International Journal of Scientific Research in Mathematical and Statistical Sciences
Vol.**11**, Issue.**4**, pp.**24-28**, August **2024**
E-ISSN: 2348-4519
Available online at: www.isroset.org

Research Article

A Novel Area-Biased Distribution With Essential Statistical Properties

Asgar Ali[1], Aafaq A. Rather[2*], Berihan R. Elemary[3]

[1]Dept. of Statistics, K. K. Das College, Garia, Kolkatta-700084, India
[2]Symbiosis Statistical Institute, Symbiosis International (Deemed University), Pune-411004, India
[3]Dept. of Applied Mathematical and Statistics, Faculty of Commerce, Damietta University-34511, Egypt

**Corresponding Author: aafaq7741@gmail.com*

Received: 13/Jun/2024; **Accepted**: 15/Jul/2024; **Published**: 31/Aug/2024

Abstract— In this paper, we introduce a novel model called the area biased Transmuted Mukherjee-Islam (ABTMI) distribution, which generalizes the Transmuted Mukherjee-Islam distribution through an area biased transformation approach. We thoroughly explore the probability density function (PDF) and the cumulative distribution function (CDF) of the ATMI distribution. Additionally, we investigate the distinctive structural properties of the proposed model, including the survival function, conditional survival function, hazard function, cumulative hazard function, mean residual life, moments, moment generating function (MGF), characteristic function (CF), cumulant generating function (CGF), entropy measures, and Bonferroni and Lorenz curves. The model parameters are estimated using the maximum likelihood estimation method.

Keywords—Transmuted Mukherjee-Islam distribution, Area biased distribution, Reliability analysis, Maximum likelihood estimator

1. Introduction

In various applied sciences, including engineering, agricultural science, biological science, biomedicine, ecology, and social science fields such as economics, finance, and population science, the modelling and analysis of lifetime data are of paramount importance. Numerous lifetime distributions have been employed to characterize such data, and the effectiveness of statistical analysis procedures relies significantly on the chosen probability model or distribution. Consequently, substantial efforts have been made to create extensive classes of standard probability distributions and corresponding statistical methodologies. Despite these advancements, significant challenges persist, as real-world data often deviates from classical or standard probability models. Thus, the development of new forms of probability distributions remains a common objective in statistical theory. To extend the applicability of probability distributions, the literature proposes several methods to introduce additional parameters to established baseline probability models. This enhances the flexibility of the models to capture the complexity of the data, leading to several generalized classes such as the Pearson Family, Burr Family, Exponentiated Family, Marshall-Olkin Family, $T - X$ Family, Transmuted Family, and Weighted Family.

In this paper, we introduce a novel model termed as area biased Transmuted Mukherjee-Islam (ABTMI) distribution, which generalizes the Transmuted Mukherjee-Islam distribution using an area biased transformation approach. The weighted family of distributions originated from the pioneering work of Fisher in 1934 [1] and was further refined and formalized by Rao in 1965 [2]. This concept has become an essential tool in statistical theory, particularly in scenarios where observations are derived from non-experimental, non-replicated, and non-random conditions.

Researchers and scholars have extensively explored area biased probability models, along with the application of these models across various domains. Rather and Ozel [3] proposed the weighted power Lindley distribution, demonstrating its effectiveness in analyzing lifetime data. Further, Ahmad et al. [4], Qayoom and Rather [5] and Ahmad et al. [6] discussed on different probability distributions.

2. Probability density function (PDF) and cumulative distribution function (CDF)

The Transmuted Mukherjee-Islam distribution had explored by Rather and Subramanian [7] using the quadratic rank transmutation map studied first by Shaw and Buckley in 2007 [8]. The probability density function of a random variable say

Z following Transmuted Mukherjee- Islam distribution with parameters say (ε,ν,ω) is given by

$$f(z;\varepsilon,\nu,\omega)=\frac{\varepsilon}{\nu^{\varepsilon}}z^{\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right) \tag{1}$$

such that

$0<z<\nu,\varepsilon>0,\nu>0,-1\le\omega\le1$

And the corresponding cumulative distribution function is

$$F_Z(z)=\left(\frac{z}{\nu}\right)^{\varepsilon}\left(1+\omega-\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right) \tag{2}$$

Using the weighted transformation approach, the PDF $y(z)$ of a non-negative random variable Z is given by

$$y_w(z)=\frac{w(z)\,y(z)}{E\big(w(z)\big)};\qquad z>0$$

Where $w(z)$ be a non-negative weight function and

$$E\big(w(z)\big)=\int_{-\infty}^{\infty}w(z)\,y(z)\,dz<\infty$$

.

