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Double-Framed Created Retrenchment N-Fuzzy Soft Ideal Lattice Structures

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Abstract: In this article, we introduce a created double-framed retrenchment structure which is the generalization of negative-valued function. Using this double-framed negative-valued structure, we have applied in a B-algebra and hence introduced the notion of a double-framed negative- valued B-algebra, double-framed N-fuzzy soft structures. Also the characterization of double-framed N- fuzzy soft ideal lattice is presented.

Keywords: B-algebra, fuzzy set, soft set, N-fuzzy soft ideal lattice, double-framed N- structure, retrenchment, created N-fuzzy soft ideal.

I. INTRODUCTION

The algebraic structure of soft set theory dealing with uncertainties has also been studied in more detail. Aktas and Cagman [1] in traduced a definition of soft groups and derived their basic properties. Park et. al [15] worked on the notion of soft WS-algebras, soft sub algebras and soft deductive systems. Jun [4] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. Park [5] presented the notion of soft ideals, idealistic soft ideal and idealistic soft BCK/BCI-algebras. Jun et. al [6] applied soft set theory to commutative ideals in BCK-algebras. Jun et. al [10] initiated to introduce double framed soft sets and presented its applications in BCK/BCI algebras. Jun and Park [**7**] make application of soft sets in Hilbert algebras. Jun et .al [8, 9] presented Pseudo d-algebras and applied soft set theory to ideals in d-algebras. Roy and Maji [11] gave some application soft fuzzy sets. Aktas and Cagman [1] compared soft sets with the related concepts of fuzzy sets and rough sets. Molodtsov [14] introduced the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. Molodtsov applied this theory to several directions [14] and then formulated the notions of soft number, soft derivative, soft integral, etc. in [11,12,13]. The soft set theory has been applied to many different fields with great success. [2] introduced the notion of ideals in subtraction algebras and discussed characterization of ideals. In [3], Y. B. Jun et. al established the ideal generated by a set, and discussed related results. To solve complicated problems in economics, engineering and environment, we can't successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties can't be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics and theory of rough sets. The fundamental concept of fuzzy set is a mapping from non-empty set S to unit closed interval was published by Zadeh in his paper [16] in 1965, was applied to generalize some of the basic concepts of algebra. The fuzzy algebraic structures play a prominent role in mathematics with wide applications in many other branches such as theoretical physics, computer science, control engineering, information sciences, coding theory, topological spaces, logic, set theory, group theory, groupoids, real analysis, measure theory, etc. The fuzzification of ideal theory in subtraction algebras were discussed detail in [10].In this paper, we introduce a created double-framed retrenchment structure which is the generalization of negative-valued function. Using this double-framed negative-valued structure, we have applied in a B-algebra and hence introduced the notion of a double-framed negative-valued B-algebra, double-framed N-fuzzy soft structures. Also the characterization of double-framed N-fuzzy soft ideal lattice is presented.

II. PRELIMINARIES

In this section, we cite the fundamental definitions that will be used in the sequel.

Definition 2.1. A non empty set X together with a binary operations "+", "-" is said to be a B-algebra if it satisfies the following

(B_1) $a - 0 = 0$

(B_2) $(0 - a) + a = 0$

(B_3) $(a - b) - c = a - (c + b)$, for all a,b,c ∈ X.

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Example 2.2. Let A = {0, 1, 2, 3} be a set. Then (A, +, −) is a B-algebra.

-	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

The ordered set (X : ≤) is a semi-boolean-algebra in the sense of [J.C.ABBOTT], that is, it is a meet semi-lattice with zero in which every interval [0, a] is a Boolean algebra with respect to the induced order. Here $a \wedge b = a - (a + b)$, the complement of an element b ∈ [0, a] is a + b and if b, c ∈ [0, a], then

$$b \vee c = (b^j \wedge c^j) = a - ((a - b) \wedge (a - c))$$
$$= a - ((a - b) - ((a - b) + (a - c))).$$

Therefore (A, +, -) is a B-algebra.

