International Journal of Scientific Research in Mathematical and Statistical Sciences **Research Paper**
Vol.6, Issue.1, pp.251-254, February (2019) **E-ISSN:** 2348-4519
DOI: https://doi.org/10.26438/ijsrmss/v6i1.251254

The Generalized Zagreb Index of Capra Operation on Cycle

Keerthi G. Mirajkar [1], Pooja B[2 *] and M. R. Farahani [3]

[1,2] Department of Mathematics, Karnatak University's Karnatak Arts College, Dharwad-580 001, Karnataka, India
[3] Department of Applied Mathematics, Iran University of Science and Technology, Narmak, Tehran 16844, Iran

[*] *Corresponding Author: bkvpooja@gmail.com, Tel.: +91-8762530317*

Available online at: www.isroset.org

Received: 18/Jan/2019, Accepted: 11/Feb/2019, Online: 28/Feb/2019

Abstract— The Generalized Zagreb index of a graph $\boldsymbol{G}$ is defined for arbitrary non-negative integer $\boldsymbol{r}$ and $\boldsymbol{s}$ as $\boldsymbol{M_{(r,s)}(G)} = \sum_{uv\in E(G)} [\boldsymbol{d_G^r(u)d_G^s(v) + d_G^s(u)d_G^r(v)}]$. In this paper, we obtain expressions for generalized Zagreb index of Capra operation of cycle $\boldsymbol{C_n}$ on $\boldsymbol{n}$ vertices, which generalizing the existing results of Sardar et al. (Open J. Math. Sci. 1 (2017) 44-51). As an application, this result enables us to find first Zagreb index, second Zagreb index and $\boldsymbol{F}$-index of Capra operation of cycle $\boldsymbol{C_n}$ on $\boldsymbol{n}$ vertices.

Keywords*— Capra Operation, Generalized Zagreb Index.*

I. INTRODUCTION

Let $G = (V, E)$ be a simple graph with $n = |V|$ vertices and $m = |E|$ edges. As usual, n is said to be an order and m the size of G. Let $d_G(v)$ be the degree of vertex v in G. We refer to [14] for unexplained terminology and notation. A graph invariant is any function on a graph that does not depend on a labeling of its vertices. Such quantities are also called topological indices. Hundreds of different invariants have been employed to date (with unequal success) in quantitative structure-activity relationship (QSAR) and quantitative structure-property relationship (QSPR) studies. In 1972, Gutman and Trinajstic$'$ [13] introduced first Zagreb index M_1 and second Zagreb index M_2 of a graph G to study the structure-dependency of the total π-electron energy (ε), respectively, defined as

$$M_1(G) = \sum_{u\in V(G)} d_G(u)^2$$
and
$$M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v).$$

In the same paper, another topological index, defined as sum of cube of degrees of vertices of the graph was also shown to influence ε. However this topological index was never again investigated and was left to oblivion, except in a recent paper by Furtula and Gutman [11] where they named this index as forgotten topological index or F-index. With this notion, the forgotten topological index is defined as

$$F(G) = \sum_{v\in V(G)} d_G(v)^3.$$

There are widely studied degree based topological indices and their polynomials due to their applications in chemistry, for details see [1,2,3,7,8,9,11,15,16].

In 2011, M. Azari and A. Iranmanesh [1] introduced the generalized Zagreb index of a connected graph G, based on degree of vertices of G. The Generalized Zagreb index of G is defined for arbitrary non-negative integer r and s as follows:

$$M_{(r,s)}(G) = \sum_{uv\in E(G)} [d_G^r(u)d_G^s(v) + d_G^s(u)d_G^r(v)].$$

A mapping is a new drawing of an arbitrary planar graph G on the plane. In graph theory, there are many different mappings (or drawing); one of them is Capra operation. This method enables one to build a new structure of a planar graph G. Let G be a cyclic planar graph. Capra map operation is achieved as follows:

(1) insert two vertices on every edge of G;
(2) add pendant vertices to the above inserted ones and
(3) connect the pendant vertices in order $(-1; +3)$ around the boundary of a face of G.

By runing these steps for every face/cycle of G, one obtains the Capra-transform $Ca(G)$ of G, see Figure 1. This method was introduced by M.V. Diudea and used in many papers [4,5,6,7,8,9,10,12].

