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Solutions of Pell's Equation Involving Jarasandha Numbers

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Abstract—We look for discovering non-trivial integer solutions to the pell's equation involving jarasandha numbers. Also, we acquire the recurrence relations among the solutions.

Keywords—*Jarasandha numbers, Pell's equation, brahmagupta lemma.*

I. INTRODUCTION

Number theorists scrutinize the properties of integers. The initial phase in building a sparkly, new, mathematical theory, be that as it may, is making a hypothetical inquiry about number connections [1-3]. Pell's equation is a diophantine equation of the form $x^2 = dy^2 + 1$ where n is a given positive non-square integer and integer solutions exists for x and y. Pell's equation is identified with a few other important subjects in mathematics. However, several different procedures are required to solve this equation [4&9].

In Indian epic Mahabharatha, we come across a person named 'jarasandha'. He had a boon that if he was split into 2 parts and thrown apart, the parts would rejoin and return to life. In the field of Mathematics, we have numbers exhibiting the same property as Jarasandha [5-8]. In this communication, we search for non-trivial integer solutions to the pell's equation involving jarasandha numbers. Further recurrence relations on the solutions are inferred.

II. METHODOLOGY

This paper concerns with the pell's equation

$$y^2 = Dx^2 + J \qquad (1)$$

where $D = p^2 - 1, \quad p > 1$ and J is a jarasandha number.

The initial solution of (1) is (x_0, y_0) &given by

$$x_0 = \sqrt{J}; \quad y_0 = \sqrt{J(D+1)}$$

To find the other solutions of (1), consider the pell equation

$$y^2 = Dx^2 + 1$$

whose initial solution $(\tilde{x}_s, \tilde{y}_s)$ is given by

$$\tilde{x}_s = \frac{1}{2\sqrt{D}} g_s$$

$$\tilde{y}_s = \frac{1}{2} f_s$$

where $f_s = (\sqrt{D+1} + \sqrt{D})^{s+1} + (\sqrt{D+1} - \sqrt{D})^{s+1}$

$$g_s = (\sqrt{D+1} + \sqrt{D})^{s+1} - (\sqrt{D+1} - \sqrt{D})^{s+1}, \qquad s = 0,1,2...$$

Applying Brahmagupta's lemma between the solutions (x_0, y_0) and $(\tilde{x}_s, \tilde{y}_s)$, the sequence of non-zero distinct integer solutions to (1) are obtained as

$$x_s = \frac{1}{2\sqrt{D}}\left(\sqrt{JD}\, f_s + \sqrt{J(D+1)}\, g_s\right)$$

$$y_s = \frac{1}{2}\left(\sqrt{J(D+1)}\, f_s + \sqrt{JD}\, g_s\right)$$

and their recurrence relations are found to be

$$x_{s+2} - 2\sqrt{D+1}\, x_{s+1} + x_s = 0$$

$$y_{s+2} - 2\sqrt{D+1}\, y_{s+1} + y_s = 0, \qquad s = 0,1,2...$$

III. RESULTS AND DISCUSSION

Some numerical examples for the choice of p & J satisfying the pell equation and their solutions are presented in the accompanying table:

Table.1

S.No	p	J	Pell equation	Sequence of Integer solutions	Recurrence relation
1.	2	81	$y^2 = 3x^2 + 81$	$x_s = \frac{1}{2\sqrt{3}}\left(9\sqrt{3} f_s + 18 g_s\right)$ $y_s = \frac{1}{2}\left(18 f_s + 9\sqrt{3} g_s\right)$	$x_{s+2} - 4x_{s+1} + x_s = 0$ $y_{s+2} - 4y_{s+1} + y_s = 0$
2.	2	2025	$y^2 = 3x^2 + 2025$	$x_s = \frac{1}{2\sqrt{3}}\left(45\sqrt{3} f_s + 90 g_s\right)$ $y_s = \frac{1}{2}\left(90 f_s + 45\sqrt{3} g_s\right)$	$x_{s+2} - 4x_{s+1} + x_s = 0$ $y_{s+2} - 4y_{s+1} + y_s = 0$
3.	3	3025	$y^2 = 8x^2 + 3025$	$x_s = \frac{1}{2\sqrt{8}}\left(55\sqrt{8} f_s + 110 g_s\right)$ $y_s = \frac{1}{2}\left(110 f_s + 55\sqrt{8} g_s\right)$	$x_{s+2} - 6x_{s+1} + x_s = 0$ $y_{s+2} - 6y_{s+1} + y_s = 0$
4.	4	9801	$y^2 = 15x^2 + 9801$	$x_s = \frac{1}{2\sqrt{15}}\left(99\sqrt{15} f_s + 198 g_s\right)$ $y_s = \frac{1}{2}\left(198 f_s + 99\sqrt{15} g_s\right)$	$x_{s+2} - 8x_{s+1} + x_s = 0$ $y_{s+2} - 8y_{s+1} + y_s = 0$
5.	5	88209	$y^2 = 24x^2 + 88209$	$x_s = \frac{1}{2\sqrt{24}}\left(297\sqrt{24} f_s + 594 g_s\right)$ $y_s = \frac{1}{2}\left(594 f_s + 297\sqrt{24} g_s\right)$	$x_{s+2} - 10x_{s+1} + x_s = 0$ $y_{s+2} - 10y_{s+1} + y_s = 0$

IV. REMARKABLE NOTE:

If we consider the equation $y^2 = Dx^2 + m^2$ where $D = p^2 - 1,\quad p > 1$ and $m \in z$, then initial solution is given by $x_0 = m;\ y_0 = m\sqrt{(D+1)}$ & applying brahmagupta's lemma, the sequence of non-zero distinct integer solutions are obtained as

$$x_s = \frac{m}{2\sqrt{D}}\left(\sqrt{D} f_s + \sqrt{(D+1)} g_s\right)$$

$$y_s = \frac{m}{2}\left(\sqrt{(D+1)} f_s + \sqrt{D} g_s\right)$$

and their recurrence relations are observed to be the equivalent as in the above case.

V. CONCLUSION AND FUTURE SCOPE

In this paper, we have presented non-zero distinct integer solutions to the pell's equation involving jarasandha numbers. In such a way that, one may search for the solutions to the pell's equation with other suitable numbers.

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