Definition 2.3. [Y.B. Jun et.al] A non empty subset A of a B-algebra X is called an ideal of X; denoted by A of X; if it satisfies

(I_1) $a - x \in A$ for all $a \in A$ and $x \in X$.

(I_2) for all a, b ∈ A, whenever $a \vee b$ exists in X, then $a \vee b \in A$.

Proposition 2.4. [Y.B.Jun et.al] A non empty subset A of a B-algebra X is an ideal of X if and only if it satisfies

(i) $0 \in A$

(ii) $x - y \in A \Rightarrow x \in A$ for all x, y ∈ A.

Proposition 2.5. [Y.B.Jun et.al] Let X be a B-algebra and let x, y ∈ X. If p ∈ X is an upper bound for x and y, then the element $x \vee y = p - ((p - y) - x)$ is a least upper bound for x and y.

Definition 2.6. [Y.Ceven et.al] Let X be a B-algebra and Y be a non-empty subset of X. Then Y is called a sub algebra of X if x + y ∈ Y, whenever x, y ∈ Y.

Definition 2.7. [Zadeh] Let X be any set. A mapping A : X → [0, 1] is fuzzy set in X.

Example 2.8. Let X = {1, 2, 3}. Then the fuzzy set is

X	1	2	3
P (X)	0.7	0.8	0.9

Definition 2.9. [Molodtsov] A soft set is a parameterized family of collection of all subsets.

Denote by F (X, [−1, 1]), the collection of functions from a set X to [−1, 0] (briefly, N-fuzzy function on X).

By an N-structure we mean an ordered pair (X, δ) of X and an N-function δ on X. For any N-function δ on X and t ∈ [-1,0], the set $C(\delta^*, t)$ is $\delta^-(x) \geq m$ and $\delta^-(x) \leq n$ is called closed (δ(m,n))-cut of (X,δ).

Definition 2.10. [Y.B.Jun et.al] By an ideal (resp.,sub algebra) of X based on N-fuzzy function (briefly N-fuzzy ideal) (resp.,N-fuzzy sub algebra), we mean on N-fuzzy structure (X, δ) in which every non empty closed (δ, t) - cut of (X, δ) is an ideal (resp.,sub algebra) of X, for all t ∈ [−1, 0].

Example 2.11. Let A = {0, 1, 2} be a B-algebra and define the set (A, -) is given in the following table

Theorem 2.12. Let (X, δ) be an N-fuzzy structure of X. Then (X, δ) is called a N-fuzzy sub algebra of X if and only if it satisfies

(NFS) $\delta(x - y) \leq \max\{\delta(x), \delta(y)\}$, whenever x, y ∈ X.

Proof. The proof is straightforward from the definition (2.10).

Theorem 2.13. An N-fuzzy structure (X, δ) is an N-fuzzy soft ideal of X if and only if it satisfies the following assertions:

($NFSI_1$): $\delta(x-y) \leq \delta(x)$

($NFSI_2$): $\delta(x \vee y) \leq \max\{\delta(x), \delta(y)\}$, for all x, y ∈ X and there exists of $x \vee y$.

Theorem 2.14. Let (X,δ) be an N-fuzzy structure of X. Then the following conditions hold:

(i) $(\forall x, a, b \in X), (a - x) \leq b \Rightarrow \delta(x) \leq \max\{\delta(a), \delta(b)\}$, then (X, δ) is an N-fuzzy sub algebra.

(ii)Any N-fuzzy ideal (X,δ) of X is an N-fuzzy sub algebra of X.

Proof. (i)Let x, y ∈ X be such that $x - (x - y) \leq y$. Thus $\delta(x-y) \leq \max\{\delta(x),\delta(y)\}$, Hence δ is an N-fuzzy sub algebra of X.

