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# Various Distance Measures on Pythagorean Vague Sets

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***Abstract***— In this paper, we introduced and study a new concept of Pythagorean vague sets and deduce some of its properties. Based on these definition varies form of Pythagorean vague distances are proposed and studied with suitable illustration.

**Keywords—** In Pythagorean vague Sets, Hamming Pythagorean Vague Measures, Normalized Hamming Pythagorean Vague Measures, Euclidean Pythagorean Vague Measures, Normalized Euclidean Pythagorean Vague Measures.

## I. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [11] in 1965. The membership of an element to a fuzzy set is a single value between 0 and 1. In 1968 the theory of fuzzy topology was introduced by C.L. Chang [2]. Atanassov [1] initiated the concept of intuitionistic fuzzy set (IFS), which is a generalization of Zadesh's fuzzy set in 1986.

The theory of vague set was proposed by Gaw and Buchere [5] as an extension of fuzzy set theory. The idea of vague set is defined by a truth membership ($t_v$) and false membership function ($f_v$). The value of $t_v(x)$ and $f_v(x)$ are both defined on the closed interval [0,1] with each point in a basic set X, where $t_v(x) + f_v(x)$ 1.

In 2013 Yagar [6] introduced the model of Pythagorean fuzzy sets (PFS) characterized by a membership and non membership degree, which satisfies the condition that, the square sum of its membership and non membership degree is less than or equal to 1.

After that, Yaga and Abbasov [7] presented the concept of Pythagorean membership grades and ideas related to PFS and also introduced the relationship between the Pythagorean membership grades and the complex numbers. Yagar [8] initiated Pythagorean membership grades in multi criteria decision making in 2014. Later, Yagar and Zahand and Xu [12] gave some basic operations for Pythagorean Fuzzy Numbers. Pythagorean Fuzzy Numbers be the elements of Pythagorean Fuzzy Set. In 2016 Gou, Xu and Ren [9] presented the properties of continuous Pythagorean fuzzy information. Peng and Yang [10] studied some results of Pythagorean fuzzy sets in 2015. After that Khaista Rahman et.al [4] studied the idea of some interval valued fuzzy sets, weighted aggregation operators and decision making in 2018. In 1997 Szmidt and Kacprzyk [3] defined different types of distances between intuitionistic fuzzy sets .

The purpose of this paper is to introduce the concept of Pythagorean vague sets and obtain some of their Pythagorean vague properties, also obtain measuring distance between Pythagoren vague sets.

Section I contains the introduction of Pythagorean Sets , Section II contains preliminaries, Section III contain the, Pythagorean Vague Set, Section IV contain the Distance for Pythagorean Vague Set.

## II. PRELIMINARIES

**Definition 1[1]:**

A fuzzy set A = {< u, $\mu_A(u)$ > |u U} in a universe of discourse U is characterized by a membership function, $\mu_A$, as follows: $\mu_A : U \rightarrow [0, 1]$.

**Definition 2[2]:**

Let A and B be fuzzy sets in a space X = {x}, with the grades of membership of x in A and B denoted by $\mu_A(x)$ and $\mu_B(x)$, respectively. Then

A = B iff $\mu_A(x) = \mu_B(x)$

$A \in B$ iff $\mu_A(x) \leq \mu_B(x)$.

$A \cup B = Max[\mu_A(x), \mu_B(x)]$ for all x X.

$A \cap B = Min[\mu_A(x), \mu_B(x)]$ for all x X.

$A^c = 1 - \mu_A(x)$ for all x X.

**Definition 3[1]:**

Let X be a non-empty set. Then A is called an Intuitionistic Fuzzy set (in short, IFS) of X , if it is an object having the form A = { < x, $\mu_A$ ,$\gamma_A$ > | $x \in X$} where the function $\mu_A : X \rightarrow [0,1]$ and $\gamma_A : X \rightarrow [0,1]$ denote the degree of membership $\mu_A(x)$ and degree of non membership

$\gamma_A(x)$ of each element $x \in X$ to the set A and satisfies the condition that, $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

**Definition 4[1]:**

If A and B are two IFSs of the set X, then

$A^c = \{< x, \nu_A(x), \mu_A(x) > | x \in X\}$,

$A \in B$ iff $\forall x \in X, \mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$,

$A = B$ iff $\forall x \in X, \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$,

$A \cup B = \{< x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) > | x \in X\}$, and

$A \cap B = \{< x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) > | x \in X\}$.

