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Endorsement of Optimal ordering cycle implementing Quick Switching System with Single Sampling Plan with Permissible delay in Payments involving non-destructive testing

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Abstract— Quick Switching System-1 intensifies the likelihood of identifying the defects at the same time curtails the sample size. It consists of two intensity of inspection namely, normal and tightened inspection based on the number of defects. Tightened inspection with less acceptance number is employed when there is a possibility of high number of defects otherwise normal inspection is applied. Economic ordering quantity with permissible delay in payment enables the consumer optimal ordering size with the least possible cost and permissible late payment. Application of QSS-1 in EOQ model in permissible delay in payment has the benefits of cost-effective, reliable quality products. Case study on casting defects is given. Numerical illustrations are also provided to validate the results.

Keywords— Quick Switching System-1; Normal inspection; Tightened inspection; Economic Ordering Quantity; Permissible delay in payments.

I. INTRODUCTION

Acceptance Sampling is inevitably a protective system, originated as a defensive measure against the peril of the degeneration of the quality. When the sole purpose of the inspection is to either accept or reject the lot, based on the constancy of the standards acceptance sampling plan by attributes is employed. The main objective of the acceptance sampling plan is to curtail the inspection cost simultaneously assures the consumer that the consumer that satisfactory number of products are inspected. Acceptance sampling system refers to a collection of acceptance sampling plans or acceptance sampling scheme together with criteria by which appropriate plans or schemes may be chosen. Quick Switching System-1 is simple of all sampling schemes proposed by Romboski [1]. It consists of two sampling procedures with switching rules between normal and tightened inspection. Romboski has presented extensively a system of immediate switching to tightened inspection when the rejection comes under normal inspection and vice versa[1]. Due to rapid switching between normal and tightened plans, this system is referred as Quick Switching System-1(QSS-1). QSS-1(n,c_N,c_T) consists of two sampling procedures (n,c_N) & (n,c_T) where n= sample size; c_N=acceptance number under normal inspection; c_T=acceptance number under tightened inspection(c_N<c_T).The probability of acceptance of Quick Switching System-1 is

$$P_a = \frac{P_T}{(1-P_N)+P_T} \qquad (1)$$

Where,
P_N = proportions of lots expected to be accepted using (n, c_N).
P_T = proportions of lots expected to be accepted using (n, c_T).
The modus operandi of QSS-1 is

1. At the outset, start using normal inspection with c_N.
2. If a lot is rejected(d>c), then switch to tightened inspection with c_T.
3. When on the tightened inspection, switch to normal inspection after a lot is accepted (d<c).
4. Alternate to and fro as imposed by the switching rules. The sample size n,c_N,c_T are obtained from the table of Romboski.

The inspection is based on non-destructive testing. Tables are developed based on the QSS-1 with single sampling plan as reference plan under the conditions of Poisson distribution. It exactly deals with the defects per unit. The np value also known as unity value aids in construction and evaluation on the basis of the operating ratio idlest p_2/p_1. The sampling plan parameters are derived with the objective of satisfying both the producer risk α and consumer risk β. For the predetermined sampling strength (p_1,α,p_2,β) is the acceptance number and sample size is determined by applying the following procedure

- Calculate the operating ratio R= p_2/p_1,where p_1=Acceptable Quality Level(AQL);p_2=Limiting Quality Level.
- Select a value of R from the table which is equal or just lesser than the desire value in terms of assuring both the risk.
- The acceptance number c can be obtained by using the closet value of the operating ratio
- The sample size n is obtained by calculating np_1/p_1 or np_2/p_2, whichever is larger; always round off the sample size.

5. For instance, consider α=0.05& β=0.10, p_1=0.01;p_2=0.04; Then the operating ratio R=4; By applying the table the value which is equal or just less than the desired value is 4.057 with c_N=4;c_T=4and np_1=1.97& np_2=7.994. Then, the size of the sample n=1.97/0.01=197 or 7.994/0.04=199.85=200.usually the larger sample size is selected. Therefore n=200. The values are extracted in Schilling as Table T17.1 .

The efficiency of the QSS-1 is it requires less acceptance number only when there is high number of defects. The result is that a succeeding lot is more likely to be rejected if proceeding lots have been rejected and more likely to be accepted if proceeding lots have been accepted. Application of the acceptance sampling by attributes has extended its core of its application. Tsao considered acceptance sampling plan in the EOQ model under the permissible delay in payments [3]. Pradeepa Veerakumari and Aruna developed economic ordering models under the conditions of permissible delay in payments with single sampling plan under the conditions of IPD with three payment privileges. Pradeepa Veerakumari and Aruna proposed an EOQ model with c=0 single sampling plan with inspection errors. The present study incorporates Quick Switching System-1 with Economic Ordering Quantity with permissible delay in payments. The study is advantageous as it is cost-effective and less time consuming.

