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Some Results on Contra Harmonic Mean Labeling of Graphs

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Abstract- In this paper we prove that some standard graphs like Comb, Ladder, Crown, Triangular, snake, Quadrilateral snake, Alternate Triangular snake, Alternate Triangular snake and Alternate Quadrilateral snake are Contra Harmonic mean graphs.

Keywords- Graph, Contra Harmonic mean graph, Comb, Ladder, Crown, Triangular snake, Alternate Triangular, snake, Quadrilateral snake, Alternate Quadrilateral snake.

1. Introduction

The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. In the intervening years various labeling of graphs such as grace labeling, harmonious labeling, magic labeling, prime labeling, cordial labeling etc., have been studied in over 2000 papers. For standard terminology and notation we refer to Bondy and Murthy[1]. For all detailed survey of graph labeling we refer to Gallian. J.A (2016) [2]. Also we refer to Harary [3] . Some basic concepts of mean labelings are taken from [4- 6].We have proved Contra Harmonic mean labeling of some graphs in [7].In this paper we prove that some standard graphs admits Contra Harmonic mean labeling. The following definition are useful for our present study.

Definition: 1.1 A path P_n is obtained by joining u_i to the consecutive vertices u_{i+1}, for $1\le i\le n$.

Definition: 1.2 A closed path is called a cycle (C_n).

Definition: 1.3 The graph obtained by joining a single pendant edge to each vertex of a path is called a Comb.

Definition: 1.4 The product $P_m x P_n$ is called a planar grid and $C_m x P_n$ is called a prism. The product $P_2 x P_n$ is called a ladder (L_n).

Definition: 1.5 Any cycle with a pendant edge attached at each vertex is called a crown.

Definition: 1.6 A Triangular snake T_n is obtained from a path $u_1 \ldots . u_n$ by joining u_i and to a vertex v_i, for $1\le i\le n-1$.

Definition: 1.7 A Quadrilateral snake Q_n is obtained from a path $u_1 \ldots . u_n$ by joining u_i and u_{i+1} to new vertices v_i, w_i, $1\le i\le n-1$.
Definition: 1.8 An Alternate Triangular snake $A(T_n)$ is obtained from a path $u_1 \ldots .. u_n$ by join u_{i+1} (Alternatively) to new vertex v_i .

2. Main Results

Definition 2.1

A graph G=(V,E) with p vertices and q edges is called a Contra Harmonic mean graph if it is possible to label the vertices $x\in V$ with distinct elements f(x) from 0,1,…,q in such a way that when each edge e=uv is labeled with, f (e=uv) = $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ with distinct edge labels. Here f is called a Contra Harmonic mean labeling of G.

Remark: 2.2

- If G is a Contra Harmonic mean graph then the vertex x labels from 0,1,…,q and the edges labels from 1,...,q.
- If G is a Contra Harmonic mean graph then 1 must be a label of one of the vertices of G, since an edge should get label 1.

Theorem 2.3 Combs are Contra Harmonic mean graphs.

Proof

Let G be a comb obtained by joining the vertices u_i of a path to v_i ,1≤*i*≤n.

Define a function f: V(G) →{0,1,…,q} by,

$f(u_i) = 2i-2$, $1 \le i \le n$

$f(v_i) = 2i-1$, $1 \le i \le n$

Edges are labeled as,

$f(u_iu_{i+1}) = 2i$, $1 \le i \le n-1$

$f(u_iv_i) = 2i-1$, $1 \le i \le n$

Then f is a Contra Harmonic mean labeling of G.

The Contra Harmonic mean labeling of $P_7 \odot K_1$ is

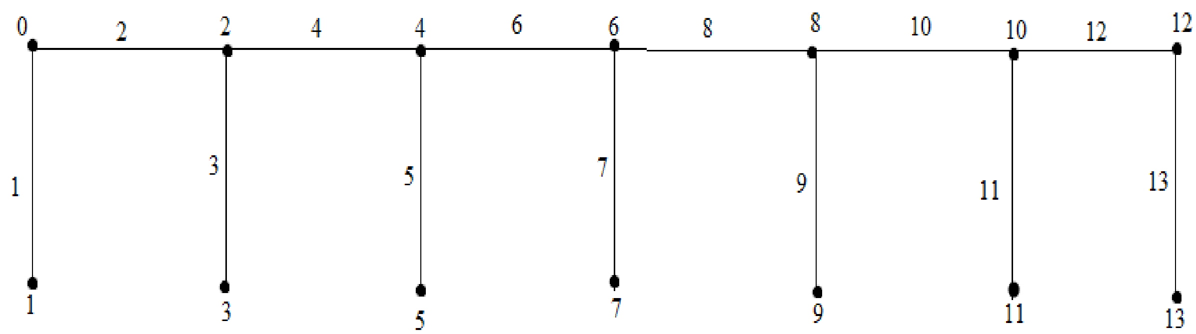

Figure: 1

Theorem 2.4

A graph obtained by attaching pendant edges to both sides of each vertex of a path is a Contra Harmonic mean graph.

