International Journal of Scientific Research in Mathematical and Statistical Sciences
**Research Paper**
Vol.6, Issue.2, pp.193-202, April (2019)
**E-ISSN:** 2348-4519
*DOI: https://doi.org/10.26438/ijsrmss/v6i2.193202*

# A Modification of Pranav Distribution using Quadratic Rank Transmutation Map Approach

**C. C. Odom[1*], E. C. Nduka[2] and M.A. Ijomah[3]**

[1,2,3]Dept. of Mathematics and Statistics, University of Port Harcourt, Nigeria.

**Corresponding Author: odom_cc@yahoo.com, Tel: +234 816 077 0913*

**Available online at: www.isroset.org**

Received: 11/Apr/2019, Accepted: 26/Apr/2019, Online: 30/Apr/2019

***Abstract*** - A modification of the Pranav distribution is derived using the Quadratic Rank Transmutation Map (QRTM) approach and named Transmuted Pranav distribution (TPD). Mathematical properties of the new distribution are also derived. The new distribution is applied to real life data from the behavioral sciences and the method of maximum likelihood is used to estimate the parameters. The goodness of fit of the proposed distribution is compared with that of some competing distributions and the results show the superior performance of the proposed distribution.

***Keywords****: Quadratic Rank Transmutation Map, Pranav distribution, Moment Generating Function, Rényi Entropy, Order Statistics, Reliability Analysis, Maximum Likelihood Estimation*

.

## I. INTRODUCTION

Statistical distributions play a very important role in the description and prediction of real life phenomena. Data from some application areas such as Engineering, Physics, Biological sciences and actuarial sciences are often considered and interpreted as lifetimes and there is always the need to model these lifetime data sets. Over the years, some well-known distributions such as Gamma, Weibull, Lindley and Exponential distributions have been applied to real lifetime data. However, studies have shown the inferiority of some of these distributions in modelling some lifetime data sets when compared with some newer distributions. As more data become available through sophistication in technology, the need for more distributions with better fitting to real life data and more flexibility keeps arising. Researchers have been motivated by this need to develop new distributions.

According to Ghitany et al. (2008), the attractiveness of the exponential distribution seemingly eclipsed the possible interest of researchers in the Lindley distribution. They found many of the mathematical properties of the Lindley distribution to be more flexible than those of the exponential distribution. Shanker (2015) proposed the Akash Distribution and claimed that the mean residual life function and the hazard rate function of the Akash distribution exhibited more flexibility than those of the Lindley and exponential distributions. Adamidis and Loukas (1998) proposed a model belonging to the family of distributions introduced by Marshall and Olkin (1997) and suggested that the model is ideal for modeling time to the first failure. Yilmaz et al. (2016) compounded the modified discrete Lindley distribution and exponential distribution and showed the resulting new distribution to outperform the exponential-poisson distribution and the exponential-geometric distribution in one data set which consists of the times between successive earthquakes in North Anatolia fault zone. Nofal et al. (2017) argued that although the classical Weibull distribution is popular and versatile for modeling real life data, there are cases where the distribution fails to capture the underlying phenomenon under study. They transmuted the geometric-Weibull distribution and demonstrated the flexibility and superiority of their proposed distribution over some competing distributions like Weibull, Kumaraswamy-Weibull, Exponentiated Generalized-Weibull and WeibullWeibull using real world data. Bhatti et al. (2018) transmuted the geometric- quadratic hazard rate (G-QHR) distribution and suggested that the resulting distribution is suitable for survival analysis and application in actuarial sciences.

The last few decades have witnessed the proposal and study of several new distributions. Among the recently proposed distributions include: Akash Distribution (Shanker, 2015), Sujatha distribution (Shanker, 2016a), Discrete Shanker distribution (Borah and Hazarika, 2017) and Ishita distribution (Shanker and Shukla, 2017a). More recently, Pranav Distribution (a newly proposed distribution by Shukla (2018)) was demonstrated to give better fit over some of these afore mentioned new distributions for some real life data sets.

Given the huge number of existing and evolving real life data sets and the inability of existing probability distributions to perfectly model these data sets, this work sets out to propose a new probability distribution derived using the Quadratic Rank Transmutation Map approach of Shaw and Buckley (2009) and termed Transmuted Pranav Distribution (TPD). The transmutation approach to generalizing distributions enjoys vibrant research interests as evidenced in literature and a chronological ordering of over fifty transmuted distributions were reported by Tahir and Cordeiro (2016). Some of the mathematical properties of the proposed distribution are derived and its goodness of fit is compared with other distributions using real life data.

The rest of the paper is organized as follows: section II introduces the newly derived distribution, section III presents the mathematical properties, section IV contains the entropy, section V contains the order statistics, section VI presents the reliability analysis, section VII presents the maximum likelihood estimation, section VIII contains the application to data from the behavioral sciences and section IX concludes the paper.