The study is organized as the section one consists of introduction, section 2 involves the model and formulation, Section 3 has optimal conditions, section 4 consists of the numerical illustrations and conclusion is provided at the end of the study.

II. MODEL ASSUMPTIONS AND FORMULATION

The assumptions made in the model are as follows

- QSS-1(n,c_N,c_T) with single sampling as reference plan is applied.
- Replenishment cycle time is instantaneous.
- Single product inventory is considered.
- Demand is known and invariable throughout the year.
- Lead time is zero
- $W<S, I_e<I_p$. Whereas, W is the price at which the product is sold to buyer and S is the price that sold back. I_e is the Interest earned and I_p is the interest paid.

Consider that the consumer places an order of size Q to the producer. It is assumed that the products in the rejected lots are sold at a reduced rate G per unit before the shipment of the next lot. The ordering cost is denoted as C_o and the demand rate is D. Then the modus operandi of the single sampling plan is applied. If the lot is accepted, the vendor offers the buyer, the permissible delay in the payments to owe the amount paid to the vendor. In practice, no interest is levied on the balance amount within the permissible delay in payments. But, if the payment is not paid after the closure of the permissible delay in payments, then interest is levied on the balance amount. Therefore leads to a valid point to the customer to delay the payments. On the contrast, permissible delay in payments attracts more new customers and already established customers will make more order to get the advantage of permissible delay in payments. So the permissible delay in payment is both favorable to the consumer and the producer. Total costs included in the model are the cost of placing orders C_o, inspection cost C_i, cost of holding the products C_h, cost of interest charges for unsold items, interest earned from the sales revenue. The model consists of two case, T≥M,T≤M where T is the replenishment cycle time and M is the permissible delay payment period. The total cost function consists of the following costs

- Annual Ordering Cost= $\frac{C_o}{T}$ (2)
- Annual Inspection cost= $\frac{C_i.n}{T}$ (3)
- Annual cost for inventory holding = $\frac{DTC_h}{2}$ (4)
- Annual Interest earned involves two cases

Case 1
Suppose that T≥M,

Annual Interest earned= $\frac{S.I_e DM^2}{2T}$ (5)

Case 2:
Suppose that T≤M,

Annual Interest earned= $\frac{S.I_e DT}{2} + S.I_e.D(M-T)$ (6)

- Annual Interest Paid

Case 1

Suppose that T≥M

Annual Interest paid = $\frac{WD.I_p.(T-M)^2}{T}$ (7)

Case 2

When T≤M

Annual Interest charge =0. In this case, no interest charge is paid for the items.

Total cost, suppose that T≥M

At first, if the lot is ordered, it is subjected to the inspection, if the lot is rejected, then the expected total cost,

$$E(TC_1) = \frac{C_o}{T} + \frac{C_i.n}{T} - GD \quad (8)$$

The process is continued until the lot is accepted then, the total expected cost

$$E(TC_1) = P_a \sum_{x=0}^{\infty} \left(\left(x+1 \left(\frac{C_o}{T} + \frac{C_i.n}{T} \right) \right) - xGD + \frac{DTC_h}{2} + \frac{W.D.I_p(T-M)^2}{T} - \frac{S.I_e DM^2}{2T} \right) (1-P_a)^x \quad (9)$$

Then the series can be rewritten as,

$$E(TC_1) = P_a \left(\left(\sum_{x=0}^{\infty} (x+1) \left(\frac{C_o}{T} + \frac{C_i.n}{T} \right) \right) - \sum_{x=0}^{\infty} (1-P_a)^x xGD + \sum_{x=0}^{\infty} (1-P_a)^x \left(\frac{DTC_h}{2} + \frac{W.D.I_p(T-M)^2}{T} - \frac{S.I_e DM^2}{2T} \right) \right) \quad (10)$$

Then the infinite series expansion is

$$\sum_{x=0}^{\infty} [(1+x)(1-P_a)^x] = \frac{1}{P_a^2} \quad (11)$$

$$\sum_{x=0}^{\infty} x(1-P_a)^x = \frac{1-P_a}{P_a^2} \quad (12)$$

$$\sum_{x=o}^{\infty} (1-P_a)^x = \frac{1}{P_a} \quad (13)$$

Substituting the values of 11 to 13 in eqn 10, total expected cost function becomes,

$$(TC_1) = \left(\left(\frac{C_o}{T} + \frac{C_i.n}{T} \right) - (1-P_a)GD + P_a \left(\frac{DTC_h}{2} + \frac{W.D.I_p(T-M)^2}{T} - \frac{S.I_e DM^2}{2T} \right) \right) \quad (14)$$

Similarly, the total function, *suppose that T≤M*, is

$$(TC_2) = \left(\left(\frac{C_o}{T} + \frac{C_i.n}{T} \right) - (1-P_a)GD + P_a \left(\frac{DTC_h}{2} - \frac{S.I_e DT}{2} - S.I_e.D(M-T) \right) \right) \quad (15)$$