Proof

Let G be a graph obtained by attaching pendant edges u_iv_i, u_iw_i to both sides of each vertex of a path u_i, 1≤*i*≤n.

Define a function f: V(G) →{0,1,…,q} by,

$f(u_i) = 3i-2$, $1 \le i \le n$, $f(v_i) = 3i-3$, $1 \le i \le n$, $f(w_i) = 3i-1$, $1 \le i \le n$

Edges are labeled as,

$f(u_iu_{i+1}) = 3i$, $1 \le i \le n-1$, $f(u_iv_i) = 3i-2$, $1 \le i \le n$, $f(u_iw_i) = 3i-1$, $1 \le i \le n$

Then f is a Contra Harmonic mean labeling of G.

The Contra Harmonic mean labeling is shown below

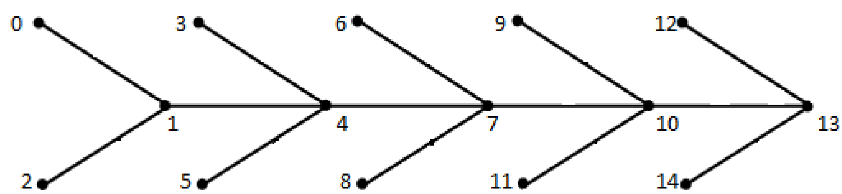

Figure: 2

Theorem 2.5

Any Ladder is a Contra Harmonic mean graph.

Proof

Let L_n denote the ladder graph.

Define a function $f:V(L_n) \to \{0,1,\ldots,q\}$ by

$f(u_1) = 0$, $f(u_2) = 4$, $f(u_3) = 5$

$f(u_4) = 9$ and $f(u_i) = f(u_{i-2}) + 6$, $5 \le i \le n$.

$f(v_1) = 1$, $f(v_2) = 2$ and $f(v_i) = 3i-2$, $3 \le i \le n$.

Edges are labeled as

$f(u_1u_2) = 4$, $f(u_iu_{i+1}) = 3i-1$, $2 \le i \le n-1$

$f(u_1v_1) = 1$, $f(v_1v_2)=3$, $f(u_iv_i) = 3i-2$, $3 \le i \le n$

$f(v_1v_2) = 2$, $f(v_iv_{i+1}) = 3i$, $2 \le i \le n-1$

Then f is a Contra Harmonic mean labeling.

The Contra Harmonic mean labeling of P_2xP_7 is

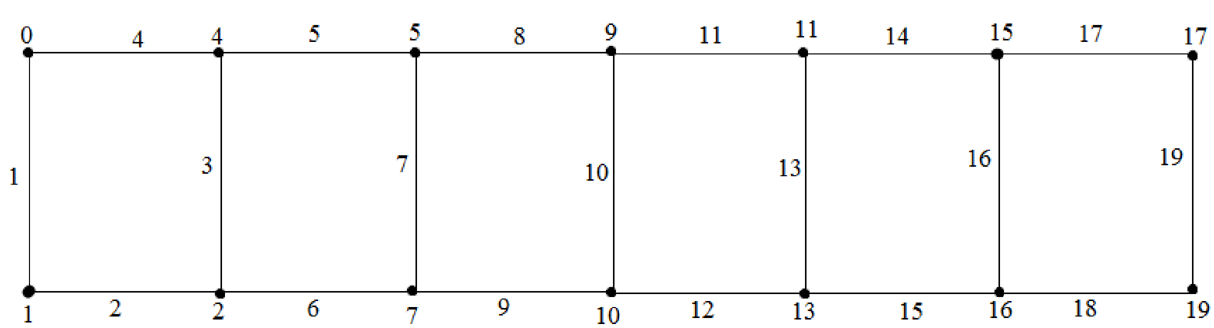

Figure: 3

Theorem : 2.6

A Crown $C_n \odot K_1$ is a Contra Harmonic mean graph for all n≥3.

Proof:-

Let C_n be the cycle $u_1 \ldots u_n$ and v_i be the pendant vertices adjacent to u_i,for $1 \le i \le n$.