(ii) Assume that (X, δ) is an N-fuzzy ideal of X. Then, there exist x, y ∈ X such that

$x - y \leq x \leq p - ((p - y) - x) = x \vee y$, for all x, y, p ∈ X. Thus $NFSI_2$

is $\delta(x - y) \leq \delta(x \vee y) \leq \max\{\delta(x), \delta(y)\}$.

Hence (X, δ) is N-fuzzy sub algebra of X.

-	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

Example 2.15. Let X = {0, 1, 2} be a B-algebra with the Cayley table which is given in example (2.11). Let (X, δ) be a N -fuzzy structure defined by $\delta(0) = -0.2$,$\delta(1) = -0.4$, $\delta(2) = -0.9$.

-	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

It is easy to check that (X, δ) is both an N-fuzzy sub algebra and an N-fuzzy ideal of X.

However, the following example 2.16 shows that the converse of theorem (2.14) of (ii) is not true.

Example 2.16. Let X = {0, 1, 2, 3} be a B-algebra with the Cayley table which is given in example (2.15). Let (X, δ) be a N-fuzzy structure defined by

$\delta(0) = -0.7$, $\delta(1) = -0.4$, $\delta(2) = -0.5$, $\delta(3) = -0.6$.

It is easy to check that (X, δ) is an N-fuzzy sub algebra, but not an N-fuzzy ideal of X. Since

$\delta(2 \vee 2) = \delta(1 - ((1 - 2) + 2)) = \delta(0) = -0.7$ not belongs to $-0.5 = \max\{\delta(2), \delta(2)\}$.

III. DOUBLE-FRAMED N-FUZZY SOFT IDEALS OF B-ALGEBRA

Let X be a non-empty fixed set. A double-framed N-fuzzy structure (DFNFS in short) is an object of the form $N=\{(x,\delta_N,F_N)|x\in X\}$ where δ_N and F_N are negative valued functions from X to [−1, 0] with $-1 \leq \delta_N(x)+ F_N(x) \leq 0$ for all x ∈ X. For the sake of simplicity, we use the symbol (X, δ_N, F_N) for the DFNFS $=\{(x,\delta_N,F_N)|x\in X\}$.In what follows, let X denote B-algebra and δ_N, F_N are N-fuzzy functions on X unless otherwise specified.

For any N-fuzzy functions δ_N and F_N on X and t∈ [−1,0], the set $C=C(\delta_N,F_N;T) =\{x \in X| \delta_N(x) \leq t$ and $F_N(x) \geq t\}$ is called level set of (X, δ_N, F_N).

Definition 3.1. A double-framed N-fuzzy structure (X, δ_N, F_N) is called a double-framed N-fuzzy soft ideal lattice of X, if it satisfies

($DFNFSI_1$) : $\delta_N(x - y) \leq \delta_N(x)$ and $\delta_N(x \vee y) \leq S\{\delta_N(x), \delta_N(y)\}$

($DFNFSI_2$) : $F_N(x - y) \leq F_N(x)$ and $F_N(x \vee y) \geq T\{F_N(x), F_N(y)\}$

for all x, y ∈ X and existing of x ∨ y.

Example 3.2. Let X = {0, 1, 2, 3} be a B-algebra with the Cayley table which is given in example (2.2). Let (X, δ_N , F_N) be a double-framed N-fuzzy structure defined by

$\delta_N(0) = -0.4$ $\delta_N(1) = -0.7$ $\delta_N(2) = -0.6$ $\delta_N(3) = -0.5$

$F_N(0) = -0.3$ $F_N(1) = -0.5$ $F_N(2) = -0.2$ $F_N(3) = -0.9$

By routine calculations, we can prove that (X, δ_N, F_N) is a double-framed N-fuzzy soft ideal.

The following theorems provide the relation between the double-framed N-fuzzy soft ideal and an ideal on C(δ_N, F_N; t).

Theorem 3.3. Let 'N' be a double-framed N-fuzzy structure of X and C be a level set of X. Then 'N' is a double-framed N-fuzzy soft ideal of X if and only if C is an ideal of X for all

t ∈ [−1,0].

