

Residual FRAR Control Chart for Correlated Data

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Abstract— In this paper, the FRAR residual control chart is used to detect the using the given assignable cause shift in the process mean, using FRAR chart developed by Poojalakshmi and Venkatesan (2019). Which is the extension of the residual ARMA chart of wardell et.al (1994) and illustrated with an example.

Keywords— Residual, Control Chart, FRAR, Correlated Data, ARMA

I. INTRODUCTION

Statistical Process Control (SPC) methodologies including the process Control Charts have been widely used in industry for process mean or variability monitoring and quality improvement. The majority of statistical process control chart techniques including the Shewhart chart, the Cumulative Sum (CUSUM) chart, and the Exponentially Weighted Moving Average (EWMA) chart, assume that the observations are free of autocorrelation. The more interesting approaches to SPC for autocorrelated process was proposed by Alwan and Roberts (1988). They are introduced two charts, which the referred to as the common –cause control chart and special- cause chart. The common cause chart is a plot of forecasted values that are determined by fitting the correlated process with an autoregressive moving average (ARMA) model, according to the procedure developed by Box and Jenkins (1976). The Common-Cause chart essentially accounts for the systematic variation in the process. In most situations in practice where SPC data are correlated, the systematic variation in the data is much larger, and thus more important with respect to influencing product quality, then are special cause effects. The Special- Cause chart is a traditional Shewhart chart of the residuals. One of the most important properties associated with any SPC chart is the run length. The run length is the number of observations required to obtain an observation outside of the control limits for a given shift in the mean. Run –Length distribution permits us to compute the Average Run Length (ARL) namely, the average time until an observation falls outside the control limits. The Standard Deviation of Run Length (SDRL) we normally desire the ARL to be large when no assignable cause has occurred and small when one has occurred. The SRL gives an indication of run- length dispersion. For example, with the standard shewhart control

chart, in the absence of a shift in the mean, the ARL is about 370; however, the SDRL is also about 370, so it is possible to have observations out of control much sooner or much later than expected; even when there is no shift in the mean. Use of residual chart has the advantage that it can be applied to any autocorrelated data even if the data from nonstationary process. It needs time series modelling efforts. Although the residual charts have some advantages by using them for autocorrelated processes, there are some problems due to the detection capability.

II. RELATED LITERATURE

Many authors have developed the special cause control charts, for the correlated data.. Alwan and Roberts (1988) propose the use of time series modelling to detect assignable causes by plotting estimated residuals of the time series model on a standard control chart. A practical limitation on such a time series approach is that it requires one to have some skill in time series analysis. Wardell et al (1992) compare the performance of various control charts of the process data of processes modelled as an ARMA (1,1). By setting the in-control ARL for the control charts considered, the out of control ARL for are analyzed. They observed that the EWMA chart is good at detecting small shifts , and perform well for large shifts in the mean when the AR parameters are less than 0 and the MA parameters are greater than 0. Harris and Ross (1991) use control charts to monitor the residuals of time series models. Wardell et al (1994) indicate the regardless of the sign of the autocorrelation, positive or negative, the probability of detecting a mean shift early by using a residual control chart is substantially higher than that of a traditional control chart. English (1994) recommends that fixed control limits should be used due to their effectiveness in monitoring autocorrelated data and case in interpretation. EWMA

control charts have been frequently recommended for the monitoring of correlated observation.