## II. TRANSMUTED PRANAV DISTRIBUTION

According to the Quadratic Rank Transmutation Map approach of Shaw and Buckley (2009), given $F_1(x)$ (the c.d.f. of the base distribution) and $\lambda$ such that $|\lambda| \le 1$, the c.d.f., $F_2(x)$, of the transmuted distribution is given by

$$F_2(x) = (1+\lambda)F_1(x) - \lambda F_1(x)^2 \tag{1}$$

Consequently, the PDF of the transmuted distribution is given by

$$f_2(x) = (1+\lambda)f_1(x) - 2\lambda f_1(x)F_1(x) \tag{2}$$

We make the following definition using the Pranav distribution (proposed by Shukla (2018)) as our base distribution having cumulative distribution function (CDF) and probability density function (PDF) given respectively as

$$F(x;\theta) = 1 - \left[1 + \frac{\theta x\left(\theta^2 x^2 + 3\theta x + 6\right)}{\theta^4 + 6}\right]e^{-\theta x}\ ,\ x > 0, \theta > 0 \tag{3}$$

$$f(x;\theta) = \frac{\theta^4}{\theta^4 + 6}\left(\theta + x^3\right)e^{-\theta x}\ ,\ x > 0, \theta > 0 \tag{4}$$

**Definition 2.1:** A random variable X is said to have a Transmuted Pranav distribution if the c.d.f. and p.d.f. are given respectively as

$$G(x;\theta,\lambda) = (1+\lambda)\left\{1 - \left[1 + \frac{\theta x\left(\theta^2 x^2 + 3\theta x + 6\right)}{\theta^4 + 6}\right]e^{-\theta x}\right\} - \lambda\left[1 - \left\{1 + \frac{\theta x\left(\theta^2 x^2 + 3\theta x + 6\right)}{\theta^4 + 6}\right\}e^{-\theta x}\right]^2 \tag{5}$$

$x > 0, \theta > 0, |\lambda| \le 1$ and

$$g(x;\theta,\lambda) = \frac{\theta^4}{\theta^4 + 6}\left(\theta + x^3\right)e^{-\theta x}\left\{(1-\lambda) + 2\lambda\left[1 + \frac{\theta x\left(\theta^2 x^2 + 3\theta x + 6\right)}{\theta^4 + 6}\right]e^{-\theta x}\right\} \tag{6}$$

$x > 0, \theta > 0, |\lambda| \le 1$, where the scale and shape parameters are $\theta$ and $\lambda$ respectively.

Figure 1 and Figure 2 present the graphs of the PDF and CDF of the Transmuted Pranav distribution respectively for varying values of the parameters.

 