Define a function f : $V(C_n \odot K_1) \rightarrow \{0,1,\ldots,q\}$ by,

$f(u_i) = 2i-2,\ 1 \le i \le n-1$

$f(u_n) = 2n$

$f(v_i) = 2i-1,\ 1 \le i \le n$

Edges are labeled as

$f(u_i u_{i+1}) = 2i,\ 1 \le i \le n$

$f(u_i v_i) = 2i-1,\ 1 \le i \le n$

Then f is a Contra Harmonic mean labeling.

The Contra Harmonic mean labeling of is $C_8 \odot K_1$ is

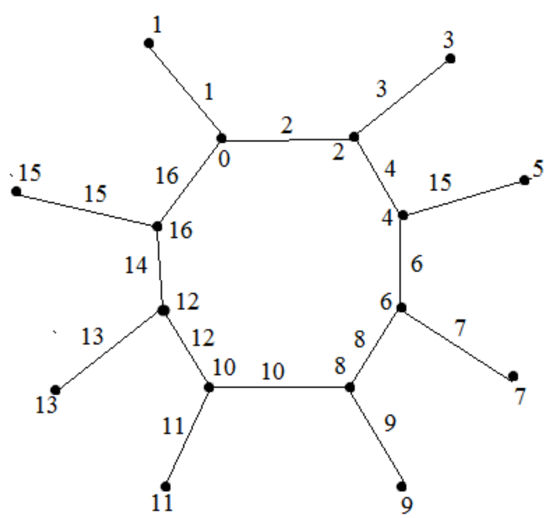

Figure: 4

Theorem 2.7

A Triangular snake T_n is a Contra Harmonic mean graph.

Proof: Let T_n be a Triangular snake obtained from a path $v_1 \ldots v_n$ by joining v_i to v_{i+1} to a new vertex w_i, for $1 \le i \le n-1$.

Define f: $V(T_n) \rightarrow \{0,1,...,q\}$ by

$f(v_1) = 1,\ f(v_i) = 3i-3,\ 2 \le i \le n$

$f(w_1) = 0,\ f(w_i) = 3i-1,\ 2 \le i \le n-1$

The edges are labeled as

$f(v_iv_{i+1}) = 3i-1,\ 1\le i\le n-1$

$f(v_iw_i) = 3i-2,\ 1\le i\le n-1$

$f(w_iv_{i+1}) = 3i,\ 1\le i\le n-1$

Then f is a Contra Harmonic mean labeling.

The Contra Harmonic mean labeling pattern is shown below

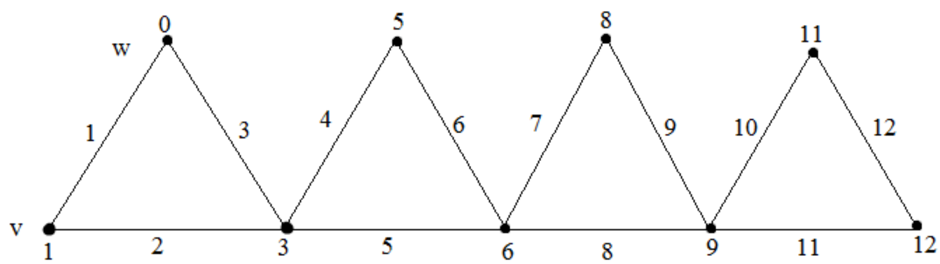

Figure: 5

Theorem 2.8:

Any Quadrilateral snake Q_n is a Contra Harmonic mean graph.

Proof:

Let Q_n be the Quadrilateral snake with 3n-2 vertices and 4n-4 edges.

Define a function $f: V(Q_n) \to \{0,1,\ldots,q\}$by

$f(u_i) = 4i-4,\ 1\le i\le n$

$f(v_i) = 4i-3,\ 1\le i\le n-1$

$f(w_i) = 4i-2,\ 1\le i\le n-1$

Edges are labeled as

$f(u_1u_2) = 4,\ f(u_iu_{i+1}) = 4i-1,\ 2\le i\le n-1$

$f(u_iv_i) =4i-3,\ 1\le i\le n-1$

$f(w_1u_2) = 3,\ f(w_iu_{i+1}) = 4i,\ 2\le i\le n-1$

Then f is a Contra Harmonic mean labeling.

