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Bayesian Approach in Control Charts Techniques

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Abstract— The major use of Bayesian methods in SPC is the area which aims to estimate the control chart parameters more efficiently, considering cost of sampling and chart performance. This issue is now being investigated for economical design of control charts and more recently for adaptive control charts. The traditional approach to a control chart design considers the classical control chart framework with the objective of determining the values of the chart parameters, namely the sample size, sampling interval, and the control limits to satisfy economic or statistical requirements. Under a Bayesian approach, the focus can be shifted to determining the optimal control policy based on the posterior probability that the process is out of control, minimizing the total expected cost over a finite horizon, or the long-run expected average cost. In this paper, the Bayesian control charts is developed.

Keywords— *Statistical Process Control, Bayesian approach, Prior and Posterior distribution*

I. INTRODUCTION

In ISO 9000 "Quality" is defined as the "degree to which a set of inherent characteristics fulfills requirements" and quality control is referred to all operational activities that are used to fulfill the requirements for quality.
SPC is used for process monitoring and is a proactive approach. Unlike acceptance sampling which is generally applied only at the end of the process and in which product nonconformities have already occurred, SPC is used to signal when a process is out of control, and then institute necessary corrective and preventive actions to preclude potential product nonconformities. Specifically, it is an application of statistical methods for collecting and charting of data, and monitoring the variability of a particular process of interest over time relative to the upper and lower control limits normally set at above and below three standard deviations from the process mean.

One of the exciting new paradigms in industrial SQC is the use of Bayesian approaches and methods. Bayesian inference is an approach to statistics in which many forms of uncertainty are included in the model and expressed in terms of probability. To conclude a Bayesian approach, an estimated degree of the hypothesis is numerically estimated by calculating the degree of belief in hypothesis after observing the evidence and not before observing it. In the SQC context, this implies revision of belief about a process and product/service after observation of data, and the possibility of dynamic updating of estimated parameters and control charts components as new data are gathered.

Developed computational methods and software in Bayesian statistics pave the way for extending current approaches to capture and model importance sources of uncertainty under a new outlook. Bayesian models aim to define parameters of interest in the system as variables which behave under an unknown probability distribution. A Bayesian models thus presents a structure of observable and unobservable variables, parameters and their dependencies. This structure considers more flexibility for and tries to infer deeply about parameters of monitored system process of the manufacturing.

Consideration of the capabilities of the Bayesian approach in SQC and achievements obtained in an industrial context has motivated the present research, which aims to develop Bayesian models and methods for monitoring and improving the quality.

Acceptance sampling plans and rectifying inspection might ensure the quality of in-coming/outgoing data, but they do not lead to improvement in data collection. The data must be produced, transferred and stored accurately and completely. In a more general framework, to obtain the desirable quality outcomes.

Several authors extended the MLE approach to the situation in which no prior knowledge of the change type exists. The only assumption they made was that the form of shifts belongs to the set of monotonic effects. The organization of this paper is as follows.

Bayesian approach is provided in section 2. Section 3 provides the Bayesian quality control. Whereas the Bayesian Control Chart is provided in section 4. A numerical example is provided in section 5 whereas section 6 provides the Conclusion.

II. BAYESIAN APPROACH

A Bayesian approach to statistical modeling and analysis allows estimates to be based on a synthesis of prior distributions and current sample data. In a statistical theory viewpoint, a statistical process is obtained by finding the mean performance over all the possible data. This is the contrary for a Bayesian process, for more importance is drawn on how a certain process performs at a given situation. Further, in contrast to the Frequentist procedures, Bayesian approaches formally use information available from sources other than the statistical investigation. Such information, available through expert judgment, past experience or literature, is described by a probability distribution on the set of all possible values of the un- known parameter of the statistical model at hand. Bayesian processes provide an absolute model for both statistical decisions made in doubt and for statistical conclusions. Bayesian approaches have many of the frequently used statistical procedures which are used in solving many problems faced by standard statistical methods and extend the relevancy of statistical methods. Statistical deductions for a precise interest in a Bayesian framework are known as the adjustments of the unpredictability caused by the values in the evidence. The Bayesian theorem states clearly how this adjustment should be made:

$$Posterior \propto Likelihood \times Prior$$

Where "*Prior*" is the precedent knowledge of the quantity of interest of a probability distribution; "*likelihood*" is a prototype of the observation; and "*Posterior*" is later knowledge of the quantity of data observed.

Applying the Bayesian structure and getting a later distribution of parameters of interest allows us to construct a probability-based interval around estimated parameters.

The confidence interval (CI) is the interval based on the posterior probability that contains the highest probability values at the posterior density of the parameter being evaluated.