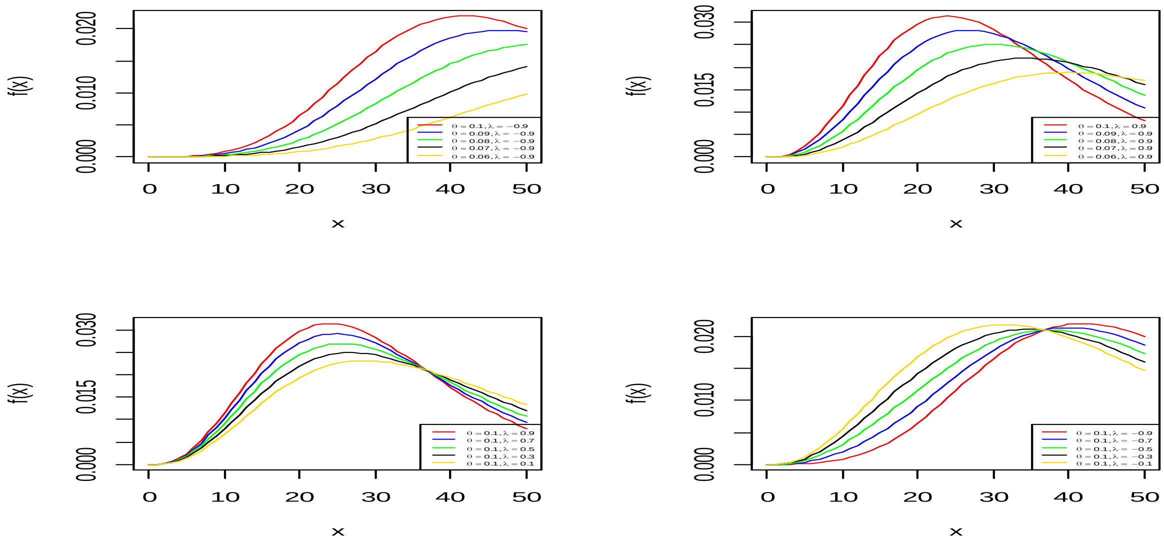


Figure 1. *The PDF of the Transmuted Pranav Distribution*

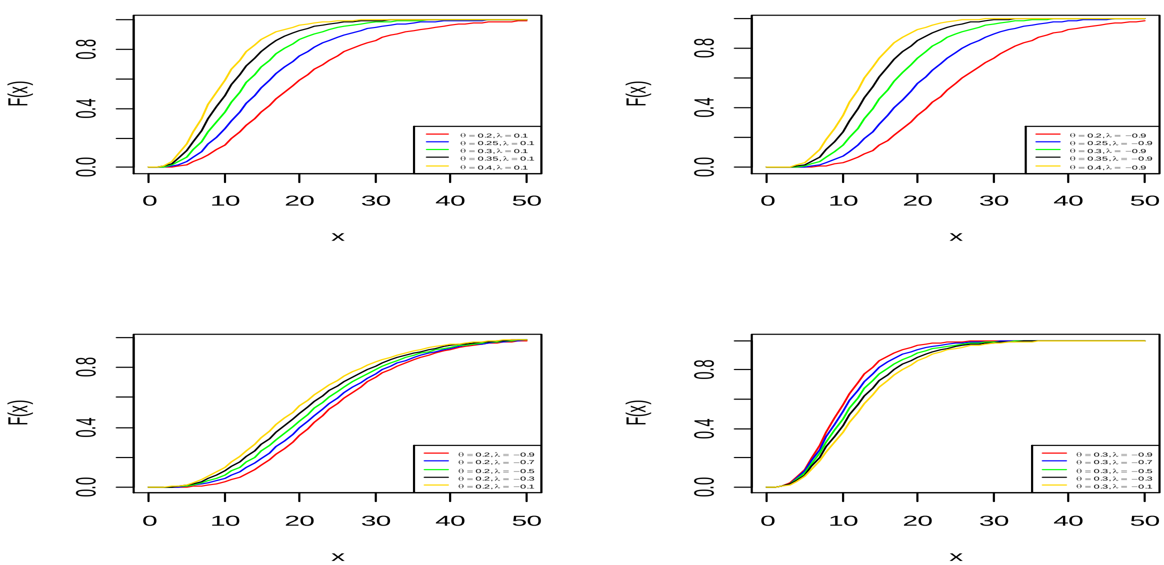


Figure 2. *The CDF of the Transmuted Pranav distribution*

## III. MATHEMATICAL PROPERTIES

### A. Moments

The $r^{th}$ moment about the origin, $E\left(X^r\right)$, of a random variable following the Transmuted Pranav distribution is given by

$$E\left(X^r\right) = \int_0^\infty x^r g(x;\theta,\lambda)dx \tag{7}$$

 

$$= \int_0^\infty \frac{x^r\theta^4}{\theta^4+6}\left(\theta+x^3\right)e^{-\theta x}\left\{(1-\lambda)+2\lambda\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\}dx$$

$$= \int_0^\infty \frac{\theta^5x^r}{\theta^4+6}e^{-\theta x}\left\{(1-\lambda)+2\lambda\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\}dx$$

$$+ \int_0^\infty \frac{\theta^4x^{r+3}}{\theta^4+6}e^{-\theta x}\left\{(1-\lambda)+2\lambda\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\}dx$$

$$= \frac{\theta^5(1-\lambda)}{\theta^4+6}\int_0^\infty x^r e^{-\theta x}dx+\frac{2\lambda\theta^5}{\theta^4+6}\int_0^\infty\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]x^r e^{-2\theta x}dx$$

$$+ \frac{\theta^4(1-\lambda)}{\theta^4+6}\int_0^\infty x^{r+3} e^{-\theta x}dx+\frac{2\lambda\theta^4}{\theta^4+6}\int_0^\infty\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]x^{r+3} e^{-2\theta x}dx$$

Recall that $\int_0^\infty x^r e^{-\theta x}dx=\frac{\Gamma(r+1)}{\theta^{r+1}}$ and $\Gamma(r)=(r-1)!$

Applying these identities and simplifying we obtain

$$E\left(X^r\right) = \frac{\theta^4(1-\lambda)r!}{\left(\theta^4+6\right)\theta^r}+\frac{\lambda\theta^4 r!}{\left(\theta^4+6\right)(2\theta)^r}+\frac{\lambda\theta^4(r+3)!}{8\left(\theta^4+6\right)^2(2\theta)^r}+\frac{3\lambda\theta^4(r+2)!}{4\left(\theta^4+6\right)^2(2\theta)^r}+\frac{3\lambda\theta^4(r+1)!}{\left(\theta^4+6\right)^2(2\theta)^r}$$
$$+\frac{(1-\lambda)(r+3)!}{\left(\theta^4+6\right)\theta^r}+\frac{\lambda(r+3)!}{8\left(\theta^4+6\right)(2\theta)^r}+\frac{\lambda(r+6)!}{64\left(\theta^4+6\right)^2(2\theta)^r}+\frac{3\lambda(r+5)!}{32\left(\theta^4+6\right)^2(2\theta)^r}+\frac{3\lambda(r+4)!}{8\left(\theta^4+6\right)^2(2\theta)^r} \quad (8)$$