Harris and Ross (1991) state that the EWMA is a flexible smoothing approach that provides the theoretical framework for dealing with correlated observations. Montgomery and Mastrangelo (1991) support the same theory by stressing that the use of EWMA is best for process observations that are positively autocorrelated at low lags and a process mean that does not drift rapidly. However, Wardell et al (1994) show that the ARL of the control chart for residuals is relatively smaller when the process is negatively rather than positively autocorrelated. The reason behind this finding is that when the process is negatively autocorrelated and the shift in process mean occurs, the one-step-ahead forecast moves in the opposite direction of the shift. This causes the residual to be very large, and hence the shift is detected earlier. By comparing the ARL of the residual control chart to the ARL of the Shewhart chart and EWMA chart, they conclude that the residual chart is not necessarily the best chart to use for every type of autocorrelated process in some cases, Shewhart and EWMA charts are at least as good in terms of the resulting ARLs when the process is positively autocorrelated. Runger et al (1995) have explored the use of CUSUM control chart for monitoring residuals. Tseng and Adams (1994) generate in-control ARLs with simulation for EWMA forecast errors. They compared the Shewhart, EWMA and CUSUM control charts and conclude that the Shewhart control chart is preferred for monitoring residuals. Hu and Roan (1996) present time series- based control charts and investigated their usefulness in detecting mean shifts or sporadic spikes for AR(1), ARMA(2,1) and ARMA(1,1) models.

Zhang (1997) showed that the detection capability of a \bar{X} -residual chart was poor for small mean shifts compared to the detection capability of the \bar{X} -chart and EWMA chart. Zhang (1998) proposed the EWMAST chart, which is constructed by charting the EWMA statistic for stationary processes. EWMAST chart apply to general stationary process data.. Lu and Reynolds (1999a) showed that the ARL behaviour for the EWMA chart of the residual is better for large shifts. However, they are suggested using the EWMA of the observations because this chart is easier compared to EWMA charts of the residual. The full range autoregressive model is a family of time series model this model has the advantages of completely avoid the problem of order determination more details can be found in venkatesan et al (2017). The FRAR control charts are developed by Poojalakshmi and Venkatesan (2019) detect the mean shift of the control limits and the FRAR model is very quickly detect the shift in control charts.

An outline of this paper, Review of related literature is provided in section 2. In section 3. Model descriptions for

ARL properties of EWMA and ARMA model discussion. In section 4. The FRAR residual control chart performance of the process model. In section 5. Numerical results and compare the EWMA, ARMA, and FRAR residual control charts and Conclusion is provided in section.6

III. MODEL DESCRIPTIONS

The average run length is an important characteristics for any SPC chart. It is defined as the expectation of the time before the control chart gives a false alarm that an in-control process has run out-of-control (ARL₀). A second important characteristic for a SPC chart is the Average Delay Time (ARL₁). To design of efficient chart the ARL₀ should be large and the ARL₁ should be small, more details can be found in Yupaporn Areepong (2013).

Let $E_{\mu}(x)$ denote the expectation that the change-point occurs in a random observations generated from a distribution function $F(x, \mu, \sigma^2)$. At one point of time

the distribution mean is change from $\mu = \mu_0$ to $\mu = \mu_1$,.

A typical condition imposed on an ARL₀ is

$$E_{\theta}(\tau) = T, \theta \leq \infty \quad (\text{in-control state})$$

Where T is given value. For a given distribution function and chart, this condition determine the choices of UCL and LCL and the typical practical constraint is ARL₀ =T. The ARL₁ is

$$ARL_1 = E_1(\tau | \tau \geq 1),$$

Means that change point occurs at One could expect that a sequential control chart has a near optimal performance if its ARL₁ is close to a minimal value. There are many other criteria that could be used for designing optimal SPC charts. For any Shewhart control chart, ARL can be expressed as

$$ARL = \frac{1}{P(\text{one point plots out of control})}$$

ARL properties of EWMA

There are two main approaches for computing ARL for an EWMA sequence. The first approach is based on the fact that ARL must satisfy the Fridholm integral equation (see Crowder (1987)). The second approach is based on the flexible and relatively easy to use Markov chain approach, originally proposed by Brook and Evans in (1972). The most popular properties are used Lucas and Saccucci (1998) developed the Markov- chain approach. The properties of an EWMA control scheme can be approximated using a procedure- similar to the described by Brook and Evans (1972). They are used second approach to calculate the ARLs of EWMA and GMA control schemes. This procedure involves dividing the interval between LCL and UCL into $P=2m+1$ subintervals of width 2δ .

Where $\delta = (UCL - LCL) / (2p)$. When the number of subintervals p is sufficiently large the finite approach provides an effective method that allows ARL to be effectively evaluated. The EWMA statistics (z_t) is said to be

in transient state j at time t if $H_j - \delta < z_t < H_j + \delta$ for $j = -m, \dots, -1, 0, +1, \dots, +m$.