The choice of prior distribution is very critical as it essentially indicates how we believe the parameter would behave if we had no data from which to make our decision. In other words, a prior is often the purely subjective assessment of an expert. An informative prior expresses specific, definite information about the parameter of interest. Therefore the posterior of the parameter of interest is largely determined by the prior, where there is minimal information. When there is no a priori knowledge on the parameter an uninformative distribution, or diffuse prior, can be used. In this setting, the posterior is heavily affected by the observed data. In either cases it is common to apply conjugate priors to make calculation of the posterior distribution easier by giving a closed-form expression for the posterior. (More details Gelman *et al.*, 2004).

Bayes' structure is expandable to multiple levels which allows enriching the model by capturing all kind of uncertainties for data observed from the process as well as priors. Bayesian has been increasingly recognized as a powerful approach for analyzing complex phenomena. Bayesian models are now commonly used both within and outside the statistics literature, and are widely lauded for their capacity to synthesize data from different sources, to accommodate complicated dependence structures, to handle irregular features of data such as missingness and censoring, and to incorporate scientifically based process information (More details Craigmile *et al.*, 2009).

III. BAYESIAN QUALITY CONTROL

In a Bayesian framework statistical inference about a quantity of interest is described as the modification of the uncertainty about its value in the light of evidence, and Bayes' theorem precisely specifies how this modification should be made. In the SPC context, this implies revision of belief about a process after observation of data, and the possibility of dynamic updating of control charts as new data are gathered.

There appears to be no literature on the application of Bayesian methods in the design of control charts that considers both economical and statistical issues simultaneously. Hence this gap needs to be investigated for different specifications in terms of cost and risk.

IV. BAYESIAN CONTROL CHART

In this Bayesian chart the probability of two out of control levels are plotted and used to make decisions about the state of a process. This requires more knowledge about process structure than most popular charts but acquiring this knowledge can yield real benefits (More details Amin and Venkatesan, 2017).

The later distribution as an agreement between the prior and the data. This can be well illustrated by the common relationships

$$E\left[\theta\right] = E\left[E\left(\theta \mid y\right)\right] \quad \textbf{(1)}$$

$$Var\left[\theta\right] = E\left[Var\left(\theta \mid y\right)\right] + Var\left[E\left(\theta \mid y\right)\right] \quad \textbf{(2)}$$

The first equation says that our prior mean is the average of all possible posterior means (averaged over all possible data sets of the process control).
The second one says that the later variance is, on average, smaller than the preceding variance.

The size of the difference depends on the variability of the posterior means. This can be exhibited more precisely using examples.
By using Binomial Model - Conjugate prior

$$P\left(\pi\right) \sim \beta\left(a,b\right)$$

$$P\left(y \mid \pi\right) \sim B\left(n,\pi\right)$$

Then $P\left(\pi \mid y\right) \sim \beta\left(a+y;b+n-y\right)$

Then the Prior mean is given by:

$$E\left[\pi\right] = \frac{a}{a+b} = \pi$$

And the MLE is then given by:

$$\pi = \frac{y}{n}$$

Then the posterior mean satisfies:

$$E\left[\pi \mid y\right] = \frac{a+b}{a+b+n}\pi + \frac{n}{a+b+n}\pi = \bar{\pi}$$

a weighted average of the prior mean and the sample proportion (MLE).
Hence, the later variance can be obtained by:

$$Var\left[\pi \mid y\right] = \frac{\bar{\pi}\left(1-\bar{\pi}\right)}{a+b+n+1}$$

So if the sample size n is large enough, the posterior variance will be smaller than the prior variance.
Therefore the control limits can be given as:

$$UCL = \bar{\pi} + k\sqrt{\frac{\bar{\pi}\left(1-\bar{\pi}\right)}{a+b+n+1}}$$

$$CL = \bar{\pi}$$

$$LCL = \bar{\pi} - k\sqrt{\frac{\bar{\pi}\left(1-\bar{\pi}\right)}{a+b+n+1}}$$

V. NUMERICAL EXAMPLE

Let in a manufacturing, we are collecting a sample of size n = 5 (Montgomery, 2012), y = 2, a = 4, b =3

Therefore,

$$E\left[\pi\right] = \frac{4}{7}, \quad \pi = \frac{2}{5}, \quad E\left[\pi \mid y\right] = \bar{\pi} = \frac{6}{12} = 0.5$$