**Definition 5[5]:**

A vague set V in a universe of discourse X is characterized by a true membership function $t_v$, and a false membership function $f_v$, as follows: $t_v : U \to [0, 1]$, $f_v: U \to [0, 1]$, and $t_v+f_v \le 1$, where $t_v(x)$ is a lower bound on the grade of membership of x derived from the evidence for x, and $f_v(x)$ is a lower bound on the grade of membership of the negation of x derived from the evidence against x. The vague set A is written as, $A=\{ < x, t_A(x), 1-f_A(x)> | x \in X\}$.

**Definition 6 [5]:**

A vague topology (VT in short) on X is a family A of vague sets( VS in short) in X satisfying the following axioms:

(1) $0,1 \in A$;

(2) $G_1 \cap G_2 \in A$ for any $G_1, G_2 \in A$.

(3) $\cup G_i \in A$ for any family $\{G_i : i \in N\} \subseteq A$.

In this case the pair (X, A) is called a vague topological space (VTS in short) and any vague set in A is known as a vague open set (VOS) in X. The complement $A^c$ of a VOS A in a VTS (X,A) is called a vague closed set (VCS in short) in X.

**Definition 7[5]:**

Let $(X,\tau)$ be VTS and $A=\{ < x, t_A(x), 1-f_A(x)>\}$ be VS in X. Then the vague interior and vague closure are defined by

$vint(A) = \cup \{G/G$ is aVOS in X and $G \subseteq A\}$

$vcl(A) = \cap \{ K / K$ is aVCS in X and $A \subseteq K\}$

**Definition 8[5]:**

Let A and B be vague sets of the form $A=\{ < x, t_A(x), 1-f_A(x)> | x \in X\}$ and $B=\{ < x, t_B(x), 1-f_B(x)> | x \in X\}$.Then

$A \subseteq B$ if and only if $t_A(x) \le t_B(x)$ And $1-f_A(x) \le 1-f_B(x)$.

$A=B$ if and only if $A \subseteq B$ and $B \subseteq A$.

$A^c=\{ < x, 1-f_A(x), t_A(x) > | x \in X\}$.

$A \cup B = \{ < x, \max(t_A(x), t_B(x)), \max(1-f_A(x), 1-f_B(x)) > | x \in X\}$.

$A \cap B = \{ < x, \min(t_A(x), t_B(x)), \min(1-f_A(x), 1-f_B(x)) > | x \in X\}$.

**Definition 9[6]**

Let X be a universe of discourse. An Pythagorean Fuzzy Set (PFS) P in X is given by $P = \{< x, \mu_P(x), \nu_P(x) > | x \in X\}$, where $\mu_P : X \to [0,1]$ denotes the degree of membership and $\nu_P : X \to [0,1]$ denotes the degree of non membership of the element $x \in X$ to the set P, respectively, with the condition that $0 \le (\mu_P(x))^2 + (\nu_P(x))^2 \le 1$.

**Definition 10[10]:**

Let $p = (\mu, \nu)$, $p_1 = (\mu_1, \nu_1)$, and $p_2 = (\mu_2, \nu_2)$ be three PFNs, then their operations are defined as follows:

(1) $p_1 \cup p_2 = (\max\{\mu_1,\mu_2\}, \min\{\nu_1, \nu_2\})$

(2) $p_1 \cap p_2 = (\min\{\mu_1,\mu_2\}, \max\{\nu_1, \nu_2\})$

(3) $p^c = (\nu,\mu)$.

**Definition 11[3]:**

Let A and B are two intuitionistic fuzzy sets in X

**Hamming Distance :**

$$d_{IFS}(A,B)= \sum_{i=1}^{n}\left(|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x)-\nu_B(x_i)|+|\pi_A(x_i)-\pi_B(x_i)|\right)$$

**Normalized Hamming Distance:**

$$l_{IFS}(A,B)= \frac{1}{n}\sum_{i=1}^{n}\left(|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|+|\pi_A(x_i)-\pi_B(x_i)|\right)$$

**Euclidean Distance:**

$$E_{IFS}(A,B)= \sqrt{\sum_{i=1}^{n}\left((\mu_A(x_i)-\mu_B(x_i))^2+(\nu_A(x_i)-\nu_B(x_i))^2+(\pi_A(x_i)-\pi_B(x_i))^2\right)}$$

**Normalized Euclidean Distance :**

$$Q_{IFS}(A,B)= \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left((\mu_A(x_i)-\mu_B(x_i))^2+(\nu_A(x_i)-\nu_B(x_i))^2+(\pi_A(x_i)-\pi_B(x_i))^2\right)}$$

## III. PYTHAGOREAN VAGUE SETS (PVS)

**Definition 3.1**

Let X be a universe of discourse. A Pythagorean Vague Set (PVS) A in X is given by $A = \{< x, t_A(x), 1-f_A(x) > | x \in X\}$, where $t_A(x): X \to [0,1]$ denotes the truth value and $1-f_A(x) : X \to [0,1]$ denotes the false value of the element $x \in X$ to the set A, respectively, with the condition that $0 \le (t_A(x))^2 + (1-f_A(x))^2 \le 1$.

