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Semi Pre Zc-open sets in Generalized Topology

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***Abstract*—** In this paper, we introduce the concept of μ-*spZc*-open sets in generalized topology by using the concept of *Zc*-open sets in general topology. In this paper we have discussed some properties and theorems using μ-*spZc*-continuous, μ-*spZc*-irresolute in GTS through μ-*spZc* –open sets.

***Keywords*—** μ-spZc -open, μ-spZc –closed, μ-spZc-mapping, μ-spZc-continuous, μ-spZc-irressolute.

I. INTRODUCTION

Generalized topology is an important generalization of topological spaces. Many interesting results have been obtained. Using various forms of open sets, many authors have introduced and studied various types of continuity in it. Generalized topology was first introduced by Csaszar [1].

Section I contains the introduction of Generalized topology, Section II contains the preliminaries, Section III contains the main results with some theorems, Section IV contain the conclusion and future scope, followed by References in the last section.

II. PRELIMINARIES

Let X be a set. A subset μ of exp X is called a generalized topology (GT) on X and (X, μ) is called a generalized topological space [1] (GTS) if μ has the following properties:
(i) $\phi \in \mu$,
(ii)Any union of elements of μ belongs to μ.
For a GTS (X, μ), the elements of μ are called μ-open sets and the complement of μ-open sets are called μ-closed sets.
For $A \subset X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, that is the smallest μ-closed set containing A, and by $i_\mu(A)$, the union of all μ-open sets contained in A, that is the largest μ-open set contained in A.

Definition 2.1[2]: A subset A of a topological space X is *Zc*-open`if for each $x \in A \in ZO(X)$,there exists a closed set F such that $x \in F \subset A$. A subset A of a space X is *Zc*-closed if $X-A$ is *Zc*-open. The family of all Zc-open (resp. *Zc*-closed) subsets of a topological space (X,τ) is denoted by $ZcO\ (X,\tau)$ or $ZcO(X)$ (resp. $ZcC\ (X,\tau)$ or $ZcC(X)$).

Example 2.2 [2]: Consider $X=\{a,b,c,d\}$with $\tau=\{\emptyset,X,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},\{a,b,c\}\}$. Then the family of closed sets are $\{\emptyset,X,\{b,c,d\},\{a,b,d\},\{a,c,d\},\{c,d\},\{a,d\},\{b,d\}, \{d\}\}$.
The family of Z-open sets are :
$ZO(X)=\{\emptyset,X,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},\{a,d\},\{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\}, \{a,b,d\},\{b,c,d\}\}$
The family of *Zc*-open sets are : $ZcO(X)=\{ \emptyset,X, \{a,d\},\{b,d\},\{c,d\},\{a,b,d\},\{b,c,d\},\{a,c,d\}\}$.
The family of *Zc*-closed sets are:
$ZcC(X)=\{ \emptyset,X,\{b,c\},\{a,c\},\{a,b\},\{c\},\{a\},\{b\}\}$.

Definition 2.3 [3]: Let (X,τ) be a topological space. Then
(i) The union of all *Zc*-open sets contained in A is called the *Zc-interior* of A and is denoted by *Zc -Int*(A).
(ii)the intersection of all *Zc*-closed sets containing A is called the *Zc-closure* of A and is denoted by *Zc-cl*(A).
(iii) Let A be a subset of a topological space (X,τ).Then a point $p\in X$ is called a *Zc* -limit point of a set $A\subseteq X$ if every *Zc*-open set $G \subseteq X$ containing p contains a point of A other than p. The set of all *Zc*-limit points of A is called a *Zc*-derived set of A and is denoted by *Zc-d*(A).
(iv) Let $A \subseteq X$. Then the *Zc*-boundary of A is defined by $Zc\text{-}b(A) = Zc\text{-}cl(A) \cap Zc\text{-}cl((X \setminus A)$ and is denoted by *Zc-b*(A).

Definition 2.4 [4]: A function $f : X \to Y$ is
(i) *Zc*-continuous if $f^{-1}(V)$ is *Zc*-open in X for every open set V in Y.
(ii) *Zc*-irresolute if for every *Zc*-open set V in Y, $f^{-1}(V)$ is *Zc*-open in X.

Definition 2.5 [5]: A subset A of (X,τ) is called
(i) *spZc* open if $A \subseteq Zccl(Zc\ int(Zccl(A)))$ and is denoted by $spZcO(X)$

(ii) *spZc* closed if $X-A$ is *spZc* open and is denoted by $spZcC(X)$.