Here, we will put the weight function as $w(z)=z^{s}$ and PDF of the random variable Z to be Transmuted Mukherjee- Islam distribution to derive the PDF of Weighted Transmuted Mukherjee- Islam distribution. The PDF of Weighted Transmuted Mukherjee- Islam distribution distribution is given by

$$g(z;\varepsilon,\nu,\omega,s)=\frac{z^{s}f(z;\varepsilon,\nu,\omega)}{E(z^{s})} \tag{3}$$

Now

$$E(z^2)=\int_0^{\nu}z^2\frac{\varepsilon}{\nu^{\varepsilon}}z^{\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)dz \tag{4}$$

In this paper we will introduce area biased technique. For that, we take s=2 in equation (3), we get

$$E(z^2)=\frac{\varepsilon}{\nu^{\varepsilon}}\left((1+\omega)\int_0^{\nu}z^{\varepsilon+1}dz-\frac{2\omega}{(\nu)^{\varepsilon}}\int_0^{\nu}z^{2+2\varepsilon-1}dz\right) \tag{5}$$

After simplification we get

$$E(z^2)=\frac{\varepsilon\nu^2\big(2(1-\omega)+2\varepsilon\big)}{(2+\varepsilon)(2+2\varepsilon)} \tag{6}$$

Using (1) and (6) in (3) we get

$$g(z;\varepsilon,\nu,\omega)=\frac{z^2\frac{\varepsilon}{\nu^{\varepsilon}}z^{\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)}{\frac{\varepsilon\nu^2\big(2(1-\omega)+2\varepsilon\big)}{(2+\varepsilon)(2+2\varepsilon)}} \tag{7}$$

$$g(z;\varepsilon,\nu,\omega)=\frac{(2+\varepsilon)(2+2\varepsilon)z^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)}{\nu^{2+\varepsilon}\big(2(1-\omega)+2\varepsilon\big)} \tag{8}$$

The corresponding CDF of ABTMI distribution is given by

$$G_Z(z)=\int_0^{z}\left(\frac{(2+\varepsilon)(2+2\varepsilon)z^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)}{\nu^{2+\varepsilon}\big(2(1-\omega)+2\varepsilon\big)}\right)dz \tag{9}$$

$$G_Z(z)=\frac{(2+\varepsilon)(2+2\varepsilon)}{\nu^{2+\varepsilon}\big(2(1-\omega)+2\varepsilon\big)}\left((1+\omega)\int_0^{z}z^{2+\varepsilon-1}dz-\frac{2\omega}{(\nu)^{\varepsilon}}\int_0^{z}z^{2+2\varepsilon-1}\right) \tag{10}$$

After simplification we get

$$G_Z(z)=\frac{(2+2\varepsilon)(1+\omega)(\nu)^{\varepsilon}z^{2+\varepsilon}-2\omega(2+\varepsilon)z^{2+2\varepsilon}}{\nu^{2+2\varepsilon}\big(2(1-\omega)+2\varepsilon\big)} \tag{11}$$

3. Reliability Analysis

3.1 Survival function

The survival function of ABTMI distribution is given by

$$R_T(t)=1-\frac{(2+2\varepsilon)(1+\omega)(\nu)^{\varepsilon}t^{2+\varepsilon}-2\omega(2+\varepsilon)t^{2+2\varepsilon}}{\nu^{2+2\varepsilon}\big(2(1-\omega)+2\varepsilon\big)}$$

$$R_T(t)=\frac{\nu^{2+2\varepsilon}\big(2(1-\omega)+2\varepsilon\big)-(2+2\varepsilon)(1+\omega)(\nu)^{\varepsilon}t^{2+\varepsilon}}{\nu^{2+2\varepsilon}\big(2(1-\omega)+2\varepsilon\big)}+\frac{2\omega(2+\varepsilon)t^{2+2\varepsilon}}{\nu^{2+2\varepsilon}\big(2(1-\omega)+2\varepsilon\big)}$$