**B. Moment Generating Function**

The moment generating function, $M_X(t)$, of a random variable, X, following the Transmuted Pranav distribution is given by

$$M_X(t) = E\left(e^{tx}\right)=\int_0^\infty e^{tx}g(x;\theta,\lambda)dx \quad (9)$$

$$= \int_0^\infty \frac{\theta^4e^{tx}}{\theta^4+6}\left(\theta+x^3\right)e^{-\theta x}\left\{(1-\lambda)+2\lambda\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\}dx$$

$$= \int_0^\infty \frac{\theta^5e^{-x(\theta-t)}}{\theta^4+6}\left\{(1-\lambda)+2\lambda\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\}dx$$

$$+ \int_0^\infty \frac{\theta^4x^3e^{-x(\theta-t)}}{\theta^4+6}\left\{(1-\lambda)+2\lambda\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\}dx$$

$$= \frac{\theta^5(1-\lambda)}{\theta^4+6}\int_0^\infty e^{-x(\theta-t)}dx + \frac{2\lambda\theta^5}{\theta^4+6}\int_0^\infty\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-x(2\theta-t)}dx$$

$$+ \frac{\theta^4(1-\lambda)}{\theta^4+6}\int_0^\infty x^3e^{-x(\theta-t)}dx + \frac{2\lambda\theta^4}{\theta^4+6}\int_0^\infty\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]x^3e^{-x(2\theta-t)}dx$$

Recall that

$$\int_0^\infty x^n e^{-\theta x}dx = \begin{cases}\frac{\Gamma(n-1)}{\theta^{n+1}} & (n>-1, \theta>0)\\ \frac{n!}{\theta^{n+1}} & (n=0,1,2,\cdots,\ \ \theta>0)\end{cases}$$

Therefore, applying the identity and simplifying, we obtain

$$M_X(t) = \frac{\theta^5(1-\lambda)}{\left(\theta^4+6\right)(\theta-t)} + \frac{2\lambda\theta^5}{\left(\theta^4+6\right)(2\theta-t)} + \frac{12\lambda\theta^8}{\left(\theta^4+6\right)^2(2\theta-t)^4} + \frac{12\lambda\theta^7}{\left(\theta^4+6\right)^2(2\theta-t)^3} + \frac{12\lambda\theta^6}{\left(\theta^4+6\right)^2(2\theta-t)^2}$$

$$+\frac{6\theta^4(1-\lambda)}{\left(\theta^4+6\right)(\theta-t)^4} + \frac{12\lambda\theta^4}{\left(\theta^4+6\right)(2\theta-t)^4} + \frac{1440\lambda\theta^7}{\left(\theta^4+6\right)^2(2\theta-t)^7} + \frac{720\lambda\theta^6}{\left(\theta^4+6\right)^2(2\theta-t)^6} + \frac{288\lambda\theta^5}{\left(\theta^4+6\right)^2(2\theta-t)^5}$$

$$= \frac{(1-\lambda)\theta^4}{\left(\theta^4+6\right)}\sum_{k=0}^\infty\left(\frac{t}{\theta}\right)^k + \frac{\lambda\theta^4}{\left(\theta^4+6\right)}\sum_{k=0}^\infty\left(\frac{t}{2\theta}\right)^k + \frac{3\lambda\theta^4}{4\left(\theta^4+6\right)^2}\sum_{k=0}^\infty\binom{k+3}{k}\left(\frac{t}{2\theta}\right)^k + \frac{3\lambda\theta^4}{2\left(\theta^4+6\right)^2}\sum_{k=0}^\infty\binom{k+2}{k}\left(\frac{t}{2\theta}\right)^k$$

$$+\frac{3\theta^4\lambda}{\left(\theta^4+6\right)^2}\sum_{k=0}^\infty\binom{k+1}{k}\left(\frac{t}{2\theta}\right)^k + \frac{6(1-\lambda)}{\left(\theta^4+6\right)}\sum_{k=0}^\infty\binom{k+3}{k}\left(\frac{t}{\theta}\right)^k + \frac{3\lambda}{4\left(\theta^4+6\right)}\sum_{k=0}^\infty\binom{k+3}{k}\left(\frac{t}{2\theta}\right)^k$$

$$+\frac{45\lambda}{4\left(\theta^4+6\right)^2}\sum_{k=0}^\infty\binom{k+6}{k}\left(\frac{t}{2\theta}\right)^k + \frac{45\lambda}{4\left(\theta^4+6\right)^2}\sum_{k=0}^\infty\binom{k+5}{k}\left(\frac{t}{2\theta}\right)^k + \frac{9\lambda}{\left(\theta^4+6\right)^2}\sum_{k=0}^\infty\binom{k+4}{k}\left(\frac{t}{2\theta}\right)^k$$

$$M_X(t) = \sum_{k=0}^\infty\left(\frac{(1-\lambda)\theta^4k!}{\left(\theta^4+6\right)\theta^k} + \frac{\lambda\theta^4k!}{\left(\theta^4+6\right)(2\theta)^k} + \frac{\lambda\theta^4(k+3)!}{8\left(\theta^4+6\right)^2(2\theta)^k} + \frac{3\lambda\theta^4(k+2)!}{4\left(\theta^4+6\right)^2(2\theta)^k} + \frac{3\lambda\theta^4(k+1)!}{\left(\theta^4+6\right)^2(2\theta)^k}\right.$$

$$\left.+\frac{(1-\lambda)(k+3)!}{\left(\theta^4+6\right)\theta^k} + \frac{\lambda(k+3)!}{8\left(\theta^4+6\right)(2\theta)^k} + \frac{\lambda(k+6)!}{64\left(\theta^4+6\right)^2(2\theta)^k} + \frac{3\lambda(k+5)!}{32\left(\theta^4+6\right)^2(2\theta)^k} + \frac{3\lambda(k+4)!}{8\left(\theta^4+6\right)^2(2\theta)^k}\right)\frac{t^k}{k!} \quad (10)$$

### IV. ENTROPY

Entropy is an important property of probability distributions. It is a quantity which appropriately measures the randomness or uncertainty in a probability distribution. The Shannon entropy and the Rényi entropy are the two popularly known proposed forms of entropy. However, this work considers Rényi entropy, which is widely used in literature.