Proof. Necessary Part:

Let 'N' be a double-framed N-fuzzy soft ideal of X and C be a level set in X.

(i)For all x,y ∈ C,∀ t ∈ [−1,0], then $\delta_N(x-y) \leq \delta_N(x) \leq t$ and $F_N(x - y) \geq F_N(x) \geq t$.

This implies x − y ∈ C. This satisfies the condition ($DFNFSI_1$).

(ii)For all x, y ∈ C,∀ t ∈ [−1, 0], then

$\delta_N(x \vee y) \leq S\{\delta_N(x), \delta_N(y)\} \leq t$,

$F_N(x \vee y) \geq T\{F_N(x), F_N(y)\} \geq t$.

This implies that there exists x ∨ y ∈ C. Thus 'C' is an ideal of N.

Suppose that N is a double-framed N-fuzzy soft ideal of X, that is, every non-empty C of N is an ideal of X for all t ∈ [−1, 0]. If there are a, b ∈ X such that $\delta_N(a - b) > \delta_N(a)$ and $F_N(a-b) < F_N(a)$, then $\delta_N(a-b) > t_0 \geq \delta_N(a)$.

Thus a ∈ C, but a−b∉ C. This is a contradiction and so $\delta_N(x-y) \leq \delta_N(x)$ and

$F_N(x - y) \geq F_N(x)$, for all x, y ∈ X.

Thus 'C' satisfies ($DFNFSI_1 - DFNFSI_2$).

Sufficient Part:

Assume that there exist a, b ∈ X such that a ∨ b exists. Here $\delta_N(a \vee b) > S\{\delta_N(a), \delta_N(b)\}$ and $F_N(a\vee b) < S\{F_N(a),F_N(b)\}$. Then

$\delta_N(a\vee b) > t_1 \leq S\{\delta_N(a),\delta_N(b)\}$,

$F_N(a\vee b) < t_1 \geq T\{F_N(a),F_N(b)\}$,

for some $t_1\in [-1, 0]$. It follows that a,b ∈ C and a∨ b∉ C which is a contradiction. Thus C satisfies ($DFNFSI_1-$

DFNFSI$_2$).

Hence N is a double-framed N-fuzzy soft ideal of X.

Theorem 3.4. If a double-framed N-fuzzy structure N in X is double-framed N-fuzzy soft ideal of X, then so is $\overline{\delta}_N$ of X where $\overline{\delta}_N = 1-\delta_N$.

Proof. It is sufficient to show that $\overline{\delta}_N$ satisfies condition DFNFSI$_1$.

(i) For all x, y ∈ X, we have

$\overline{\delta}_N(x-y) = 1-\delta_N(x-y) \geq 1-\delta_N(x) = \overline{\delta}_N(x)$ and there exists x ∨ y ∈ X such that

$\overline{\delta}_N(x \vee y) = 1-\delta_N(x \vee y)$

$\geq 1 - T\{\delta_N(x), \delta_N(y)\}$

$= T\{1-\delta_N(x), 1-\delta_N(y)\}$

$= T\{\overline{\delta}_N(x), \overline{\delta}_N(x)\}$.

Hence $\overline{\delta}_N$ is a double-framed N-fuzzy soft ideal of X.

Theorem 3.5. A double-framed N-fuzzy structure 'N' is a double-framed N-fuzzy soft ideal of X if and only if the N-fuzzy functions δ_N and $\overline{F}_N$ are N-fuzzy soft ideals of X.

Proof. Let 'N' be a double-framed N-fuzzy soft ideal of X. Clearly, δ_N is N-fuzzy soft ideal of X.

For all x, y ∈ X, we have

$\overline{F}_N(x-y) = 1-F_N(x-y) \leq 1-F_N(x) = \overline{F}_N(x)$,

and there exists x ∨ y ∈ X such that

$\overline{F}_N(x \vee y) = 1-F_N(x \vee y) \leq 1 - S\{F_N(x), F_N(y)\} = S\{1-F_N(x), 1-F_N(y)\} = S\{\overline{F}_N(x), \overline{F}_N(y)\}$.