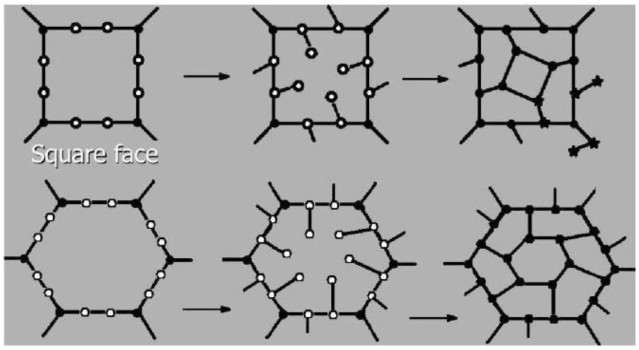

Figure 1: An example of Capra map operation on the hexagon face. Since Capra of planar benzenoid series has a very remarkable structure, we lionize it.

For a given Capra operation Ca on G, we define the iteration of Ca as follows:

(1) $Ca_0(G) = G$

(2) $Ca_k(G) = Ca(Ca_{k-1}(G))$ for $k \geq 2$.

For detailed discussions of the Capra operation, we refer the interested reader to [16] and the references cited therein. Figures 2 and 3 dipicts the graphs $Ca_k(C_6)$ for $k = 1,2,3$ and C_6.

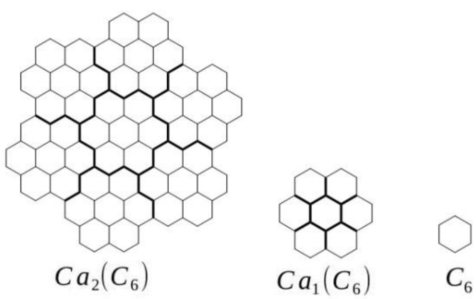

Figure 2

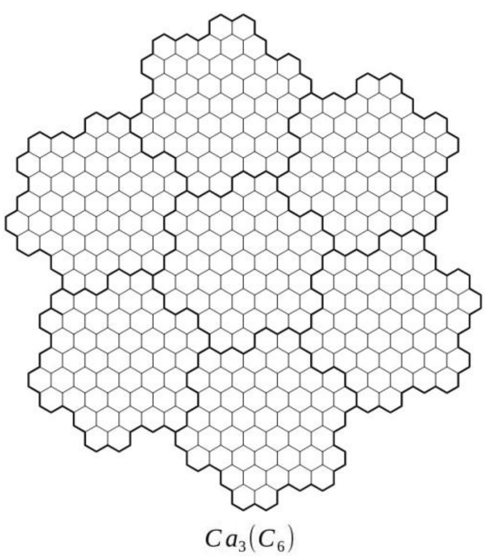

Figure 3

In [16], authors calculated the generalized Zagreb index of Capra operation of cycle C_6 on 6 vertices. In this paper, we obtain expressions for generalized Zagreb index of Capra operation of cycle C_n on n vertices, which generalizing the existing results of Sardar et al. [16].

In the next section, we obtain expressions for generalized Zagreb index of Capra operation of cycle $\boldsymbol{C_n}$ on $\boldsymbol{n}$ vertices, which generalizing the existing results of Sardar et al. (Open J. Math. Sci. 1 (2017) 44-51). As an application, this result enables us to find first Zagreb index, second Zagreb index and $\boldsymbol{F}$-index of Capra operation of cycle $\boldsymbol{C_n}$ on $\boldsymbol{n}$ vertices.

II. MAIN RESULTS

Theorem 2.1. *Consider the graph $G = Ca_k(C_n)$ as the iterative Capra of C_n. Then:*

(a) $$|V(G)| = \begin{cases} 4n & if\ \ k = 1 \\ n(2 \cdot 7^{k-1} + 3^k - 3^{k-1} + 1) & if\ \ k \geq 2. \end{cases}$$

(b) $$|\mathrm{E(G)}| = \begin{cases} 4\mathrm{n} & \text{if } \mathrm{k} = 1 \\ \mathrm{n}\left(3 \cdot 7^{\mathrm{k}-1} + 2 \cdot 3^{\mathrm{k}-1} + 2\right) & \text{if } \mathrm{k} \geq 2. \end{cases}$$