 

The Rényi entropy of a random variable, X, following the Transmuted Pranav distribution is given by

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\left\{\int_0^\infty g^\gamma(x;\theta,\lambda)dx\right\} \qquad \gamma > 0, \gamma \neq 1 \tag{11}$$

$$= \frac{1}{1-\gamma}\log\left\{\int_0^\infty \frac{\theta^{4\gamma}}{(\theta^4+6)^\gamma}(\theta+x^3)^\gamma e^{-\theta\gamma x}\left[(1-\lambda)+2\lambda\left[1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right]e^{-\theta x}\right]^\gamma dx\right\}$$

$$= \frac{1}{1-\gamma}\log\left\{\int_0^\infty \frac{\theta^{4\gamma}}{(\theta^4+6)^\gamma}\theta^\gamma\left(1+\frac{x^3}{\theta}\right)^\gamma e^{-\theta\gamma x}\left[(1-\lambda)+2\lambda\left[1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right]e^{-\theta x}\right]^\gamma dx\right\}$$

Recall the binomial expansion $(1+a)^n = \sum_{j=0}^\infty \binom{n}{j} a^j$ . Applying the binomial expansion and simplifying, we have

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\left\{\sum_{j=0}^\infty\binom{\gamma}{j}\frac{\theta^{5\gamma-j}}{(\theta^4+6)^\gamma}\sum_{k=0}^\infty\binom{\gamma}{k}(1-\lambda)^{\gamma-k}(2\lambda)^k\sum_{l=0}^\infty\binom{k}{l}\left(\frac{\theta}{\theta^4+6}\right)^l\sum_{m=0}^\infty\binom{l}{m}6^{l-m}\sum_{n=0}^\infty\binom{m}{n}3^{m-n}\theta^{m+n}\right.$$

$$\left.\cdot\int_0^\infty x^{3j+l+m+n}e^{-x\theta(\gamma+k)}dx\right\}$$

Recall that $\int_0^\infty x^n e^{-ax}dx = \frac{\Gamma(n+1)}{a^{n+1}}$ and $\Gamma(n) = (n-1)!$. Therefore

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\left\{\sum_{j=0}^\infty\binom{\gamma}{j}\frac{\theta^{5\gamma-j}}{(\theta^4+6)^\gamma}\sum_{k=0}^\infty\binom{\gamma}{k}(1-\lambda)^{\gamma-k}(2\lambda)^k\sum_{l=0}^\infty\binom{k}{l}\left(\frac{\theta}{\theta^4+6}\right)^l\sum_{m=0}^\infty\binom{l}{m}6^{l-m}\sum_{n=0}^\infty\binom{m}{n}3^{m-n}\theta^{m+n}\right.$$

$$\left.\cdot\frac{\Gamma(3j+l+m+n+1)}{[\theta(\gamma+k)]^{3j+l+m+n+1}}\right\}$$

$$= \frac{1}{1-\gamma}\log\left\{\sum_{j=0}^\infty\binom{\gamma}{j}\frac{\theta^{5\gamma-j}}{(\theta^4+6)^\gamma}\sum_{k=0}^\infty\binom{\gamma}{k}(1-\lambda)^{\gamma-k}(2\lambda)^k\sum_{l=0}^\infty\binom{k}{l}\left(\frac{\theta}{\theta^4+6}\right)^l\sum_{m=0}^\infty\binom{l}{m}6^{l-m}\sum_{n=0}^\infty\binom{m}{n}3^{m-n}\theta^{m+n}\right.$$

$$\left.\cdot\frac{(3j+l+m+n)!}{[\theta(\gamma+k)]^{3j+l+m+n+1}}\right\}$$

 

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\left\{\sum_{j=0}^{\infty}\binom{\gamma}{j}\sum_{k=0}^{\infty}\binom{\gamma}{k}\sum_{l=0}^{\infty}\binom{k}{l}\sum_{m=0}^{\infty}\binom{l}{m}\sum_{n=0}^{\infty}\binom{m}{n}\left(\frac{\theta^{5\gamma-4j-1}\lambda^k(1-\lambda)^{\gamma-k}2^{k+l-m}3^{m-n}}{(\theta^4+6)^{\gamma+l}}\right)\left(\frac{(3j+l+m+n)!}{(\gamma+k)^{3j+l+m+n+1}}\right)\right\} \quad (12)$$