Where H_j represents the midpoint of the j th subinterval.

The EWMA statistics is in the absorbing state if $z_t \notin [LCL, UCL]$ an approximation for ARL is given by $ARL = d^T Q g$. Where d is the $(p, 1)$ initial probability

vector, $Q = (I - P)^{-1}$ is the fundamental (p, p) matrix, P is the (p, p) transition probability matrix and $g = \mathbf{1}$ is a $(p, 1)$ vector of 1s. The initial probability vector d contains the probability that the statistic z_t starts in a given state.

The TPM is P contains the one-step transition probabilities. The generic element P_{ij} of P represents the probability that the statistic z_t goes from state i to state j in

one step. This probability can be calculated by,

$$P_{ij} = \Phi\left(\frac{H_j + \delta - (1 - \lambda)H_i}{\lambda}\right) - \Phi\left(\frac{H_j - \delta - (1 - \lambda)H_i}{\lambda}\right)$$

ARL properties of ARMA

For EWMA charts, run-length distributions have been studied extensively. Several approximation approaches have been proposed to analyze the ARL performance. Crowder (1987) applied the numerical method to solve an integral equation for the approximation. Lucas and Saccucci (1990) used a Markov chain model to investigate the ARL values and the design strategies. We shall approximate the ARL of the ARMA chart on an iid process using the Markov chain method. The evaluation of average run length is important designing appropriate control charts for monitoring stochastic processes. For Markov -type control charts, e.g., the EWMA chart and the CUSUM chart, several approximation approaches have been developed. One example is a Markov chain approximation which replaces the continuous control statistic by a discretized version proposed in Brook and Evans (1972).

It is easy to see that the distribution of the ARMA (1,1) statistic in equation depends on both Z_{t-1} and a_{t-1} thus Z_t does not have the Markov property. The random vector

$W_t = (Z_t, a_t)'$ is a Markov chain, however, and it can be written as

$$W_t = \lambda Y_t + (1 - \lambda)W_{t-1}$$

where $\lambda = 1 - \phi$ and $Y_t = \left[\left(\frac{\theta_0 a_t - \theta a_{t-1}}{\lambda} \right), \left(\frac{a_t - \phi a_{t-1}}{\lambda} \right) \right]'$

. To evaluate the ARL for the ARMA Chart with Control limits $\pm L_z$ we need to set large control limits $\pm L_a$ for a_t so that the in-control region can be segmented within a two-dimensional rectangle, $[-L_z, L_z] \times [-L_a, L_a]$

Following Lucas and Saccucci (1990) and Runger and Prabhu (1996), the in-control region is divided into $M = t_1 \times t_2 = (2m_1 + 1) \times (2m_2 + 1)$ subregions of width $(2\delta_1) \times (2\delta_2)$ The control variable $W_i = (Z_i, a_i)'$ is said to be in transient state (j) and (i) if $SZ_j - \delta_1 < Z_i \leq SZ_j + \delta_1$ and $Sa_j - \delta_1 < a_i \leq Sa_j + \delta_1$, for, $1 \leq j \leq M$, where SZ_j and Sa_j are the partition points of the region. Then the transition probability matrix at each run becomes

$$P = \begin{pmatrix} R & (I - R)\mathbf{1} \\ \mathbf{0}^T & 1 \end{pmatrix}$$

Where the submatrix R contains the probabilities of going from one transient state to another, I is the identity matrix, and $\mathbf{1}$ is a column vector of ones. To simplify the calculations, shift magnitudes and control limits are scaled in terms of the process standard deviation. When the process mean is μ , the transition probability matrix can be calculated as, $P_{jk} = \Pr(\text{going to } S_k | \text{in } S_j)$

$$\begin{aligned} &= \Pr \left\{ SZ_k - \delta_1 < Z_t \leq SZ_k + \delta_1, Sa_k + \delta_2 < a_t \right. \\ &\quad \left. + \mu \leq Sa_k + \delta_2, |Z_{k-1} = SZ_j, a_{t-1} = Sa_j \right\} \\ &= \Pr \{ SZ_k - \delta_1 < \phi SZ_j + \theta_0 a_t - \theta Sa_j + \theta_0 \mu \leq SZ_k \\ &\quad + \delta_1, Sa_k - \delta_2 < a_t + \mu \leq Sa_k + \delta_2 \} \\ &\Pr\{\min[b_U, \max(b_L, c_L)] < a_t \leq \max[b_L, \min(b_U, c_U)]\} \end{aligned}$$