After simplification we get

$$R_T(t)=\frac{\nu^{2+2\varepsilon}\big(2(1-\omega)+2\varepsilon\big)-(2+2\varepsilon)(1+\omega)(\nu)^{\varepsilon}t^{2+\varepsilon}}{\nu^{2+2\varepsilon}\big(2(1-\omega)+2\varepsilon\big)}+\frac{2\omega(2+\varepsilon)t^{2+2\varepsilon}}{\nu^{2+2\varepsilon}\big(2(1-\omega)+2\varepsilon\big)}$$

3.2 Hazard function

The hazard function of ABTMI distribution can be

$$H_T(t)=\frac{(2+\varepsilon)(2+2\varepsilon)(\nu)^{\varepsilon}t^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{t}{\nu}\right)^{\varepsilon}\right)}{\nu^{2+2\varepsilon}\big(2(1-\omega)+2\varepsilon\big)-t^{2+\varepsilon}\big((2+2\varepsilon)(1+\omega)(\nu)^{\varepsilon}-2\omega(2+\varepsilon)t^{\varepsilon}\big)}$$

3.3 Reverse Hazard function

The reverse hazard function of ABTMI distribution can be

$$H_r(t)=\frac{(2+\varepsilon)(2+2\varepsilon)(\nu)^{\varepsilon}t^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{t}{\nu}\right)^{\varepsilon}\right)}{(2+2\varepsilon)(1+\omega)(\nu)^{\varepsilon}t^{2+\varepsilon}-2\omega(2+\varepsilon)t^{2+2\varepsilon}}$$

3.4 Mills Ratio

The Mills ratio of ABTMI distribution is given by

$$Mills\ \ ratio = \frac{1}{H_r(t)}$$

$$\text{Mills ratio} = \frac{(2+2\varepsilon)(1+\omega)(\nu)^{\varepsilon}t^{2+\varepsilon} - 2\omega(2+\varepsilon)t^{2+2\varepsilon}}{(2+\varepsilon)(2+2\varepsilon)(\nu)^{\varepsilon}t^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{t}{\nu}\right)^{\varepsilon}\right)}$$

4. Moments

The rth raw moment about origin of ABTMI distributionis defined as

$$\mu_r' = \int_0^{\nu} z^r\, g(z;\varepsilon,\nu,\omega)dz$$

$$\mu_r' = \int_0^{\nu} z^r \frac{(2+\varepsilon)(2+2\varepsilon)z^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}dz$$

After simplification we get

$$\mu_r' = \frac{(2+\varepsilon)(2+2\varepsilon)(\nu)^r\left((1-\omega)(2+\varepsilon+r)+(1+\omega)\varepsilon\right)}{\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+r)(2+2\varepsilon+r)} \quad (12)$$

Putting $r = 1,2,3,4$ in (12) we get

$$\mu_1' = \frac{(2+\varepsilon)(2+2\varepsilon)(\nu)\left((1-\omega)(\varepsilon+3)+(1+\omega)\varepsilon\right)}{\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+1)(2\varepsilon+3)}$$

$$\mu_2' = \frac{(2+\varepsilon)(2+2\varepsilon)(\nu)^2\left((1-\omega)(2+\varepsilon+2)+(1+\omega)\varepsilon\right)}{\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+2)(2+2\varepsilon+2)}$$

$$\mu_3' = \frac{(2+\varepsilon)(2+2\varepsilon)(\nu)^3\left((1-\omega)(2+\varepsilon+3)+(1+\omega)\varepsilon\right)}{\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+3)(2+2\varepsilon+3)}$$

$$\mu_4' = \frac{(2+\varepsilon)(2+2\varepsilon)(\nu)^4\left((1-\omega)(2+\varepsilon+4)+(1+\omega)\varepsilon\right)}{\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+4)(2+2\varepsilon+4)}$$

The variance and coefficient of variance $(C.V)$ respectively are given by

$$\sigma^2 = \mu_2' - (\mu_1')^2$$

and

$$C.V = \frac{\sigma}{\mu_1'}; \quad \text{where, } \sigma = \sqrt{\mu_2' - (\mu_1')^2}$$