The Contra Harmonic mean labeling pattern is shown below

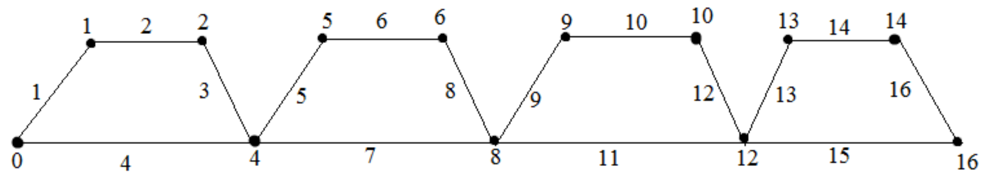

Figure: 6

Theorem 2.9

Alternate Triangular snakes are Contra Harmonic mean graph.

Proof:

Let $A(T_n)$ be the Alternate Triangular snake.

The following two cases are to be considered.

Case (i) Triangle starts from u_1

Define a function $f: V(A(T_n)) \rightarrow \{0,1,\ldots,q\}$ by

$f(u_i) = 2i-1,\ 1 \le i \le n$

$f(v_1) = 0,\ f(v_i) = 2i,\ i = 3,5,\ldots,n-1$

Edges are labeled as

$f(u_iu_{i+1}) = 2i, 1 \le i \le n-1$

$f(u_iv_i) = 2i-1,\ i = 1,3,5,\ldots,n-1$

$f(v_iu_{i+1}) = 2i+1,\ i = 1,3,5,\ldots,n-1$

Then f is a Contra Harmonic mean labeling.

The labeling pattern is a shown below

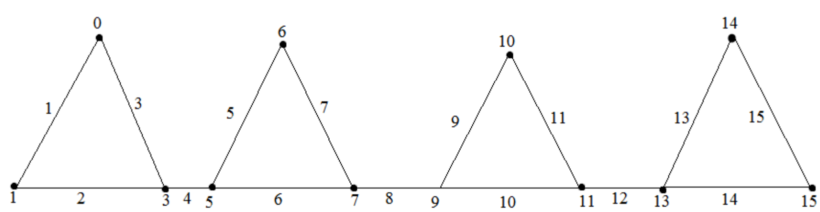

Figure: 7

Case (ii) Triangle starts from u_2

Define a function

$f: V(A(T_n)) \rightarrow \{0,1,\ldots,q\}$ by

$f(u_1) = 0,\ f(u_2) = 1,\ f(u_i) = 2i-2,\ 3 \le i \le n\ ,\ f(v_i) = 2i+1,\ i = 1,3,5,\ldots,n-2$

Edges are labeled as

$f(u_iu_{i+1}) = 2i-1, 1\leq i\leq n-1$

$f(u_{i+1}\ v_i) = 2i$, i = 1,3,5,…,n-2, $f(v_i\ u_{i+2}) = 2i+2$, i = 1,3,5,…,n-2.Then f is a Contra Harmonic mean graph.

The labeling pattern is a shown below

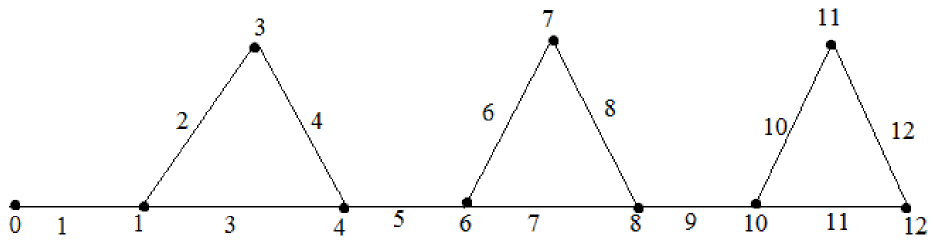

Figure: 8

From case (i) and case (ii) we conclude that Alternate triangular snakes are Contra Harmonic mean labeling.

Theorem 2: 10

Alternate Quadrilateral snake is a Contra Harmonic mean graph.

Proof:

Let $A(Q_n)$ be the Alternate Quadrilateral snake. The following two cases are to be considered.