More details can be found in Jiang et al (2000). With this notation and the same arguments of Lucas and Saccucci (1990), both the in-control and out-of-control ARL's can be evaluated as $ARL = P^T \cdot (I - R)^{-1}$ with the means being 0 and μ , respectively.

Table 2. ARMA Charts Compared with the Corresponding Optimal EWMA Chart ($\phi = .85$) for Detecting Mean Shift of $1.0\sigma_a$

| Charts | EWMA $\lambda = .15$ | θ | ARMA | | | | | |
|-------------|----------------------|----------|----------------|----------------|---------------|----------------|---------------|---------------|
| | | | -.075 | -.05 | -.03 | .03 | .10 | .30 |
| L | 2.913 | | 2.832 | 2.843 | 2.867 | 2.952 | 3.023 | 3.080 |
| $\mu = .0$ | 502 (1.00) | | 502 (1.00) | 501 (1.00) | 501 (1.00) | 500 (1.00) | 500 (1.00) | 498 (1.00) |
| $\mu = .5$ | 35.5 (.06) | | 35.0 (.06) | 34.9 (.06) | 35.0 (.06) | 36.4 (.06) | 39.9 (.07) | 62.6 (.15) |
| $\mu = 1.0$ | 10.06 (.01) | | 10.12 (.01) | 10.02 (.01) | 9.99 (.01) | 10.19 (.01) | 10.8 (.01) | 15.1 (.03) |
| $\mu = 2.0$ | 3.92 (.00) | | 4.19 (.00) | 4.06 (.00) | 3.98 (.00) | 3.88 (.00) | 3.84 (.00) | 4.16 (.01) |
| $\mu = 3.0$ | 2.54 (.00) | | 2.91 (.00) | 2.76 (.00) | 2.66 (.00) | 2.44 (.00) | 2.25 (.00) | 2.02 (.00) |
| $\mu = 4.0$ | 1.96 (.00) | | 2.37 (.00) | 2.21 (.00) | 2.10 (.00) | 1.83 (.00) | 1.57 (.00) | 1.27 (.00) |

Where the control limits for Z_t are chosen ± 4.0 as because Z_t would likely go outside the control limits whenever a_t exceeds these limits. The result from this implementation agree with the simulation results quite well. Table A.1 shows the analytical results when $\phi = .85$ Compared with table-2, the discrepancies are within 3% of the simulated ARL's.

Table A.1. ARMA Charts Compared With the Corresponding Optimal EWMA Chart ($\phi = .85$) for Detecting $\mu = 1.0$: Analytical Results

| Charts | EWMA $\lambda = .15$ | θ | ARMA | | | | | |
|-------------|-------------------------|----------|-------|-------|-------|-------|-------|-------|
| | | | -.075 | -.05 | -.03 | .03 | .10 | .30 |
| L | 2.913 | | 2.832 | 2.843 | 2.868 | 2.952 | 3.023 | 3.080 |
| $\mu = .0$ | 499 | | 503 | 498 | 496 | 501 | 503 | 505 |
| $\mu = .5$ | 36.2 | | 35.8 | 35.5 | 35.9 | 36.7 | 40.6 | 62.0 |
| $\mu = 1.0$ | 10.3 | | 10.3 | 10.2 | 10.1 | 10.8 | 11.0 | 15.6 |
| $\mu = 2.0$ | 3.97 | | 4.25 | 4.11 | 4.01 | 3.92 | 3.85 | 4.16 |
| $\mu = 3.0$ | 2.56 | | 2.94 | 2.78 | 2.69 | 2.47 | 2.25 | 2.00 |
| $\mu = 4.0$ | 2.01 | | 2.31 | 2.22 | 2.11 | 1.86 | 1.58 | 1.25 |

The ARMA chart, the control limits are defined as $|Z_t| > L_z a_z$ and $|a_t| \Rightarrow L_z \sigma_z$. The sequence is that their grides are inside a circle, whereas our grides are defined in a rectangle. More memory space is needed for implementing our algorithm.