5. Harmonic mean

The harmonic mean of ABTMI distribution can be obtained as

$$Harmonic\ mean = E\left(\frac{1}{Z}\right)$$

$$Harmonicmean = \frac{(2+\varepsilon)(2+2\varepsilon)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}\left((1+\omega)\int_0^{\nu} z^{2+\varepsilon-2}dz - \frac{2\omega}{(\nu)^{\varepsilon}}\int_0^{\nu} z^{2+2\varepsilon-2}dz\right)$$

After simplification we get

$$Harmonicmean = \frac{(2+\varepsilon)(2+2\varepsilon)\left((1-\omega)(2+\varepsilon-1)+(1+\omega)\right)}{\nu\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon-1)(2+2\varepsilon-1)}$$

6. MGF, CF and CGF

The MGF of ABTMI distributionis equal to

$$M_Z(t) = E(e^{tz})$$

$$M_Z(t) = \int_0^{\nu} e^{tz}\frac{(2+\varepsilon)(2+2\varepsilon)z^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}dz$$

$$M_Z(t) = \int_0^{\nu}\sum_{k=0}^{\infty}\frac{(tz)^k}{k!}\frac{(2+\varepsilon)(2+2\varepsilon)z^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}dz$$

$$M_Z(t) = \sum_{k=0}^{\infty}\frac{(t)^k}{k!}\int_0^{\infty} z^k g(z;\varepsilon,\nu,\omega)dz$$

$$M_Z(t) = \sum_{k=0}^{\infty}\frac{(t)^k}{k!}\mu_k'$$

$$M_Z(t) = \sum_{k=0}^{\infty}\frac{(t)^k}{k!}\frac{(2+\varepsilon)(2+2\varepsilon)(\nu)^k\left((1-\omega)(2+\varepsilon+k)+(1+\omega)\varepsilon\right)}{\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+k)(2+2\varepsilon+k)}$$

The CF of ABTMI distribution can be obtained as

$$\phi_Z(t) = E(e^{itz})$$

$$\phi_Z(t) = \int_0^{\nu} e^{\iota tz}\frac{(2+\varepsilon)(2+2\varepsilon)z^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}dz$$

$$\phi_Z(t) = \int_0^{\nu}\sum_{k=0}^{\infty}\frac{(\iota tz)^k}{k!}\frac{(2+\varepsilon)(2+2\varepsilon)z^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}dz$$

$$\phi_Z(t) = \sum_{k=0}^{\infty}\frac{(\iota)^k(t)^k}{k!}\int_0^{\infty} z^k g(z;\varepsilon,\nu,\omega)dz$$

$$\phi_Z(t) = \sum_{k=0}^{\infty}\frac{(\iota t)^k}{k!}\mu_k'$$

$$\phi_Z(t) = \sum_{k=0}^{\infty}\frac{(\iota t)^k}{k!}\frac{(2+\varepsilon)(2+2\varepsilon)(\nu)^k\left((1-\omega)(2+\varepsilon+k)+(1+\omega)\varepsilon\right)}{\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+k)(2+2\varepsilon+k)}$$

The CGF of ABTMI distribution is given by

$$\kappa_Z(t) = \log\left(M_Z(t)\right)$$

$$\kappa_Z(t)=\log\left(\sum_{k=0}^{\infty}\frac{(t)^k}{k!}\frac{(2+\varepsilon)(2+2\varepsilon)(\nu)^k\left((1-\omega)(2+\varepsilon+k)+(1+\omega)\,\varepsilon\right)}{\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+k)(2+2\varepsilon+k)}\right)$$

7. Estimation of Parameters

Suppose $z_1, z_2, z_3, \ldots, z_n$ be a random sample of size n from ABTMI distribution. Then The likelihood function is defined as the joint density of the random sample, which is given as

$$L(\varepsilon,\nu,\omega)=\prod_{l=1}^{n}\frac{(2+\varepsilon)(2+2\varepsilon)z_l^{\,2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z_l}{\nu}\right)^{\varepsilon}\right)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}$$

$$L(\varepsilon,\nu,\omega)=\frac{(2+\varepsilon)^n(2+2\varepsilon)^n}{\nu^{n(2+\varepsilon)}\left(2(1-\omega)+2\varepsilon\right)^n}\left(\prod_{l=1}^{n}z_l^{\,2+\varepsilon-1}\right)\left(\prod_{l=1}^{n}\left(1+\omega-2\omega\left(\frac{z_l}{\nu}\right)^{\varepsilon}\right)\right)$$