$$Var\left[\pi \mid y\right] = \frac{1}{52} = 0.0192$$

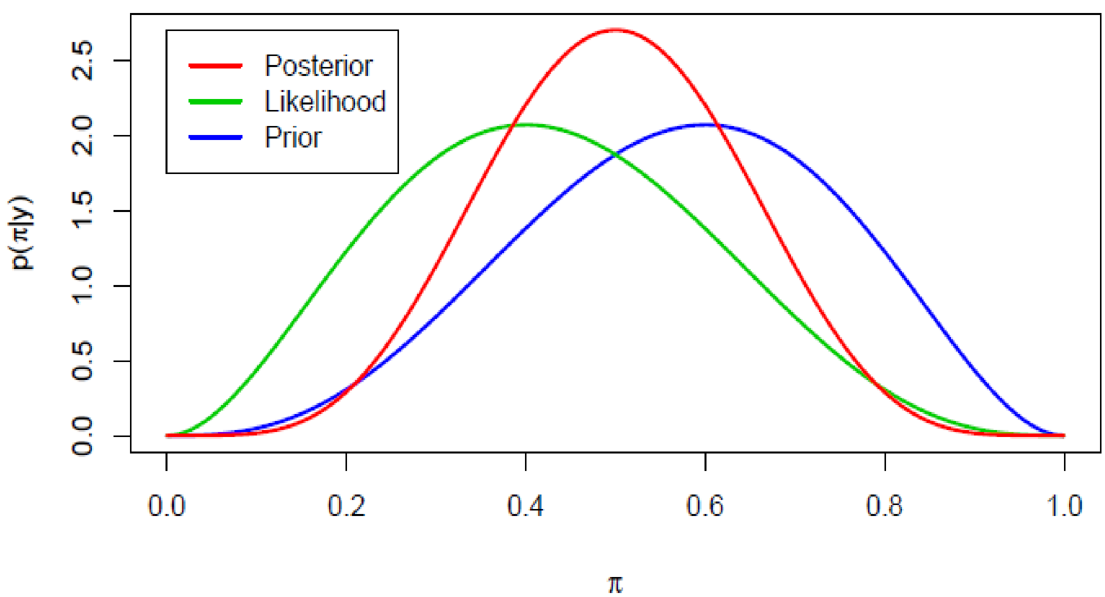
$$SD\left(\pi \mid y\right) = 0.14$$

The control limits are given then as:

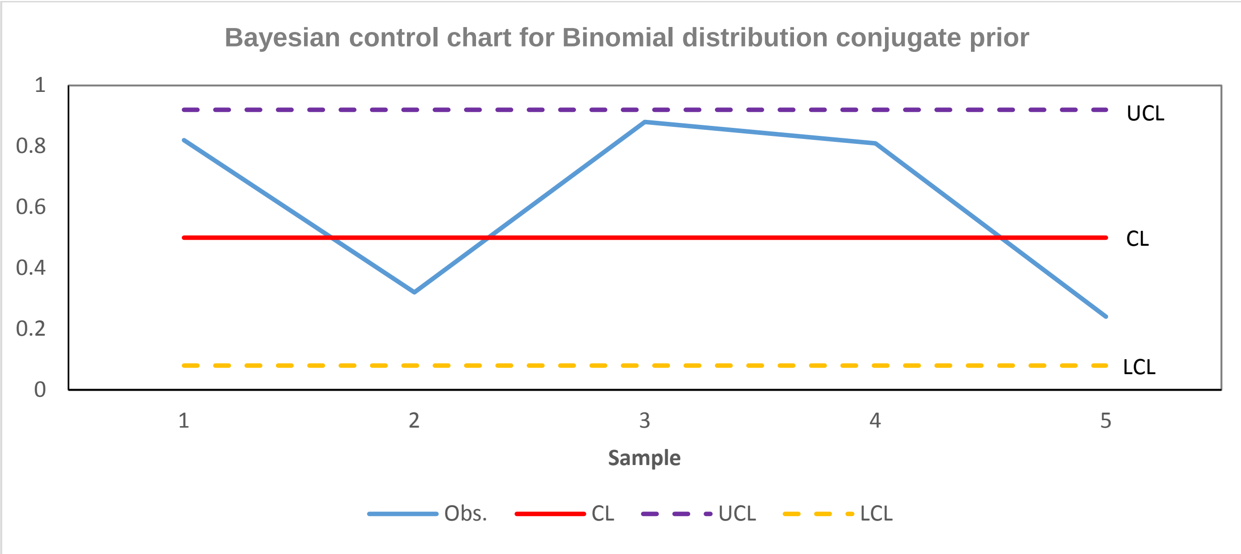
$$UCL = 0.5 + 3 \times 0.14 = 0.92$$

$$CL = 0.5$$

$$LCL = 0.5 - 3 \times 0.14 = 0.08$$



The above figure shows that the posterior mean is 0.5 with a well-shaped curve.
Therefore, the Bayesian control chart for Binomial distribution conjugate prior is given by:



VI. CONCLUSION

In the Bayesian approach preceding knowledge about unknown parameters is included into the process of deduction by assigning a prior distribution to the parameters. In order to obtain the posterior distribution, information contained in the prior is combined with the likelihood function, where the control charts using the Bayesian approach have been made. A future distribution of standardized mean is obtained by using the posterior distribution. To determine non-informative priors in multi-parameter problems is a tidy task to do. A numerical example have been given by using a Binomial distribution conjugate prior where the control limits have been found, the control chart is also given where all the process observations are in control.

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