## V. ORDER STATISTICS

Let $Y_1 \le Y_2 \le Y_3 \le \cdots \le Y_n$ be the order statistics corresponding to the random sample $X_1, X_2, X_3, \cdots, X_n$ from a Transmuted Pranav distribution. Then the cumulative distribution function (c.d.f.) and the probability density function (p.d.f.) of the $r^{th}$ order statistics, $Y_r$, are respectively given by

$$G_{Y_r}(y) = \sum_{i=r}^{n}\binom{n}{i}[G(y;\theta,\lambda)]^i[1-G(y;\theta,\lambda)]^{n-i} \quad (13)$$

and

$$g_{Y_r}(y) = \frac{n!}{(r-1)!(n-r)!}[G(y;\theta,\lambda)]^{r-1}[1-G(y;\theta,\lambda)]^{n-r}g(y;\theta,\lambda) \quad (14)$$

Substituting (5) in (13), we obtain the c.d.f. of the $r^{th}$ order statistics as

$$G_{Y_r}(y) = \sum_{i=r}^{n}\sum_{j=0}^{n-i}\binom{n}{i}\binom{n-i}{j}(-1)^j(\lambda+1)^{i+j}\left[1-\left\{1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right\}e^{-\theta x}\right]^{i+j}$$

$$\cdot\left[1-\frac{\lambda}{\lambda+1}\left\{1-\left[1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right]e^{-\theta x}\right\}\right]^{i+j} \quad (15)$$

Similarly, substituting (5) and (6) in (14), we obtain the p.d.f. of the $r^{th}$ order statistics as

$$g_{Y_r}(y) = \frac{n!\theta^4(\theta+x^3)e^{-\theta x}}{(\theta^4+6)(r-1)!(n-r)!}\sum_{j=0}^{n-r}\binom{n-r}{j}(-1)^j(\lambda+1)^{r+j-1}\left[1-\left\{1+\frac{\theta x(\theta^2x^2+3\theta x+6}{\theta^4+6}\right\}e^{-\theta x}\right]^{r+j-1}$$

$$\cdot\left[1-\frac{\lambda}{\lambda+1}\left\{1-\left[1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right]e^{-\theta x}\right\}\right]^{r+j-1}\left\{(1-\lambda)+2\lambda\left[1+\frac{\theta x(\theta^2x^2+3\theta x+6)}{\theta^4+6}\right]e^{-\theta x}\right\} \quad (16)$$

## VI. RELIABILITY ANALYSIS

Two important functions used in reliability analysis, given a probability distribution, include the survival function (reliability) and hazard rate function (failure rate).

### A. Survival function

The survival function is used to analyze the expected length of time before one or more events (such as death of an organism or failure of a mechanical system) occur. The survival function is given by

$$S(x;\theta,\lambda) = 1-G(x;\theta,\lambda) \quad (17)$$

 

$$= 1-\left\{(1+\lambda)\left\{1-\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\}-\lambda\left[1-\left\{1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right\}e^{-\theta x}\right]^2\right\}$$

$$S(x;\theta,\lambda) = 1-\left\{1-\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\}\left\{1+\lambda\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\} \qquad (18)$$

**B. Hazard rate function**

The hazard rate is the rate of failure of a life in question (say life of an organism or mechanical system) at time, t, given that it has survived up to the time, t. The hazard rate function is given by

$$h(x;\theta,\lambda)=\frac{g(x;\theta,\lambda)}{1-G(x;\theta,\lambda)}=\frac{g(x;\theta,\lambda)}{s(x;\theta,\lambda)} \qquad (19)$$

$$h(x;\theta,\lambda) = \frac{\frac{\theta^4}{\theta^4+6}\left(\theta+x^3\right)e^{-\theta x}\left\{(1-\lambda)+2\lambda\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\}}{1-\left\{1-\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\}\left\{1+\lambda\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+6\right)}{\theta^4+6}\right]e^{-\theta x}\right\}} \qquad (20)$$

The graph of the hazard rate function of the Transmuted Pranav distribution is plotted in Figure 3. The graphs show various shapes including monotonically increasing and unimodal shapes for varying values of the parameters.