Taking logarithm on both sides we get

$$\log L(\varepsilon,\nu,\omega)=n\log(2+\varepsilon)+n\log(2+2\varepsilon)-n(2+\varepsilon)\log(\nu)$$
$$-n\log\left(2(1-\omega)+2\varepsilon\right)+(2+\varepsilon-1)\sum_{l=1}^{n}\log z_l \qquad (13)$$
$$+\sum_{l=1}^{n}\log\left(1+\omega-\frac{2\omega}{(\nu)^{\varepsilon}}(z_l)^{\varepsilon}\right)$$

Differentiating equation (13) partially with respect to ε and equating to zero we get

$$\frac{n}{2+\varepsilon}+\frac{2n}{2+2\varepsilon}-n\log(\nu)-\frac{2n}{2(1-\omega)+2\varepsilon}+\sum_{l=1}^{n}\log z_l$$
$$-\sum_{l=1}^{n}\frac{2\,\omega}{\left(1+\omega-\frac{2\omega}{(\nu)^{\varepsilon}}(z_l)^{\varepsilon}\right)}\left(\frac{z}{\nu}\right)^{\varepsilon}\log\left(\frac{z}{\nu}\right)=0 \qquad (14)$$

Differentiating equation (13) partially with respect to ν and equating to zero we get

$$\sum_{l=1}^{n}\frac{2\,\omega\,(z_l)\,\varepsilon}{\left(1+\omega-\frac{2\omega}{(\nu)^{\varepsilon}}(z_l)^{\varepsilon}\right)(\nu)^{\varepsilon+1}}-\frac{n(2+\varepsilon)}{\nu}=0 \qquad (15)$$

Differentiating equation (13) partially with respect to ω and equating to zero we get

$$\frac{2n}{2(1-\omega)+2\varepsilon}+\sum_{l=1}^{n}\frac{1-2\left(\frac{z_l}{\nu}\right)^{\varepsilon}}{\left(1+\omega-\frac{2\omega}{(\nu)^{\varepsilon}}(z_l)^{\varepsilon}\right)}=0 \qquad (16)$$

Differentiating equation (13) partially with respect to s and equating to zero we get

$$\frac{n}{2+\varepsilon}+\frac{n}{2+2\,\varepsilon}-n\log(\,\nu)-\frac{n\,(1-\omega)}{2(1-\omega)+2\varepsilon}+\sum_{l=1}^{n}\log z_l=0 \qquad (17)$$

By solving equation (14), (15), (16), and (17) simultaneously, we obtain the maximum likelihood estimators of parameters involved in the given distribution. However, the above system of non-linear equations cannot be evaluated directly. So, to get the maximum likelihood estimates for the distribution parameters, we have to solve these system of equations using Newton-Raphson method, Mathematica, or Secant method.

8. Renyi entropy and Tsallis entropy measures

The Renyi entropy [9] is given by

$$R(\tau)=\frac{1}{1-\tau}\log\left(\int_0^{\nu}\left(g(z_k;\varepsilon,\nu,\omega)\right)^{\tau}dz\right)$$

$$R(\tau)=\frac{1}{1-\tau}\log\int_0^{\nu}\left(\frac{(2+\varepsilon)(2+2\varepsilon)z^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}\right)^{\tau}dz$$

$$R(\tau)=\frac{1}{1-\tau}\log\left(\left(\frac{(2+\varepsilon)(2+2\varepsilon)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}\right)^{\tau}\int_0^{\nu}\left(z^{\tau(2+\varepsilon-1)}\sum_{k=0}^{\tau}({}^{\tau}C_k)(-1)^k(1+\omega)^{\tau-k}\left(2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)^k\right)dz\right)$$

$$R(\tau)=\frac{1}{1-\tau}\log\left(\left(\frac{(2+\varepsilon)(2+2\varepsilon)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}\right)^{\tau}\sum_{k=0}^{\tau}({}^{\tau}C_k)(-1)^k(1+\omega)^{\tau-k}\left(\frac{2\omega}{(\nu)^{\varepsilon}}\right)^k\int_0^{\nu}z^{\varepsilon k+\tau(2+\varepsilon-1)}dz\right)$$