IV. THE FRAR MODEL

Full Range Autoregressive model is introduced by Venkatesan et.al (2017) is a new family of time series models, which avoid the problem of order determination and explained in the following. FRAR model is defining a family of models by a discrete-time stochastic process $\{X_t\}, t = 0, \pm 1, \pm 2, \pm 3, \dots$, called the Full Range Autoregressive (FRAR) model, by the difference equation.

$$X_t = \sum_{r=1}^{\infty} \frac{k \sin(r\theta) \cos(r\phi)}{\alpha^r} X_{t-r} + e_t$$

$$= \sum_{r=1}^{\infty} a_r X_{t-r} + e_t$$

Where

$$a_r = \frac{k \sin \sin(r\theta) \cos(r\phi)}{\alpha^r}, r = 1, 2, 3, \dots$$

k, α, θ and ϕ are real parameters. The region of identifiability of the models is given by,

$$S = \left\{ a, k, \theta, \phi \mid k \leq R, a \geq 1, \theta \in [0, \pi) \text{ and } \phi \in \left[0, \frac{\pi}{2}\right) \right\}$$

and more details can be found in venkatesan et.al (2017).

Let X_t be autocorrelated observations, then the residual forms FRAR model of x_t defined as $\hat{e}_t = x_t - x_t$. Where

x_t is the prediction of x_t at time t. Residual control chart are construct based on e_t depending on the traditional charts used. Suppose that we are monitoring an iid process e_1, e_2, \dots , with normality, an in-control mean of 0 and variance is σ_e^2 For a Shewhart residual chart the 3σ upper and lower control limits are defined by,

$$UCL = \bar{e} + L\sigma_e, \quad \text{and} \quad LCL = \bar{e} - L\sigma_e$$

Where \bar{e} is the center line of the chart with standard deviation σ_e following Lucas and Saccussi (1990) table.

V. NUMERICAL EXAMPLES

The numerical data already used by Jiang et al (2000), is proposed to utilize in this paper and also used. In this

observation, Poojalakshmi and Venkatesan(2019) have identify that the shift of the FRAR control chart and comparison of EWMA, ARMA and FRAR control chart with detecting mean shift and the same parameters. Then in this paper, we are find out the residual performance of FRAR control chart. Table-1 shows the performance of residual charts of EWMA, ARMA and FRAR. Figure-2 the residual of FRAR control chart is very sensitive to the falls out-side the control limits of shift occurred. When the data fluctuates up and down, indicating that the process is not steady. Then the FRAR residual control chart more than 4 points falls down the same side this type of chart is SCC control chart. The control limits are applied in FRAR model. Here the target mean is 0 and standard deviation is 0.9362. Following the control limits are ± 3.00 One point plot out-side the control limits are ARL in- control average run length and more than point plot out-side the control limits are ARL is

out-of- control average run length. Several points fall outside of both the upper and lower control limits, indicating that the process is seemingly out of control. The control chart is a graph used to study how a process changes over time. Data are plotted in time order. A control chart always has a central line for the average, an upper line for the upper control limit and lower line for the lower control limit. These lines are determined from historical data. By comparing current data to these lines, you can draw conclusions about whether the process variation is consistent (in control) or is unpredictable (out of control, affected by special causes of variation).