**Definition 3.2**

Let $P= <t_p, 1-f_p>$, $P_1= < t_{P_1}, 1-f_{P_1} >$, $P_2 = < t_{p_2}, 1-f_{p_2} >$ be the Pythagorean vague elements and $\lambda>0$, satisfies the following operations.

1. $P^{\lambda} = [ (t_p)^{\lambda}, \sqrt{1-(1-(1-f_P)^2)^{\lambda}} ]$,

2. $\lambda P= [ \sqrt{1-(1-t_p^{\,2})^{\lambda}}, (1-f_v)^{\lambda} ]$

3. $P_1 \oplus P_2 = \left[\sqrt{(t_{p_1})^2+(t_{p_2})^2-t_{p_1}^{\,2}.t_{p_2}^{\,2}}, (1-f_{p_1})(1-f_{p_2})\right]$

4. $P_1 \otimes P_2 = \left[\sqrt{(1-f_{p_1})^2+(1-f_{p_2})^2-(1-f_{p_1})^2.(1-f_{p_2})^2}, (t_{p_1})(t_{p_2})\right]$

**Example 3.3**

Let X={a,b} and P, $P_1$ and $P_2$ are vague sets in X where $\lambda \ge 0$

P={<x,[0.4,0.6], [0.3,0.5]>} $P_1$={<x,[0.5,0.7], [0.4,0.5]>} $P_2$={<x,[0.5,0.6],[0.3,0.4]>} and Let $\lambda$=2.

 

1.$P^{\lambda}$ = ([0.16,009] ,[0.59,0.43])
2.λP= ([0.29,0.17], [0.36,0.25] )
3. $P_1 \oplus P_2 =$ ([0.47,0.68], [0.12,0.2])
4. $P_1 \otimes P_2$ = ([0.25,0.42], [0.47,0.62])

**Theorem 3.4**

Let P= <a,b> and $P_1$ =<$a_1$,$b_1$> , $P_2$= <$a_2$, $b_2$> be the pythagorean vague sets and $\lambda, \lambda_1, \lambda_2 \in 0$ then,

1. $P_1 \oplus P_2 = P_2 \oplus P_1$
2. $P_1 \otimes P_2 = P_2 \otimes P_1$
3. $\lambda(P_1 \oplus P_2) = \lambda P_1 \oplus \lambda P_2$

**Proof**

1. $P_1 \oplus P_2 = \left[\sqrt{(a_1)^2+(a_2)^2-a_1^2.a_2^2}, (b_1, b_2)\right]$
$= \left[\sqrt{(a_2)^2+(a_1)^2-a_2^2.a_1^2}, (b_2, b_1)\right]$
$= P_2 \oplus P_1$

2.Similarly, $P_1 \otimes P_2 = \left[\sqrt{(b_1)^2+(b_2)^2-(b_1)^2.(b_2)^2}, (a_1)(a_2)\right]$
$= \left[\sqrt{(b_2)^2+(b_1)^2-(b_2)^2.(b_1)^2}, (a_2)(a_1)\right]$
$= P_2 \otimes P_1$

3.**LHS:**

$\lambda(P_1 \oplus P_2) = \lambda\left[\sqrt{(a_1)^2+(a_2)^2-a_1^2.a_2^2}, (b_1, b_2)\right]$
$= \left[\sqrt{1-(1-a_1^2)^{\lambda}(1-a_2^2)^{\lambda}}, (b_1, b_2)^{\lambda}\right]$

**RHS:**

$\lambda P_1 \oplus \lambda P_2 = \left[\sqrt{1-(1-a_1^2)^{\lambda}}, (b_1)^{\lambda}\right] \oplus \left[\sqrt{1-(1-a_2^2)^{\lambda}}, (b_2)^{\lambda}\right]$
$= \left[\sqrt{1-(1-a_1^2)^{\lambda}(1-a_2^2)^{\lambda}}, (b_1, b_2)^{\lambda}\right]$
$= \left[\sqrt{1-(1-(a_1^2+a_2^2-a_1^2a_2^2))^{\lambda}}, (b_1, b_2)^{\lambda}\right]$
$= \lambda(P_1 \oplus P_2)$