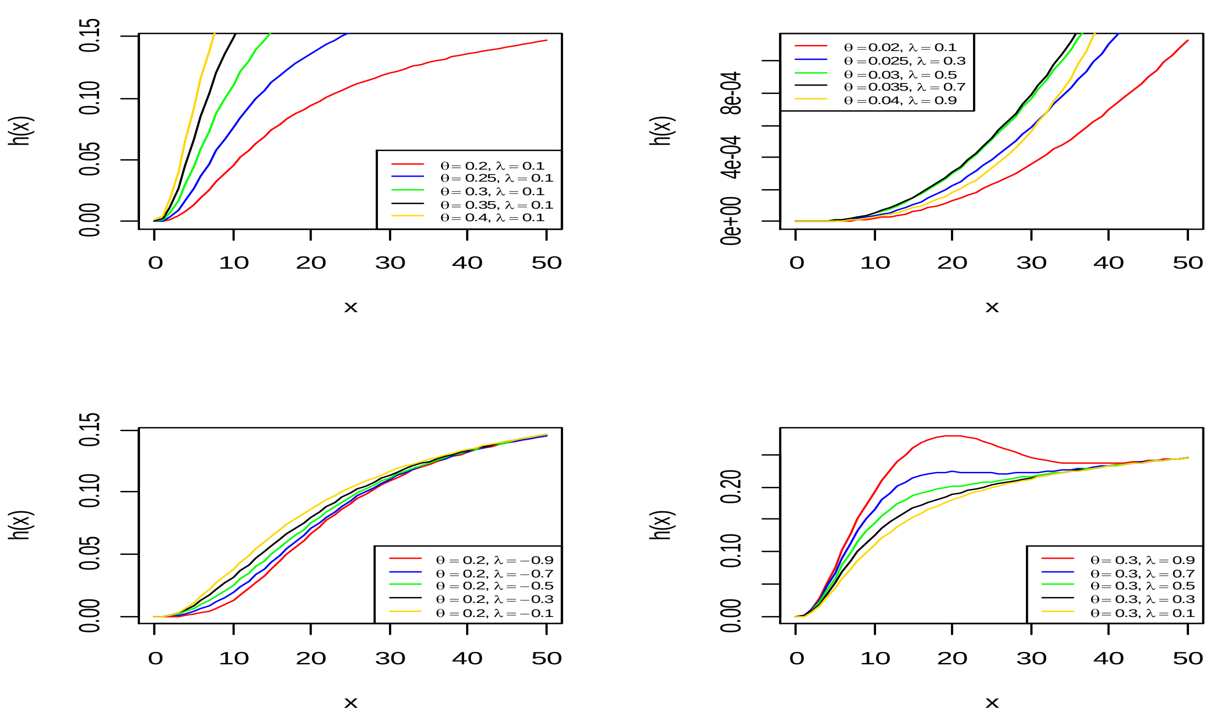


Figure 3. *The hazard rate function of the Transmuted Pranav distribution*

## VII. MAXIMUM LIKELIHOOD ESTIMATION

Let $X_1, X_2, X_3, \cdots, X_n$ be a random sample of size $n$ from a Transmuted Pranav distribution having p.d.f. as in equation (2.6). Then the likelihood function, $L$, of the Transmuted Pranav distribution is given by

$$L = \left(\frac{\theta^4}{\theta^4+6}\right)^n \prod_{i=1}^{n}\left(\theta + x_i^3\right) e^{-\theta\sum_{i=1}^{n} x_i} \prod_{i=1}^{n}\left\{(1-\lambda) + 2\lambda\left[1+\frac{\theta x_i\left(\theta^2 x_i^2 + 3\theta x_i + 6\right)}{\theta^4+6}\right]e^{-\theta x_i}\right\} \quad (21)$$

and the log likelihood function is therefore given by

$$\ln L = 4n\ln\theta - n\ln\left(\theta^4+6\right) + \sum_{i=1}^{n}\ln\left(\theta + x_i^3\right) - \theta\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\ln\left\{(1-\lambda) + 2\lambda\left[1+\frac{\theta x\left(\theta^2 x_i^2 + 3\theta x_i + 6\right)}{\theta^4+6}\right]e^{-\theta x_i}\right\} \quad (22)$$

It then follows that the maximum likelihood estimates (MLE), $\hat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}$ are the roots of the nonlinear equations

$$\frac{\partial \ln L}{\partial \theta} = \frac{4n}{\theta} - \frac{n}{\theta^4+6} + \sum_{i=1}^{n}\frac{1}{\left(\theta + x_i^3\right)} - \sum_{i=1}^{n} x_i - \frac{2\lambda}{\left(\theta^4+6\right)}\sum_{i=1}^{n}\frac{\left[\left(6\theta^3+\theta^7\right)x_i^4 + 4\theta^6 x_i^3 + 12\theta^5 x_i^2 + \left(\theta^8 + 30\theta^4\right)x_i\right]e^{-\theta x_i}}{(1-\lambda)\left(\theta^4+6\right) + 2\lambda\left(\theta^3 x_i^3 + 3\theta^2 x_i^2 + 6\theta x_i + \theta^4 + 6\right)e^{-\theta x_i}} = 0 \quad (23)$$

$$\frac{\partial \ln L}{\partial \lambda} = \sum_{i=1}^{n}\frac{2\left[\theta^3 x_i^3 + 3\theta x_i^2 + 6\theta x_i + \theta^4 + 6\right]e^{-\theta x_i} - \left(\theta^4+6\right)}{(1-\lambda)\left(\theta^4+6\right) + 2\lambda\left[\theta^3 x_i^3 + 3\theta^2 x_i^2 + 6\theta x_i + \theta^4 + 6\right]e^{-\theta x_i}} = 0 \quad (24)$$

where $\boldsymbol{\theta} = (\theta, \lambda)$ and $\hat{\boldsymbol{\theta}} = (\hat{\theta}, \hat{\lambda})$.

The values of the parameter estimates were computed using R-Software.

## VIII. APPLICATION

We demonstrate the potentials of the Transmuted Pranav distribution by application to real life data set. The distribution is compared with Pranav, Sujatha, Akash, Lindley and Exponential distributions using Goodness of fit indices such as the widely used Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) as well as $-2\ln L$. The maximum likelihood estimates of the parameters and the values of the Goodness of fit indices are computed, using R-Software, for data collected by Balakrishnan et al. (2010) on depressive conditions of children in a city located at the southern part of Chile.