Proof. (a) It is easy to observe that

$$|V(G)| = \begin{cases} n|V(Ca_{k-1}(C_6))| - n(3^{k-1} + 1) & if\ \ k = 1 \\ n|V(Ca_{k-1}(C_6))| - n(3^{k-1} + 1) + n & if\ \ k \geq 2. \end{cases}$$

It is well known [16] that

$$|V(Ca_k(C_6))| = 2 \cdot 7^k + 3^{k+1} + 1.$$

(b) Also,

$$|E(G)| = \begin{cases} n|E(Ca_{k-1}(C_6))| - n3^{k-1} & if\ \ k = 1 \\ n|E(Ca_{k-1}(C_6))| - n \cdot 3^{k-1} + 2n & if\ \ k \geq 2. \end{cases}$$

It is well known [16] that

$$|E(Ca_k(C_6))| = 3(7^k + 3^k).$$

Theorem 2.2. *Consider the graph $G = Ca_k(C_n)$ as the iterative Capra of C_n. Then:*

$$M_{(r,s)}(G) = \begin{cases} 2n[2^{r+s} + 2^r 3^s + 2^s 3^r + 2 \cdot 3^{r+s}] & if\ \ k = 1 \\ 2^{r+s} \cdot n(3^{k-1} + 1) + n(2^r 3^s + 2^s 3^r)(3^k - 3^{k-1}) & \\ +2n \cdot 3^{r+s} \cdot (3 \cdot 7^{k-1} - 3^k + \frac{5}{2} 3^{k-1} + \frac{3}{2}) & if\ \ k \geq 2. \end{cases}$$

Proof. Let $E_{(\alpha,\beta)}(G) = \{uv; d_G(u) = \alpha \text{ and } d_G(v) = \beta\}$. Now the edge set of G can be partition into $E_1 = E_{(2,2)}(G)$, $E_2 = E_{(2,3)}(G)$ and $E_3 = E_{(3,3)}(G)$. Member of $E_{(2,2)}(G)$ lies on the exterior region of G. Therefore,

$$|E_{(2,2)}(G)| = \begin{cases} n & if\ k=1 \\ n\cdot 3^{k-1} - n(\frac{3^{k-1}-1}{3-1}) & if\ k\geq 2. \end{cases}$$

$$= \begin{cases} n & if\ k=1 \\ \frac{n}{2}(3^{k-1}+1) & if\ k\geq 2. \end{cases}$$

Similarly, member of $E_{(2,3)}(G)$ lies on the exterior region of G. Let $v_2^{(k,n)}$ be the number of vertices of degree two in G.

$$v_2^{(k,n)} = \begin{cases} 2n & if\ k=1 \\ n\cdot(3\cdot\frac{3^k+3}{6}-1) & if\ k\geq 2. \end{cases}$$

$$= \begin{cases} 2n & if\ k=1 \\ \frac{n}{2}(3^k+1) & if\ k\geq 2. \end{cases}$$

Now,

$$|E_{(2,3)}(G)| = 2\cdot v_2^{(k,n)} - 2|E_{(2,2)}(G)|$$

$$= \begin{cases} 2\cdot 2n - 2n & if\ k=1 \\ 2\cdot(\frac{n}{2}(3^k+1)) - 2\cdot\frac{n}{2}(3^{k-1}+1) & if\ k\geq 2. \end{cases}$$

$$= \begin{cases} 2n & if\ k=1 \\ n(3^k-3^{k-1}) & if\ k\geq 2. \end{cases}$$

Finally,

$$|E_{(3,3)}(G)| = |E(G)| - |E_{(2,2)}(G)| - |E_{(2,3)}(G)|$$

$$= \begin{cases} 2n & if\ k=1 \\ n(3\cdot 7^{k-1} - 3^k + \frac{5}{2}3^{k-1} + \frac{3}{2}) & if\ k\geq 2. \end{cases}$$