**Definition 3.5**

Let P= {<x,(a,b)>|x ∈ X} and $P_1$ ={<x, ($a_1$,$b_1$)>| x ∈ X}, $P_2$= {<x, ($a_2$, $b_2$)>| x ∈ X} be the pythagorean vague sets and $\lambda \geq 0$ , then

*$P_1 \cup P_2$ = (max ($a_1$,$a_2$), max ($b_1$,$b_2$))
*$P_1 \cap P_2$ =(min($a_1$,$a_2$), min ($b_1$,$b_2$))
*$P^c$ = (b,a)

**Theorem 3.6**

Let P= {<x,(a,b)>|x ∈ X} and $P_1$ ={<x, ($a_1$,$b_1$)>| x ∈ X} , $P_2$= {<x, ($a_2$, $b_2$)>| x ∈ X} be the pythagorean vague sets and $\lambda \geq 0$ , then

1.$P_1 \cup P_2 = P_2 \cup P_1$
2.$P_1 \cap P_2 = P_2 \cap P_1$
3.$\lambda(P_1 \cup P_2) = \lambda P_1 \cup \lambda P_2$

**Proof**

1. $P_1 \cup P_2$= [(max $a_1$,$a_2$), max ($b_1$,$b_2$)]
= [(max $a_2$,$a_1$), max ($b_2$,$b_1$)]
= $P_2 \cup P_1$

2. $P_1 \cap P_2$= [(min $a_1$,$a_2$), min ($b_1$,$b_2$)]
= [(min $a_2$,$a_1$), min ($b_2$,$b_1$)]
= $P_2 \cap P_1$

3. **LHS :**

$\lambda(P_1 \cup P_2) = \lambda$ [(max $a_1$,$a_2$), max ($b_1$,$b_2$)]
$= \left[\sqrt{1-(1-\max(a_1^2, a_2^2))^{\lambda}}, \max(b_1^{\lambda}, b_2^{\lambda})\right]$

**RHS:**

$\lambda P_1 \cup \lambda P_2 = \left[\sqrt{1-(1-a_1^2)^{\lambda}}, b_1^{\lambda}\right] \cup \left[\sqrt{1-(1-a_2^2)^{\lambda}}, b_2^{\lambda}\right]$
$= \left[\sqrt{1-(1-\max(a_1^2, a_2^2))^{\lambda}}, \max(b_1^{\lambda}, b_2^{\lambda})\right]$
$= \lambda(P_1 \cup P_2)$.

**Theorem 3.7**

$P_1$ ={<x, ($a_1$,$b_1$)>| x ∈ X}, $P_2$= {<x, ($a_2$, $b_2$)>| x ∈ X} be the pythagorean vague sets, then

1.$P_1^c \cup P_2^c = (P_1 \cup P_2)^c$
2.$P_1^c \oplus P_2^c = (P_1 \otimes P_2)^c$

**Proof:**

1) **LHS**

$P_1^c \cup P_2^c$ = ($b_1$,$a_1$) ∪ ($b_2$,$a_2$)
= (max ($b_1$,$b_2$), max ($a_1$,$a_2$))

**RHS**

$(P_1 \cup P_2)^c$ = ( max ($a_1$,$a_2$) , max ($b_1$,$b_2$))$^c$
= (max ($b_1$,$b_2$), max ($a_1$,$a_2$))
= $P_1^c \cup P_2^c$

3. **LHS**

$P_1^c \oplus P_2^c$ = ($b_1$,$a_1$) ⊕ ($b_2$,$a_2$)
$= \left[\sqrt{b_1^2+b_2^2-b_1^2b_2^2}, a_1.a_2\right]$

**RHS**

$(P_1 \otimes P_2)^c = \left[a_1.a_2, \sqrt{b_1^2+b_2^2-b_1^2b_2^2}\right]^c$
$= \left[\sqrt{b_1^2+b_2^2-b_1^2b_2^2}, a_1.a_2\right]$
$= P_1^c \oplus P_2^c$

## IV. DISTANCE FOR PYTHAGOREAN VAGUE SETS

**Definition 4.1**

Let A= {<x,($t_A$(x), 1-$f_A$(x))>|x ∈ X } and
B ={<x, ($t_B$, 1-$f_B$(x))>| x ∈ X } pythagorean vague sets in X