Hence $\overline{F}_N$ is a N-fuzzy soft ideal of X.

Conversely, suppose that δ_N and $\overline{F}_N$ are N-fuzzy soft ideals of X. Let x,y ∈ X. Then

$1-F_N(x-y) = \overline{F}_N(x-y) \leq \overline{F}_N(x) = 1-F_N(x)$

and there exists x ∨ y ∈ X such that

$1-F_N(x \vee y) = \overline{F}_N(x \vee y) \leq S\{\overline{F}_N(x), \overline{F}_N(y)\} = S\{1-F_N(x), 1-F_N(y)\} = 1-S\{F_N(x), F_N(y)\}$.

Which implies that $F_N(x-y) \geq F_N(x)$ and $F_N(x \vee y) \geq T\{F_N(x), F_N(y)\}$. This completes the proof.

Corollary 3.6. A double-framed N-fuzzy structure 'N' is a double-framed N-fuzzy soft ideal of X if and only if the N-fuzzy functions δ_N and F_N are N-fuzzy soft ideals of X.

Proof. The proof is straightforward.

Definition 3.7. A N-fuzzy function Φ on B is called a N-fuzzy characteristic function if

$$\Phi(x) = \begin{cases} 1 & \text{if } x \in B \\ 0 & \text{otherwise} \end{cases}$$

Corollary 3.8. Let Φ_A be the N-fuzzy characteristic function of an ideal A of X. Then the

$\overline{A} = (X, \Phi_A)$ is a double-framed N-fuzzy soft ideal of X.

Proof. The proof is straightforward.

Theorem 3.9. Every double-framed N-fuzzy structure 'N' of X is double-framed N-fuzzy soft ideal if and only if it satisfies

(i) $\delta_N(x - ((x - a) + b)) \leq S\{\delta_N(a), \delta_N(b)\}$

(ii) $F_N(x-((x-a)+b)) \geq T\{F_N(a), F_N(b)\}$, for all a, b ∈ X.

Proof. Let 'N' be a double-framed N-fuzzy structure of X satisfying (i) and (ii). We have

$x - y = (x - y) - (((x - y) - x) + x)$, for all x, y ∈ X. From (i) and (ii), we have

$\delta_N(x - y) = \delta_N((x-y)-(((x-y)-x)+x)) = S\{F_N(x), F_N(y)\} = \delta_N(x)$,

$F_N(x - y) = F_N((x-y)-(((x-y)-x)+x)) = T\{F_N(x), F_N(y)\} = F_N(x)$.

Suppose x ∈ X is an upper bound for a and b, for a,b ∈ X that is x = a ∨ b, then by proposition(2.5), we have a∨b = x − ((x − a) + b).

$\delta_N(a \vee b) = \delta_N(x - ((x - a) + b)) \leq S\{\delta_N(a), \delta_N(b)\}$

and $F_N(a \vee b) = F_N(x - ((x - a) + b)) \geq T\{F_N(a), F_N(b)\}$

Thus (DFNFSI$_1$), (DFNFSI$_2$) are valid. Hence double-framed N-fuzzy structure is double-framed N-fuzzy soft ideal of X.

Thus $\delta_N(a \vee b) = \delta_N(x - ((x - a) + b)) \leq S\{\delta_N(a), \delta_N(b)\}$ and

$F_N(a \vee b) = F_N(x - ((x - a) + b)) \geq T\{F_N(a), F_N(b)\}$. Hence (i) and (ii) are valid. This completes the proof.

Proposition 3.10. Every double-framed N-fuzzy soft ideal of X satisfies the following inequalities (i)$\delta_N(0) \leq \delta_N(x)$ and (ii)$F_N(0) \geq F_N(x)$, for all x ∈ X.