Therefore,

$$M_{(r,s)}(G) = \sum_{uv\in E(G)} [d_G^r(u)d_G^s(v) + d_G^s(u)d_G^r(v)]$$

$$= \sum_{uv\in E(G)\cap E_1} [2^r2^s + 2^s2^r] + \sum_{uv\in E(G)\cap E_2} [2^r3^s + 2^s3^r] + \sum_{uv\in E(G)\cap E_3} [3^r3^s + 3^s3^r]$$

$$= \sum_{uv\in E_1} 2\cdot 2^{r+s} + \sum_{uv\in E_2} [2^r3^s + 2^s3^r] + \sum_{uv\in E_3} 2\cdot 3^{r+s}$$

$$= \begin{cases} 2n[2^{r+s} + 2^r3^s + 2^s3^r + 2\cdot 3^{r+s}] & if\ k=1 \\ 2^{r+s}\cdot n(3^{k-1}+1) + n(2^r3^s+2^s3^r)(3^k-3^{k-1}) \\ +2n\cdot 3^{r+s}\cdot(3\cdot 7^{k-1} - 3^k + \frac{5}{2}3^{k-1} + \frac{3}{2}) & if\ k\geq 2. \end{cases}$$

Corollary 2.3. $[16, Theorem\ 3.1]$ *Consider the graph $G = Ca_k(C_6)$ as the iterative Capra of C_6. Then:*

$$M_{(r,s)}(G) = 3(7^k - 2(3^{k-2}) - 1)\times[2(3^{r+s})] + 4(3^k) \times[2^r3^s + 2^s3^r] + 3(3^{k-1}+1) \times[2(2^{r+s})].$$

Following corollaries are obvious from the properties of generalized Zagreb index that is

$$M_{(1,0)}(G) = M_1(G),\ M_{(2,0)}(G) = F(G)$$

and $M_{(1,1)}(G) = 2M_2(G)$.

Corollary 2.4. *Consider the graph $G = Ca_k(C_n)$ as the iterative Capra of C_n. Then:*

$$M_1(G) = \begin{cases} 26n & if\ k=1 \\ 2n(3^{k-1}+1) + 5n(3^k - 3^{k-1}) \\ +6n(3\cdot 7^{k-1} - 3^k + \frac{5}{2}3^{k-1} + \frac{3}{2}) & if\ k\geq 2. \end{cases}$$

Corollary 2.5. *Consider the graph $G = Ca_k(C_n)$ as the iterative Capra of C_n. Then:*

$$F(G) = \begin{cases} 70n & if\ k=1 \\ 4n(3^{k-1}+1) + 13n(3^k - 3^{k-1}) \\ +18n\left(3\cdot 7^{k-1} - 3^k + \frac{5}{2}3^{k-1} + \frac{3}{2}\right) & if\ k\geq 2. \end{cases}$$

Corollary 2.6. *Consider the graph $G = Ca_k(C_n)$ as the iterative Capra of C_n. Then:*

$$\mathrm{M_1(G)} = \begin{cases} 34\mathrm{n} & \text{if k} = 1 \\ 2\mathrm{n}(3^{\mathrm{k}-1}+1) + 6\mathrm{n}(3^{\mathrm{k}} - 3^{\mathrm{k}-1}) \\ +9\mathrm{n}\left(3\cdot 7^{\mathrm{k}-1} - 3^{\mathrm{k}} + \frac{5}{2}3^{\mathrm{k}-1} + \frac{3}{2}\right) & \text{if k} \geq 2. \end{cases}$$

V. CONCLUSION

In this paper, we obtain expressions for generalized Zagreb index of Capra operation of cycle $\boldsymbol{C_n}$ on $\boldsymbol{n}$ vertices, which generalizing the existing results of Sardar et al. (Open J. Math. Sci. 1 (2017) 44-51). As an application, this result enables us to find first Zagreb index, second Zagreb index and $\boldsymbol{F}$-index of Capra operation of cycle $\boldsymbol{C_n}$ on $\boldsymbol{n}$ vertices.

ACKNOWLEDGMENT

The authors are thankful to the referee for useful suggestions. The corresponding author is thankful to Karnatak University, Dharwad, Karnataka, India, for the financial support through University Research Studentship (URS), No.KU/Sch/URS/2017-18/467, dated 3rd July 2018.

REFERENCES

[1] M. Azari, A. Iranmanesh, "*Generalized Zagreb index of graphs*", Studia Universitatis Babes-Bolyai, Chemia, Vol. 56, Issue. 3, pp. 59-70, 2011.

[2] L. B. Bhajantri, "Fuzzy Logic Based Fault Detection in Distributed Sensor Networks", International Journal of Scientific Research in Computer Science and Engineering, Vol. 6, Issue. 2, pp.27-32, 2018.