| Observation Number | Observations | EWMA | ARMA | FRAR |
|--------------------|--------------|----------|--------|--------|
| | | Residual | | |
| 1 | 1 | 0.850 | 0.880 | 1.000 |
| 2 | -0.5 | -0.553 | -0.572 | -0.617 |
| 3 | 0 | -0.045 | -0.046 | -0.079 |
| 4 | -0.8 | -0.718 | -0.743 | -0.853 |
| 5 | -0.8 | -0.610 | -0.632 | -0.741 |
| 6 | -1.2 | -0.859 | -0.889 | -1.018 |
| 7 | 1.5 | 1.565 | 1.620 | 1.834 |
| 8 | -0.6 | -0.455 | -0.471 | -0.446 |
| 9 | 1 | 0.974 | 1.008 | 1.136 |
| 10 | -0.9 | -0.787 | -0.815 | -0.910 |
| 11 | 0.95 | 0.903 | 0.935 | 0.996 |
| 12 | 0.25 | 0.173 | 0.179 | 0.197 |
| 13 | 2.35 | 2.481 | 2.487 | 3.015 |
| 14 | 0.45 | 0.027 | 0.028 | 0.066 |
| 15 | 0.85 | 0.363 | 0.376 | 0.383 |
| 16 | 1.75 | 1.074 | 1.111 | 1.234 |
| 17 | 1.15 | 0.402 | 0.417 | 0.499 |
| 18 | 1.65 | 0.767 | 0.794 | 0.928 |
| 19 | 0.55 | -0.283 | -0.293 | -0.267 |

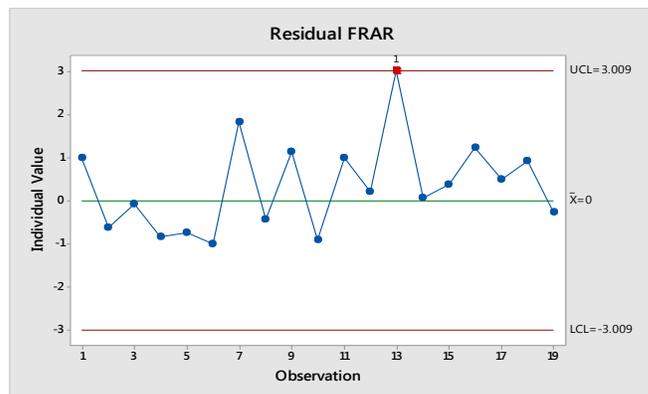


Figure-1: FRAR Residual control chart

The special-cause control chart for the same data indicating that the residuals are within the control limits. It is quite possible then that the points outside of the control limits on the original individuals chart were outside the limits because of systematic or common cause and not due to the occurrence of special causes. Thus the more traditional means of monitoring quality for this process may have yielded misleading results and suggested inappropriate corrective action, resulting in higher than necessary process variability.

Then, 19 observations are generated from each time series model. Since the observation 13, is going to out-side the control limits and in-control average run length occurred. Thus results from the fact that when the shift first occurs there is a large discrepancy between the observation and its forecasted value, giving a large residual.

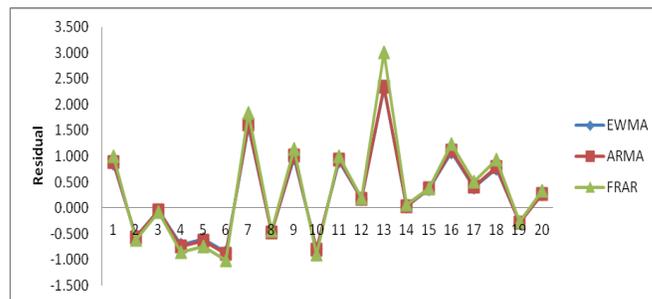


Figure-2 The comparison of EWMA, ARMA, FRAR Residual Control Charts

VI. CONCLUSION

In this paper we attempt the residual control chart of FRAR model, it is very sensitive to detecting shift in the mean of the process occurred the out-of-control points. Initially use traditional charts so that every effort is made to detect assignable causes for out-of-control points such as those in figure-1. There is a very real possibility that the autocorrelations are small and the apparent drifts in the process average quite large, thus causing the systematic behaviour. Future research of our article is on processes in which efforts to remove the systematic variation have made and autocorrelation remains.

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