Proof. By taking y = x in (DFNFSI$_1$) → (DFNFSI$_2$); we have

$\delta_N(0) = \delta_N(x - x) \leq \delta_N(x)$, $F_N(0) = F_N(x - x) \geq F_N(x)$.

Corollary 3.11. Every double-framed N-fuzzy soft ideal 'N' satisfies $x \leq y \Rightarrow \delta_N(x) \geq \delta_N(y)$, and $F_N(x) \leq F_N(y)$ for all x, y ∈ X.

Proof. Let x, y ∈ X be such that x ≤ y. Then x − y = 0 and so

$\delta_N(x) = \delta_N(x-0) = \delta_N(x-((x-y)+y)) \leq S\{\delta_N(y), \delta_N(y)\} = \delta_N(y)$

and $F_N(x) = F_N(x-0) = F_N(x-((x-y)+y))$

$\geq T\{F_N(y), F_N(y)\}$

$= F_N(y)$.

by using previous proposition (3.10). This completes the proof.

Theorem 3.12. A double-framed N-fuzzy structure 'N' is a double-framed N-fuzzy soft ideal of X if and only if it satisfies

(i) $\delta_N(x) \leq S\{\delta_N(x - y), \delta_N(y)\}$

(ii) $F_N(x) \geq T\{F_N(x - y), F_N(y)\}$, for all x, y ∈ X.

Proof. Assume that 'N' is a double-framed N-fuzzy soft ideal of X. Then by proposition (3.10) and the conditions (i) and (ii) of theorem (3.12) are valid by taking z = 0 in corollary (3.11) respectively.

Conversely, 'N' satisfying proposition (3.10) and theorem (3.12).Then

$(x-((x-a)+b))-b = (x-b) - (((x-a)+b)-b) = (x-b)-((x-a)+b) \leq x-(x-a) \leq a$.

That is $(x-((x-a)+b)) -b)-a = 0$, for all $x,a,b \in X$, it follows from proposition(3.10) that

$\delta_N(x - ((x - a) + b)) \leq S\ \{\delta_N((x - ((x - a) + b)) - b), \delta_N(b)\}$
$\leq S\ \{S\ \{\delta_N(((x - ((x - a) + b)) - b) - a), \delta_N(a), \delta_N(b)\}\}$
$=S\{S\{\delta_N(0),\delta_N(a),\delta_N(b)\}\} = S\{\delta_N(a),\delta_N(b)\}$.

and $F_N(x - ((x - a) + b)) \geq T\ \{F_N((x - ((x - a) + b)) - b), F_N(b)\}$
$\geq T\ \{T\ \{F_N(((x - ((x - a) + b)) - b) - a), F_N(a), F_N(b)\}\}$
$= T\{T\{F_N(0),F_N(a),F_N(b)\}\} = T\{F_N(a),F_N(b)\}$.

By theorem (3.5), 'N' is a double-framed N-fuzzy soft ideal of X. This completes the proof.

Definition3.13. Let (X,δ_N, F_N) and (X,p_N, r_N) be double-framed N-fuzzy structures. Then (X,δ_N,F_N) is said to be a double-framed retrenchment of (X,δ_N,F_N) if $\delta_N(x) \leq p_N(x)$ and $F_N(x) \geq r_N(x)$.

Definition 3.14. Let (X, δ_N, F_N) be a double-framed N-fuzzy structure. A double-framed N-fuzzy structure (X,p_N,r_N) is called a double-framed retrenchment N-fuzzy soft ideal of (X,δ_N,F_N) if it satisfies

(i) (X,p_N,r_N) is a double-framed retrenchment of (X,δ_N,F_N).

(ii) If (X,δ_N,F_N) is a double-framed N-fuzzy soft ideal of X, then (X, p_N, r_N) is a double-framed N-fuzzy soft ideal of X.