Definition 2.6 [5]: (i) The semi pre *Zc* interior of a subset *A* of *X* is the union of all semi pre *Zc* open sets contained in *A* and is denoted by *spZcInt* (*A*).
(ii) The semi pre *Zc closure* of a subset *A* of *X* is the intersection of all semi pre *Zc* closed sets containing *A* and is denoted by *spZcCl*(*A*).

Example 2.7 [5]: Let $X=\{a,b,c,d\}$ with $\tau=\{\emptyset,X,\{a\},\{c,d\},\{a,c,d\}\}$ then the family of *Zc*-open sets are $ZcO(X)=\{X, \emptyset, \{a,b\}\{b,c,d\}\}$ and $spZcO(X)=\{X, \emptyset, \{b\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\}, \{a,b,d\},\{b,c,d\},\{a,c,d\},\{c\},\{d\}\}$.

Definition 2.8 [5]: Let $f: X \to Y$ is called
(i) *spZc continuous* if $f^{-1}(V)$ is *spZc* open in *X* for every open set *V* in *Y*.
(ii) *spZc irresolute* if $f^{-1}(V)$ is *spZc* open in *X* for each *spZc* open set *V* in *Y*.

III. MAIN RESULTS

μ-spZc-Open sets in GTS

Definition 3.1: A subset *A* of GTS (X,μ) is *μ-Zc open* `if for each $x \in A \in ZO(X,\mu)$,there exists a closed set *F* such that $x \in F \subset A$. A subset *A* of a space (X,μ) is *μ- Zc* closed if $X-A$ is *μ-Zc* open. The family of all *μ-Zc* open (resp. *μ-Zc* closed) sub sets of a topological space (X,μ) is denoted by μ-$ZcO(X,\mu)$ (resp. μ-$ZcC(X,\mu)$).

Example 3.2: Consider $X=\{p,q,r,s\}$with $\mu=\{\emptyset,\{p\},\{r\},\{p,q\},\{p,r\},\{p,q,r\}\}$. Then the family of *μ* -closed sets are $\{X,\{q,r,s\},\{p,q,s\},\{r,s\},\{q,s\},\{s\}\}$.The family of *μ -Zc* open sets are : μ-$ZO(X)=\{\emptyset,\{p\},\{r\},\{p,q\},\{p,r\},\{r,s\},\{p,q,r\},\{p,q,s\},\{p,r,s\}\}$
The family of *μ -Zc* open sets are : μ-$ZcO(X)=\{\emptyset,\{r,s\},\{p,q,s\}\}$.
The family of *μ -Zc* closed sets are : μ-$ZcC(X)=\{X,\{p,q\},\{r\}\}$.

Definition 3.3: Let (X,μ) be a generalized topological space. Then
(i) the union of all *μ-Zc* open sets contained in *A* is called the *μ -Zc interior* of *A* and is denoted by $Zc\, i_\mu(A)$
(ii) the intersection of all *μ-Zc* closed sets containing *A* is called the *μ-Zc closure* of *A* and is denoted by $Zcc_\mu(A)$.

Definition 3.4: A subset *A* of (X,μ) is called
(i) *μ- spZc open* if $A \subseteq spZc\,c_\mu(spZc\,i_\mu(Zc\,c_\mu(A)))$ and is denoted by μ-$spZcO(X)$
(ii) *μ -spZc closed* if $X-A$ is *μ-spZc* open and is denoted by μ-$spZcC(X)$.

Definition 3.5: (i) The *μ*-semi pre *Zc* interior of a subset *A* of *X* is the union of all *μ*- semi pre *Zc* open sets contained in *A* and is denoted by $spZc\; i_\mu\;(A)$.
(ii) The *μ*-semi pre *Zc closure* of a subset *A* of *X* is the intersection of all *μ*- semi pre *Zc* closed sets containing *A* and is denoted by $spZcc_\mu(A)$.

Example 3.6: Let $X=\{a,b,c\}$ with $\mu=\{\{\emptyset,\{a\},\{b\},\{a,b\}\}\}$.The family of *μ* -Zc-open sets are :$\{\emptyset,\{b,c\},\{a,c\}\}$.Then μ-$spZcO(X)=\{\emptyset,\{c\},\{a,b\},\{b,c\},\{a,c\}\}$.