**Data Set:** Table 1 presents the data collected by Balakrishnan et al. (2010) with frequency in parenthesis. The authors conducted a study which allowed collection of real data corresponding to the scores of the "General Rating of Affective Symptoms for Preschoolers (GRASP)" scale in a city located at the southern part of Chile. The GRASP scale measures behavioral and emotional problems of children, which can be classified with or not with depressive condition according to the scale.

Table 1: Scores of the GRASP scale of children (with frequency in parenthesis) in a city located at the south part of Chile reported by Balakrishnan et al. (2010).

| | | | | | | |
|---|---|---|---|---|---|---|
| 19(16) | 20(15) | 21(14) | 22(9) | 23(12) | 24(10) | 25(6) |
| 26(9) | 27(8) | 28(5) | 29(6) | 30(4) | 31(3) | 32(4) |
| 33 | 34 | 35(4) | 36(2) | 37(2) | 39 | 42 |
| 44 | | | | | | |

Table 2: MLE's, $-2\ln L$ , AIC and BIC of the fitted distributions of the data set.

| Model | Parameter estimates (MLE) | $-2\ln L$ | AIC | BIC |
|---|---|---|---|---|
| Transmuted Pranav (TPD) | $\boldsymbol{\theta}$= 0.200562<br>λ= -1 | 885.61 | 889.61 | 895.41 |
| Pranav | $\theta$ = 0.160222 | 945.03 | 947.03 | 948.94 |
| Sujatha | $\theta$= 0.117456 | 985.69 | 987.69 | 990.29 |
| Akash | $\theta$= 0.11961 | 981.28 | 983.28 | 986.18 |
| Lindley | $\theta$= 0.077247 | 1041.64 | 1043.64 | 1046.54 |
| Exponential | λ = 0.04006 | 1130.26 | 1132.26 | 1135.16 |

It is obvious from Table 2 that the Transmuted Pranav distribution, having lowest values across the goodness of fit indices, yielded a better fit to the data than the competing distributions.

## IX. CONCLUSION

A new two-parameter distribution named Transmuted Pranav distribution is derived using the Quadratic Rank Transmutation Map (RQTM) approach and various mathematical properties of the distribution are derived. The maximum likelihood estimation of the parameters is also presented. The new distribution is fitted to the scores of the "General Rating of Affective Symptoms for Preschoolers (GRASP)" scale of children in a city located at the south part of Chile. The results highlight the better performance of the newly proposed distribution over some competing distributions.

## REFERENCES


[1]. Adamidis, K., & Loukas, S. (1998). A lifetime distribution with decreasing failure rate. *Statistics & Probability Letters*, *39*(1), 35-42.

[2]. Balakrishnan, N., Victor, L., & Antonio, S. (2010). A mixture model based on Birnhaum-Saunders distributions. *A study conducted by authors regarding the scores of the GRASP (general rating of affective symptoms for preschoolers), in a city located at south part of the Chile*.

[3]. Bhatti, F. A., Hamedani, G. G., Korkmaz, M. Ç., & Ahmad, M. (2018). The transmuted geometric-quadratic hazard rate distribution: development, properties, characterizations and applications. *Journal of Statistical Distributions and Applications, 5(1),* 4.

[4]. Borah M, Hazarika J (2017). Discrete Shanker Distribution and Its Derived Distributions. *Biom Biostat Int J 5(4): 00140. DOI: 10.15406/bbij.2017.05.00140*

[5]. Ghitany, M. E., Atieh, B., & Nadarajah, S. (2008). Lindley distribution and its application. *Mathematics and computers in simulation*, *78(4),* 493-506.

[6]. Marshall, A. W., & Olkin, I. (1997). A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families. *Biometrika*, *84*(3), 641-652.

[7]. Nofal, Z. M., El Gebaly, Y. M., Altun, E., Alizadeh, M., & Butt, N. S. (2017). The transmuted geometric-Weibull distribution: properties, characterizations and regression models. *Pakistan Journal of Statistics and Operation Research, 13(2),* 395-416.

[8]. Shanker, R. (2015). Akash distribution and its applications. *International Journal of Probability and Statistics, 4(3), 65-75.*

[9]. Shanker, R. (2016). Sujatha distribution and its Applications. *Statistics in Transition new series, 17(3)*, 391-410.

[10]. Shanker, R., & Shukla, K. K. (2017). Ishita distribution and its applications. *Biometrics & Biostatistics International Journal*, *5(2)*, 1-9.

[11]. Shaw, W. T., & Buckley, I. R. (2009). The alchemy of probability distributions: beyond Gram-Charlier expansions, and a skew-kurtotic-normal distribution from a rank transmutation map. *arXiv preprint arXiv:0901.0434.*

[12]. Shukla K. K. (2018). Pranav distribution with properties and its applications. *Biom Biostat Int J., 7(3), 244–254. DOI: 10.15406/bbij.2018.07.00215.*

[13]. Tahir, M. H., & Cordeiro, G. M. (2016). Compounding of distributions: a survey and new generalized classes. *Journal of Statistical Distributions and Applications*, *3*(1), 13.

[14]. Yilmaz, M., Hameldarbandi, M., & Kemaloglu, S. A. (2016). Exponential-modified discrete Lindley distribution. *SpringerPlus*, *5(1),* 1660.


 