Definition 3.15. Let (X,δ_N,F_N) be a double-framed N-fuzzy structure. A double-framed N-fuzzy structure (X,p_N,r_N) is called a created double-framed N-fuzzy soft ideal of (X,δ_N,F_N) if it satisfies

(i) (X,p_N,r_N) is a double-framed N-fuzzy soft ideal of X.

(ii) (X,p_N,r_N) is a double-framed retrenchment of (X,δ_N,F_N).

(iii) For any double-framed N-fuzzy soft ideal

(iv) (X,J_N,L_N) of X, if (X,J_N,L_N) is a double-framed retrenchment of (X,δ_N,F_N), then (X,J_N,L_N) is a double-framed retrenchment of (X,p_N,r_N).

Note 3.16. The created double-framed N-fuzzy soft ideal of (X, δ_N, F_N) of X will be denoted by $(X,|\delta_N|,|F_N|)$. Also it is the greatest double-framed N-fuzzy soft ideal in X which is a retrenchment of (X,δ_N,F_N).

Proposition 3.17. Every double-framed N-fuzzy soft ideal 'N' of X satisfies the following assertations;

(i) $\delta_N(x - y) \leq \delta_N((x - y) + y) \Leftrightarrow \delta_N((x - z) - (y - z)) \leq \delta_N((x - y) + z)$, for all $x, y, z \in X$.

(ii) $F_N(x-y) \geq F_N((x-y)+y) \Leftrightarrow F_N((x-z)-(y-z)) \geq F_N((x-y)+z)$, for all $x, y, z \in X$.

Proof. Assume that $\delta_N(x - y) \leq \delta_N((x - y) + y)$and $F_N(x - y) \geq F_N((x - y) + y)$, for all $x, y \in X$. Also, $((x - (y - z)) + z) - z = ((x - z) - (y - z)) - z \leq (x - y) + z$, by corollary-3.11.

Thus,

$\delta_N((x - (y - z)) + z) \leq \delta_N((x - y) + z)$
$F_N((x - (y - z)) + z) \geq F_N((x - y) + z)$.

$\delta_N((x-z)-(y-z)) = \delta_N(((x-(y-z))+z)-z) \leq \delta_N(((x - (y - z)) + z) - z)$
$\leq \delta_N((x - y) + z)$

and $F_N((x-z)-(y-z)) = F_N(((x-(y-z))+z)-z) \geq F_N(((x - (y - z)) + z) - z)$
$\geq F_N((x - y) + z)$.

Conversely,

$\delta_N(x - y) = \delta_N((x - y) + 0) = \delta_N((x - y) - (y - y)) \leq \delta_N((x - y) + y)$

and $F_N(x - y) = F_N((x - y) + 0) = F_N((x - y) - (y - y)) \geq F_N((x - y) + y)$.

This completes the proof.

Theorem 3.18. Let w be an element of X. If (X, δ_N, F_N) is a double-framed N-fuzzy soft ideal of X, then X_w is an ideal of X.

Proof. Let $0 \in X_w$. Let $x \in X$ and $y \in X_w$ be such that $x - y \in X_w$. Then

$\delta_N(x - y) \leq \delta_N(w)$ and $\delta_N(x) \leq \delta_N(w)$
and $F_N(x - y) \geq F_N(w)$ and $F_N(x) \geq F_N(w)$.

Since (X, δ_N, F_N) is a double-framed N-fuzzy soft ideal of X, we have

$\delta_N(x) \leq S\ \{\delta_N(x - y), \delta_N(y)\} \leq \delta_N(w)$,

and $F_N(x) \geq T\{F_N(x-y),F_N(y)\} \geq F_N(w)$, so that $x \in X_w$. Hence X_w is an ideal of X.

IV. CONCLUSION

A soft set is a mapping from parameter to the crisp subset of universe. However, the situation may be more complicated in real world because of the fuzzy characters of the parameters. Using this double-framed negative-valued structure, we have applied in a B-algebra and hence derived double-framed negative -valued B-algebra with double-framed N-fuzzy soft lattice.

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