Definition 3.7: Let $f:(X,\mu) \to (Y,\sigma)$ is called
(i) *μ-spZc continuous* if $f^{-1}(V)$ is *μ-spZc* open in *X* for every *μ - spZc* open set *V* in *Y*.
(ii) *μ-spZc irresolute* if $f^{-1}(V)$ is *μ-spZc* open in *X* for each *μ- spZc*-open set *V* in *Y*.

Example 3.8: Let $X=\{p,q,r,s\}$ with $\mu=\{\emptyset,\{p\},\{r\},\{p,q\},\{p,r\}\}$; $Y=\{a,b,c,d\}$ with $\sigma=\{\emptyset,\{a\},\{d\},\{a,d\}\}$.The family of *μ-spZc* –open sets are $=\{\emptyset,\{s\},\{p,r\},\{p,s\},\{q,r\},\{q,s\},\{r,s\},\{p,q,r\},\{p,q,s\},\{q,r,s\}, \{p,r,s\}\}$. Let $f:(X,\mu)\to(Y,\sigma)$ is defined by $f(p)=f(r)=a$; $f(q)=b$; $f(s)=d$. Here $f^{-1}(a)=\{p,r\}$; $f^{-1}(d)=\{s\}$; $f^{-1}(V)=\{p,q,s\}$.Then *f* is *μ-spZc* –continuous function.

Definition 3.9: Let (X,μ) be a GTS and *A* be the subsets of it. Then (i) the *μ-semi pre Zc-boundary* of *A* (briefly denoted as μ-$spZc$-$b(A)$) is defined by μ-$spZc$-$b(A)$) $= spZc\; c_\mu\;(A) \cap spZc\; c_\mu\;(X/A)$.
(ii) the *μ -semi pre Zc-border* (briefly denoted as μ-$spZc$-$Bd(A)$) is defined by μ-$spZc$-$Bd(A)) = A / spZci_\mu(A)$.
(iii) a point $p \in X$ is called *μ-semi pre Zc-limit point* of a set $A \subseteq X$ if for every *μ*-semi pre *Zc*-open set $G \subseteq X$ containing *p* contains a point of *A* other than *p*.
(iv) The set of all *μ*-semi pre *Zc* limit points of *A* is called a *μ -semi pre Zc derived set* of *A* and is denoted by μ-$spZc$-$d(A)$).
(v) *N* is *μ-semi pre Zc-neighbourhood* (briefly denoted as *μ-spZc-nbd*) of a point $p \in X$ if there exists a *μ-spZc*-open set *W* such that $p \in W \subseteq N$. The class of all *μ-spZc-nbd* of $p \in X$ is called the *μ-spZc-neighbourhood system* of *p* and is denoted by μ-$spZc$-N_p

Theorem 3.10: Let $f:(X,\mu) \to (Y,\sigma)$ be a single valued function. Then the following are equivalent.
(i) The function *f* is *μ* -semi pre *Zc*-continuous.
(ii) For each point $p \in X$ and each open set *V* in *Y* with $f(p) \in V$, there is a *μ-spZc* open set *U* in *X* such that $p \in U$, $f(U) \subseteq V$.

(iii) The inverse of each closed set is μ -*spZc*-closed.
(iv) For each $x \in X$, the inverse of every μ-nbd of $f(x)$ is a μ - *spZc-nbd* of x.
(v) For each $x \in X$, and each μ-nbd ,μ- N_x of $f(x)$, there is a μ- *spZc-nbd* V of x such that $f(V) \subseteq N_x$.