Case (i) Quadrilateral starts from u_1

Define a function f: $V(A(Q_n))\rightarrow\{0,1,...,q\}$ by $f(u_1) = 0$, $f(u_2)= 4$,

$f(u_i) = f(u_{i-2})+5, 3\leq i\leq n$

$f(v_i) = f(u_i)+1$, i=1,3,5,…,n-1

$f(w_i) = f(v_i) +1$, i=1,3,5,…,n-1

Edges are labeled as

$f(u_1u_2) = 4$, $f(u_2u_3) = 5$, $f(u_3u_4) = 8$

$f(u_iu_{i+1}) = f(u_{i-2}\ u_{i-1}) + 5, \ 4\leq i\leq n-1$

$f(v_1w_1) = 2$, $f(v_1w_1) = f(v_{i-2}\ w_{i-2}) + 5$, i= 3,5,…,n-1

$f(u_1v_1) =1$, $f(u_iv_i) = f(u_{i-2}\ v_{i-2}) + 5$, i= 3,5,…,n-1

$f(w_1u_2) = 3$, $f(w_iu_{i+1}) = 3i$, i = 3,5, …,n-3, $f(w_{n-1}u_n) = 2n+2$

The labeling pattern is

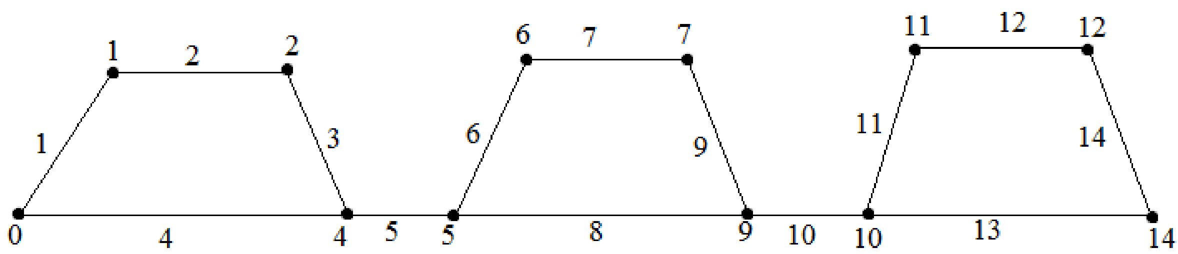

Figure: 9

Case (ii) **Quadrilateral starts from u_2**

Define a function f: $V(A(Q_n) \rightarrow \{0,1,\ldots,q\}$ by $f(u_1) = 0$, $f(u_2) = 1$,

$f(u_i) = f(u_{i-2})+5$; $3 \leq i \leq n$

$f(v_2) = 2$, $f(v_i) = f(v_{i-2}) +5$, i= 3,5,…,n-3

$f(w_i) = f(v_i)+1$, i= 3,5,…,n-3

Edges are labeled as,

$f(u_1u_2) =1$, $f(u_2u_3) = 5$, $f(u_3u_4) = 6$

$f(u_4u_5)=9$, $f(u_iu_{i+1}) = f(u_{i-2}\, u_{i-1})+5$, $5 \leq i \leq n-1$

$f(v_1w_1) =3$, $f(v_iw_i) = f(v_{i-2}\, w_{i-2}) + 5$, i= 3,5,…,n-3

$f(u_2v_1) = 2$, $f(u_{i+1}v_i) = f(u_{i-1}\, v_{i-2}) + 5$, i= 3,5,…,n-3

$f(w_1u_3) = 4$, $f(w_3u_5) = 10$, $f(w_iu_{i+2}) = f(w_{i-2}\, u_i) + 5$, i= 5,7,…,n-3

Then f is a Contra Harmonic mean labeling

Contra Harmonic mean labeling pattern is given below

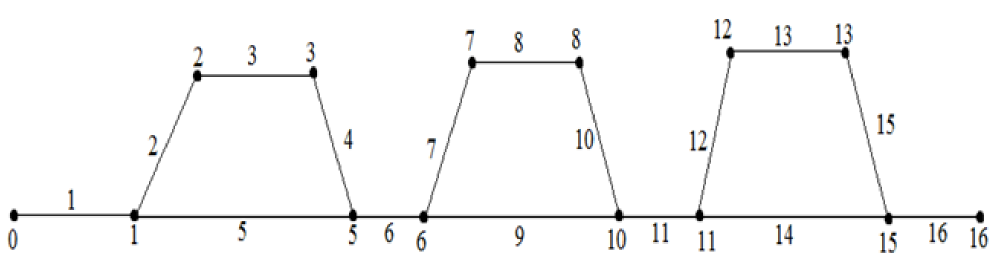

Figure:10

From case (i) and case (ii), we conclude that the Alternate Quadrilateral snakes are Contra Harmonic mean graph.

3. Conclusions

Graphs like Double Triangular snake, Double Quadrilateral snake, Alternate Double Triangular snake, Alternate Double Quadrilateral snake, Triangular ladder etc., do not admit Contra Harmonic mean labeling. Various concepts of graph labeling are to be explored in future.

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