**1. Hamming Distance**

$H_{PV}$(A,B)=

$$\sum_{i=1}^{n}\left(|t_A(x_i)-t_B(x_i)|+|[1-f_A(x\ )]-[1-f_B(x_i)]|+|\pi_A(x_i)-\pi_B(x_i)|\right)$$

**2.Normalized Hamming Distance**

$N_{PV}$(A,B)=

$$\sum_{i=1}^{n}\left(|t_A(x_i)-t_B(x_i)|+|[1-f_A(x_i)]-[1-f_B(x_i)]|+|\pi_A(x_i)-\pi_B(x_i)|\right)$$

**3.Euclidean Distance**

$E_{PV}$(A,B)=

$$\sqrt{\sum_{i=1}^{n}\left((t_A(x_i)-t_B(x_i))^2+([1-f_A(x_i)]-[1-f_B(x_i)])^2+(\pi_A(x_i)-\pi_B(x_i))^2\right)}$$

 

Where $\pi_A(x_i)$ =1-$t_A(x_1)$- (1-$f_A(x_i)$), $\pi_B(x_i)$ =1-$t_B(x_1)$-(1-$f_B(x_i)$) be the degree of indeterminacy of x in A and B.

**4. Normalized Euclidean Distance**

$NE_{PV}$(A,B)=

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left((t_A(x_i)-t_B(x_i))^2+([1-f_A(x_i)]-[1-f_B(x_i)])^2+(\pi_A(x_i)-\pi_B(x_i))^2\right)}$$

Distance measures satisfies the following conditions:

$$0 \le H_{VS}(A,B) \le 2n$$
$$0 \le N_{VS}(A,B) \le 2$$
$$0 \le E_{VS}(A,B) \le \sqrt{2n}$$
$$0 \le NE_{VS}(A,B) \le \sqrt{2}$$

**Example 4.2:**

Let X={1,2} and let A and B are the Pythagorean fuzzy set in X defined by A={<x, (0.4,0.5), (0.3,0.5)>} B={<x,(0.2,0.4),(0.1,0.5)>}

**1.Hamming Distance**

$H_{PV}$(A,B)=

$$\sum_{i=1}^{n}\left(|t_A(x_i)-t_B(x_i)|+|[1-f_A(x\ )]-[1-f_B(x_i)]|+|\pi_A(x_i)-\pi_B(x_i)|\right)$$

=|0.4-0.3| + |0.2-0.1| + |0.5-0.5| + |0.4-0.5| =0.3.

**2. Normalized Hamming Distance**

$N_{PV}$(A,B)=

$$\sum_{i=1}^{n}\left(|t_A(x_i)-t_B(x_i)|+|[1-f_A(x_i)]-[1-f_B(x_i)]|+|\pi_A(x_i)-\pi_B(x_i)|\right)$$

= 0.15

**3.Euclidean Distance**

$E_{PV}$(A,B)=

$$\sqrt{\sum_{i=1}^{n}\left((t_A(x_i)-t_B(x_i))^2+([1-f_A(x_i)]-[1-f_B(x_i)])^2+(\pi_A(x_i)-\pi_B(x_i))^2\right)}$$

= $(0.1)^2$ + $(0.1)^2$ + $(0.1)^2$ =0.17

**4. Normalized Euclidean Distance**

$NE_{PV}$(A,B)=

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left((t_A(x_i)-t_B(x_i))^2+([1-f_A(x_i)]-[1-f_B(x_i)])^2+(\pi_A(x_i)-\pi_B(x_i))^2\right)}$$

= 0.12

Distance in Pythagorean vague sets should be calculated by taking truth membership and false membership function and it also satisfies the following conditions.

- 0≤ $H_{PV}$(A,B) ≤ 2n
  0≤ 0.3≤ 4
- 2.0≤ $N_{PV}$(A,B) ≤ 2
  0≤ 0.15≤ 2
- 3.0≤ $E_{PV}$(A,B) ≤ $\sqrt{2n}$
  0≤ 0.17≤ $\sqrt{2n}$
- 4. 0≤ $NE_{PV}$(A,B) ≤ $\sqrt{2}$
  0≤ 0.12≤ $\sqrt{2}$ .

## V. CONCLUSION

In this paper we define new concept of Pythagorean vague sets and verified their addition and multiplication properties and also we find distance between Pythagorean vague sets, and verified by the example.

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