Proof: (i) ⇒ (ii)
Let $f(p) \in V$ and $V \subseteq Y$ be an μ-open set then $p \in f^{-1}(V) \in \mu$-*spZc*O(X). Since '*f* ' is μ-*spZc* continuous ,let us assume $U = f^{-1}(V)$ then $p \in U$ and $f(U) \subseteq V$. Conversely, let V be μ-open in Y and $p \in f^{-1}(V)$ then $f(p) \in V$, there exists $U_p \in \mu$-*spZc*O(X) such that $p \in U_p$ and $f(p) \subseteq V$. Then $p \in U_p \subseteq f^{-1}(V)$ and $f^{-1}(V) = \cup U_p$ .Since every continuous function is μ- semi pre *Zc*-continuous function but not the converse, we get $f^{-1}(V) \in \mu$-*spZc*O(X),which implies that '*f* ' is μ - semi pre *Zc*-continuous.
(i) ⇒ (iii)
Assume '*f* ' is μ-semi pre *Zc*-continuous. Let B be a μ-closed subset of Y. Then $Y-B$ is $\mu -$open in Y and $f^{-1}(Y-B) = X - f^{-1}(B) \in \mu$-*spZc*O($X$) which implies that $f^{-1}(B)$ is μ -*spZc* –closed.
Conversely, Assume (iii).Let G be an μ-open set in Y, so that $Y-G$ is μ-closed set in Y. Then $f^{-1}(Y-G) = X - f^{-1}(G)$. Thus $f^{-1}(G)$ is μ-*spZc*O(X) in X which implies that f is μ-semi pre *Zc*-continuous.
(iii) ⇒ (iv)
Assume (iii) for $x \in X$, let V be the μ-nbd of $f(x)$ so that $f(x) \in W \subseteq V$, where $W = Y - F$ and F is μ-closed in Y. Consequently , $f^{-1}(W)$ is a μ-semipre *Zc* open set in X. Since $f^{-1}(W) = f^{-1}(Y - F) = X - f^{-1}(F)$. By hypothesis , $f^{-1}(F)$ is μ - *spZc*- closed and $x \in f^{-1}(W) - f^{-1}(V)$. Then by definition, $f^{-1}(V)$ is a μ - *spZc-nbd* of X.
(iv) ⇒ (v)
Let $x \in X$ and N_x be a nbd of $f(x)$. Then $V = f^{-1}(N_x)$ is a μ- *spZc-nbd* of x and $f(V) = f(f^{-1}(N_x)) \subseteq N_x$
(v) ⇒ (ii)
For $x \in X$, let W be an μ-open set containing $f(x)$. Then W be an μ –open set containing $f(x)$. Then W is a μ-nbd V of x , so that $x \in V$ and $f(V) \subseteq W$ .Hence there exists a μ -semi pre *Zc*-open set A in X such that $x \in A \subseteq V$. Consequently , $f(A) \subseteq f(V) \subseteq W$.

Remark 3.11: Every μ-semipre *Zc* irresolute map is μ-semi pre *Zc*-continuous but not the converse.

Theorem 3.12: Let $f : (X,\mu) \rightarrow (Y,\sigma)$. Then the following are equivalent.
(i) The function f is μ -semi pre *Zc*-irresolute.
(ii) For each point $x \in X$ and each μ- *spZc-nbd* , $U(x)$ such that $f(U) \subseteq V$.
(iii) For each point $x \in X$ and each $V \in \mu$-*spZc*O($f(x)$), there exists $U \in \mu$-*spZc*O(X) such that $f(U) \subseteq V$.

Proof: (i) ⇒ (ii)
Let $x \in X$. Let V is a μ- semipre *Zc* open set in Y containing $f(x)$.Since '*f* ' is μ-semipre *Zc* irresolute and let $W = f^{-1}(V)$ be a μ-semipre *Zc* open set in X containing x and hence $f(W) \subseteq f(f^{-1}(V)) \subseteq V$.
(ii) ⇒ (iii)
Assume that $V \subseteq Y$ is a μ-semipre *Zc* open set containing $f(x)$.Then by (ii) there exists a μ-semipre *Zc* open set G such that $x \in G \subseteq f^{-1}(V)$. Thus, $x \in f^{-1}(V) \subseteq \mu$-*spZc*cl($f^{-1}(V)$. Thus μ-*spZc*cl($f^{-1}(V)$) is a μ-semipre *Zc* nbd of x.
(iii) ⇒ (i)
Let V be a μ - semipre *Zc* open set in Y, then μ-*spZc*cl($f^{-1}(V)$) is a μ- *spZc-nbd* of each $x \in f^{-1}(V)$.Thus for each x there is a μ- semipre *Zc* interior point of *spZc* c_μ (f^{-1} (V)) which implies that $f^{-1}(V) \subseteq$ *spZc* i_μ (*spZc* $c_\mu((f^{-1}(V)) \subseteq$ *spZc*c_μ(*spZc*i_μ(*spZc*c_μ($f^{-1}(V)$)))).Thus, $f^{-1}(V)$ is a μ-semipre *Zc* open set in X and hence f is a μ-semipre *Zc* irresolute map.

IV. CONCLUSION AND FUTURE SCOPE

The aim of the paper is to find the main results in μ-*spZc* open sets in Generalized topology via the notion of *Zc* open sets in General topology.Some properties of μ -*spZc* continuous functions are also discussed.Future work will be done by applying these sets to connected spaces and separation spaces.

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