[3] B. Basavanagoud, Praveen Jakkannavar, *"On the Zagreb polynomials of transformation graphs"*, International Journal of Scientific Research in Mathematical and Statistical Sciences, Vol. 5, Issue. 6, pp. 328-335, 2018.

[4] A. Dress, G. Brinkmann, *"Phantasmagorical fulleroids"*,

MATCH Commun. Math. Comput. Chem, Vol. 33, pp. 87-100, 1996.
[5] M. V. Diudea, *"Nanoporous carbon allotropes by septupling map operations"*, Journal of chemical information and modeling, Vol. 45, Issue. 4, pp. 1002-1009, 2005.
[6] M. V. Diudea, M. Stefu, P. E. John, A. Graovac, *"Generalized operations on maps"*, Croatica chemica acta, Vol. 79, Issue. 3, pp. 355-362, 2006.
[7] M. R. Farahani, M. P. Vlad, *"On the Schultz, Modifed Schultz and Hosoya polynomials and Derived Indices of Capra-designed planar Benzenoids"*, Studia UBB Chemia, Vol. 57, Issue. 4, pp. 55-63, 2012.
[8] M. R. Farahani, *"Computing ABC4 index of Capra-designed planar benzenoid series Cak (C6)"*, Advances in Materials and Corrosion, Vol. 1, Issue. 1, pp. 61-64, 2012.
[9] M. R. Farahani, *"On the Schultz polynomial and Hosaya polynomial of Circumcoronene series of benzenoid"*, Journal of Applied Mathematics & Informatics, Vol. 31, Issue. 5-6, pp. 595-608, 2013.
[10] M. R. Farahani, *"A new version of Zagreb index of circumcoronene series of benzenoid"*, Chemical Physics Research Journal, Vol. 6, Issue. 1, pp. 27-33, 2013.
[11] B. Furtula, I. Gutman, *"A forgotten topological index"*, J. Math. Chem. Vol. 53, pp. 1184-1190, 2015.
[12] M. Goldberg, *"A class of multi-symmetric polyhedra"*, Tohoku Mathematical Journal, First Series, Vol. 43, pp. 104-108, 1937.
[13] I. Gutman, N. Trinajstic′, *"Graph theory and molecular orbitals:Total π-electron energy of alternant hydrocarbons"*, Chem. Phys. Lett. Vol. 17, pp. 535-538, 1972.
[14] F. Harary, *"Graph Theory"*, Addison-Wesley, Mass 1969.
[15] H. H. Patel, Purvi Prajapati, "Study and Analysis of Decision Tree Based Classification Algorithms", International Journal of Computer Sciences and Engineering, Vol. 6, Issue. 10, pp.74-78, 2018.
[16] M. S. Sardar, S. Zafar, M. R. Farahani, *"The generalized Zagreb index of capra-designed planar benzenoid series $Ca_k(C_6)$"*, Open J. Math. Sci. Vol. 1, pp. 44-51, 2017.

AUTHORS PROFILE

Dr. Keerti G. Mirajkar is an Assistant Professor in the department of Mathematics, Karnatak University's Karnatak Arts College, Dharwad, Karnataka, India. She obtained her Ph. D degree from Karnatak University, Dharwad, Karnataka, India, under the supervision of Prof. B. Basavanagoud. She has guided one student for their Ph. D. At present 4 students are working for their Ph. D. She has attended more than 40 Conferences / Seminars and has more than 40 research publications in reputed National/International journals.

Pooja B received her M. Sc. Degree from Karnatak University, Dharwad in 2016. Currently she is pursuing Ph. D in Graph Theory under the guidance of Dr. Keerti G. Mirajkar, Assistant Professor, Department of Studies in Mathematics, Karnatak University's Karnatak Arts College, Dharwad, Karnataka, India. She has published few research papers in National/ International journals.

Mohammad R. Farahani was born in the city of Tehran, Iran on April . 5, 1988. He got his bachelor degree on Applied Mathematics from Iran University of Science and Technology (IUST) in 2010 as best department student. Now, as a researcher in applied mathematics, he work with faculties of this department, in applied mathematics, operation research, mathematics chemistry, Graph Theory.