III. Optimal Conditions for the Optimal Replenishment Cycle Time

Optimal value of T^* with minimum variable cost is obtained by using the concepts of maxima and minima in differential calculus. Differentiate Total variable cost with respect to T. Since for, the maximum or minimum value of total variable cost its first order derivative should be zero. By equating first order derivative of total cost to zero, the optimal replenishment order cycle time T^* is obtained. To ensure the global minimum of T^*, verify that second order derivative of total cost with respect to T is positive for any finite value of Q>0.The first order derivative of TC_1 is

$$\frac{dTC_1}{dT}=\frac{-2(C_o+C_i.n)+P_a\begin{pmatrix}(DT^2(C_h+W.I_p))\\ -DM^2(W.I_p-S.I_e)\end{pmatrix}}{2T^2} \quad (16)$$

Second order derivative of TC_1 is

$$\frac{d^2TC_1}{dT^2}=\frac{2(C_o+C_i.n)-P_a\left(DM^2(W.I_p-S.I_e)\right)}{T^3} \quad (17)$$

The first order derivative of TC_2 is

$$\frac{dTC_2}{dT}=\frac{-2(C_o+C_i.n)+P_a\left(DT^2(C_h+S.I_e)\right)}{2T^2} \quad (18)$$

Second order derivative of TC_2 is

$$\frac{d^2TC_2}{dT^2}=\frac{2(C_o+C_i.n)}{T^3} \quad (19)$$

From equation (18) that the TC_1 is minimum only when, $2(C_o+C_i.n)-P_a(DM^2(W.I_p-S.I_e)>0$, equation(20) clearly indicates that the TC_2 is global minimum. The optimal replenishment cycle time T_1* associated with the TC_1 is

$$T_1^*=\sqrt{\frac{2(C_o+C_i.n)+P_a.DM^2(W.I_p-S.I_e)}{D(C_h+W.I_p).P_a}} \quad (20)$$

Similarly, the optimal replenishment cycle time T_2^* is obtained

$$T_2^*=\sqrt{\frac{2(C_o+C_i.n)}{D(C_h+S.I_e).P_a}} \quad (21)$$

The optimal condition for T>M is obtained by equating the value of the T_1^*>M then by taking square roots on both sides.

$$T_1^*=\sqrt{\frac{2(C_o+C_i.n)+P_a.DM^2(W.I_p-S.I_e)}{D(C_h+W.I_p).P_a}}>M \quad (22)$$

$$\Delta=2(C_o+C_i.n)-DM^2(C_h+S.I_e)P_a>0 \quad (23)$$

By applying this methodology, the optimal condition for T<M.i.e. T_2^*<M is obtained as Δ<0 and if T=M i.e. $T_1^* = T_2^*$is Δ=0.

IV. Numerical Illustrations

Casting defect is the unacceptable flaw in the metal casting process. The casting defects are categorized into five types viz, gas porosity, shrinkage defects, mold material defects, pouring metal defaults and metallurgical defects. Casting defects can be negatively impact the bottom line of the foundry. Foundry manufacturing process includes preparation of moulds, molding, casting etc. The products are examined using visual inspection by the experts with the sensory enhancing equipment for instance, stethoscope, magnifiers, and tooth pick. Various types of defects may be found during the inspection on the final product for instance, acceptable defects, remediable defects and major defects. It is assumed that the products manufactured are with acceptable and remediable defects and if the lot is rejected it is sold at the discounted rate of \$1. The demand for the iron cast is 1000. The price of the iron cast is \$12 and the consumer sold it at \$13. Then, I_e=10% and I_p=15%;M=0.2;C_0=\$35;C_h=\$0.5 per unit ;C_i=\$1 per unit; p=0.01; The sample size and the acceptance number are achieved as shown in the section 1 with c=4;n=200.The value of $\Delta=2(C_0+C_i.n)-DM^2(C_h+S.I_e)P_a$ is 174 which is greater than zero. Then the optimal replenishment cycle interval time is T_1^*= 0.4613 and the associated least possible cost is \$662.60.

V. Conclusion

Applying single sampling plan by attributes in visual inspection of welds helps to identify the defects with non-destructive testing of the products leads to minimized cost and less time consuming. QSS-1 carries the advantage of having two intensity of inspection; tightened inspection with less acceptance number is applied only when there is a possible of the high number of defects otherwise normal inspection is applied. As a consequence of the permissible delay in payments, the replenishment cycle interval & ordering cost usually rise to some extent, but there is a considerable decrease in the total annual cost. Thus the study is cost-effective and less time. Future scope of the research includes the probability demand as the model considers deterministic demand. In the view of acceptance sampling, it may be extended to Skip-lot sampling plan of type 2, Quick Switching System-1 with destructive testing.

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