 \leq i \leq n$ ,  $1 \leq j_1 \leq \frac{n}{2}$ ,  $\frac{n}{2} + 1 \leq j_2 \leq n - 1$  we have  $f(u_i u_{1j_1}) = 2(j_1 - 1)(n + 1) + 1$  and  $f(u_{ij_2} u_{i(j_2+1)}) = (2i - 1) + (2j_2 - 1)n$

But  $2(j_1 - 1)(n + 1) + 1 \leq n^2 - n - 1$  and  $(2i - 1) + (2j_2 - 1)n \geq n^2 + n + 1$ . Since  $n^2 - n - 1 < n^2 + n + 1$ , it follows that the edge labels of  $E_1$  and  $E_6$  are all distinct.

**Claim xvi.** For some  $j_1$  in the interval  $\frac{n}{2} + 1 \leq j_1 \leq n - 1$ , and for some  $j_2$  in  $1 \leq j_2 \leq \frac{n}{2}$ , it is evident that

$$\begin{aligned}
 & f(u_i u_{1j_1}) \neq f(u_{ij_2} u_{i(j_2+1)}) \text{ since } (2j_1 - 3)n + 2 \text{ is even and} \\
 & 2i + 2(j_2 - 1)(n + 1) + 1 \text{ is odd.}
 \end{aligned}$$

$\Rightarrow$  The edge labels of  $E_2$  and  $E_5$  are unique.

**Claim xvii.** For some  $j_1$  and  $j_2$  in the interval  $\frac{n}{2} + 1 \leq j \leq n - 1$ ,

it is evident that  $f(u_i u_{1j_1}) \neq f(u_{ij_2} u_{i(j_2+1)})$  since  $(2j_1 - 3)n + 2$  is an even value and  $(2i - 1) + (2j_2 - 1)n$  is odd

$\Rightarrow$  The edge labels of  $E_2$  and  $E_6$  are unique.

**Claim xviii.** For some  $j_1$  and  $j_2$  in the interval  $1 \leq j_1 \leq n - 1$  and  $1 \leq j_2 \leq \frac{n}{2}$ , it is evident that  $f(u_{j_1i} u_{(j_1+1)i}) \neq f(u_{ij_2} u_{i(j_2+1)})$  since  $2[j_1 + (i - 1)(n - I)]$  is even and  $2i + 2(j_2 - 1)(n + 1) + 1$  is odd.

$\Rightarrow$  The edge labels of  $E_3$  and  $E_5$  are unique.

**Claim xix.** For some  $j_1$  in the interval  $1 \leq j_1 \leq n - 1$  and for some  $j_2$  in  $\frac{n}{2} + 1 \leq j_2 \leq n - 1$ , it is evident that  $f(u_{j_1i} u_{(j_1+1)i}) \neq f(u_{ij_2} u_{i(j_2+1)})$  since  $2[j_1 + (i - 1)(n - I)]$  is even and  $(2i - 1) + (2j_2 - 1)n$  is odd.

$\Rightarrow$  The edge labels of  $E_3$  and  $E_6$  are unique.

**Claim xx.** For some  $j_1$  and  $j_2$  in the interval  $1 \leq j_1 \leq n - 1$  and  $1 \leq j_2 \leq \frac{n}{2}$  respectively, it is evident that

$$\begin{aligned}
 & f(u_{j_1i} u_{(j_1+1)i}) \neq f(u_{ij_2} u_{i(j_2+1)}) \text{ since } 2(j_1 + 1) + (2n - \\
 & 3)n \text{ is even and} \\
 & 2i + 2(j_2 - 1)(n + 1) + 1 \text{ is odd.}
 \end{aligned}$$

$\Rightarrow$  The edge labels of  $E_4$  and  $E_5$  are unique.

**Claim xxi.** For some  $j_1$  and  $j_2$  in the interval  $1 \leq j_1 \leq n - 1$ ,  $\frac{n}{2} + 1 \leq j_2 \leq n - 1$ , it is evident that

$$\begin{aligned}
 & f(u_{j_1i} u_{(j_1+1)i}) \neq f(u_{ij_2} u_{i(j_2+1)}) \text{ since } 2(j_1 + 1) + (2i - \\
 & 3)n \text{ is even and} \\
 & (2i - 1) + (2j_2 - 1)n \text{ is odd.}
 \end{aligned}$$

$\Rightarrow$  The edge labels of  $E_4$  and  $E_6$  are unique.

From the claims discussed above, it is clear that the Mongolian Tent graph  $M_n$  on  $n$  vertices ( $n = 2^k$ ) admits vertex antimagic edge labeling. Thus  $M_n$  admits vertex antimagic edge labeling for all values of  $n$  ( $n \geq 4$ ).

#### IV. VERTEX ANTIMAGIC EDGE LABELING FOR $M_n$ ( $n = 3$ )

Consider the Mongolian Tent graph  $M_n$  formed on 10 vertices and 15 edges. Then  $M_n$  ( $n = 3$ ) admits vertex antimagic edge labeling if a function  $f: E \rightarrow R$  is defined as follows;  $f(u_1 u_{11}) = 1, f(u_1 u_{12}) = 7, f(u_1 u_{13}) = 13, f(u_{11} u_{12}) = 2, f(u_{12} u_{13}) = 8, f(u_{11} u_{21}) = 3, f(u_{12} u_{22}) = 9, f(u_{13} u_{23}) = 14, f(u_{21} u_{22}) = 4, f(u_{22} u_{23}) = 10, f(u_{21} u_{31}) = 5, f(u_{22} u_{32}) = 11, f(u_{23} u_{33}) = 15, f(u_{31} u_{32}) = 6, f(u_{32} u_{33}) = 12$ .

By this method of labeling, it follows that the Mongolian Tent graph admits vertex antimagic edge labeling. Hence the  $M_n$  admits vertex antimagic edge labeling for all values of  $n \geq 3$ .

#### V. CONCLUSION

In this paper, we have proved that the Mongolian Tent graph  $M_n$  formed out of a grid  $P_n \times P_n$  admits vertex antimagic edge labeling for  $n \geq 3$ . The proof has been given in three cases with the first case being the graph constructed on  $n \times n$  vertices where  $n \neq 2^k$  for all  $k$  and the second the proof has been given for  $n \times n$  vertices where  $n = 2^k$ . The third case being the edge labeling for  $M_n$  ( $n = 3$ ), discussed separately. Further, the vertex antimagic edge labeling can be established for different grid graphs like the Mongolian Tent graph  $M_n$  like taking the grid  $P_m \times P_n$  ( $m \neq n$ ), extended grids etc.

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