After simplification we get

$$R(\tau)=\frac{1}{1-\tau}\log\left(\frac{\left((2+\varepsilon)(2+2\varepsilon)\right)^{\tau}(\nu)^{1-\tau}}{\left(2(1-\omega)+2\varepsilon\right)^{\tau}}\sum_{k=0}^{\tau}\frac{({}^{\tau}C_k)(-1)^k(1+\omega)^{\tau-k}(2\omega)^k}{\varepsilon k+\tau(2+\varepsilon-1)+1}\right)$$

Similarly, the Tsallis entropy [10] associated with the given distribution is given by

$$T_s(\xi)=\frac{1}{\xi-1}\left(1-\int_0^{\nu}\left(g(z_k;\varepsilon,\nu,\omega)\right)^{\xi}dz\right)$$

$$T_s(\xi)=\frac{1}{\xi-1}\left(1-\frac{\left((2+\varepsilon)(2+2\varepsilon)\right)^{\xi}(\nu)^{1-\xi}}{\left(2(1-\omega)+2\varepsilon\right)^{\xi}}\sum_{k=0}^{\xi}\frac{({}^{\xi}C_k)(-1)^k(1+\omega)^{\xi-k}(2\omega)^k}{\varepsilon k+\xi(2+\varepsilon-1)+1}\right)$$

9. Bonferroni and Lorenz curves

The Bonferroni curve [11] of the given distribution is given by

$$\Psi(\zeta)=\frac{1}{\zeta\,\mu_1'}\int_0^{\varphi}z\,g(z;\varepsilon,\nu,\omega)dz$$

Where

$$\mu_1'=\frac{(2+\varepsilon)(2+2\varepsilon)(\nu)\left((1-\omega)(2+\varepsilon+1)+(1+\omega)\,\varepsilon\right)}{\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+1)(2+2\varepsilon+1)}$$

and $\quad \varphi=F^{-1}(\zeta)$

$$\Psi(\zeta)=\frac{1}{\zeta\ \mu_1'}\int_0^{\varphi} z\,\frac{(2+\varepsilon)(2+2\varepsilon)z^{2+\varepsilon-1}\left(1+\omega-2\omega\left(\frac{z}{\nu}\right)^{\varepsilon}\right)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}$$

$$\Psi(\zeta)=\frac{1}{\zeta\ \mu_1'}\frac{(2+\varepsilon)(2+2\varepsilon)}{\nu^{2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}\left((1+\omega)\int_0^{\varphi} z^{2+\varepsilon}dz-\frac{2\omega}{(\nu)^{\varepsilon}}\int_0^{\varphi} z^{2+2\varepsilon}dz\right)$$

$$\Psi(\zeta)=\frac{1}{\zeta\ \mu_1'}\frac{(2+\varepsilon)(2+2\varepsilon)}{\nu^{\backslash 2+\varepsilon}\left(2(1-\omega)+2\varepsilon\right)}\left((1+\omega)\left(\frac{(\varphi)^{2+\varepsilon+1}}{2+\varepsilon+1}\right)-\frac{2\omega}{(\nu)^{\varepsilon}}\left(\frac{(\varphi)^{2+2\varepsilon+1}}{2+2\varepsilon+1}\right)\right)$$

After simplification we get

$$\Psi(\zeta)=\frac{(2+\varepsilon)(2+2\varepsilon)(\varphi)^{2+\varepsilon+1}\left((1+\omega)(2+2\varepsilon+1)(\nu)^{\varepsilon}-2\omega(2+\varepsilon+1)(\varphi)^{\varepsilon}\right)}{\zeta\ \mu_1'\nu^{2+2\varepsilon}\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+1)(2+2\varepsilon+1)}$$

Also, the Lorenz curve [12] of the given distribution is given by

$$\Phi(\zeta)=\zeta\Phi(\zeta)$$

$$\Phi(\zeta)=\zeta\left(\frac{(2+\varepsilon)(2+2\varepsilon)(\varphi)^{2+\varepsilon+1}\left((1+\omega)(2+2\varepsilon+1)(\nu)^{\varepsilon}-2\omega(2+\varepsilon+1)(\varphi)^{\varepsilon}\right)}{\zeta\ \mu_1'\nu^{2+2\varepsilon}\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+1)(2+2\varepsilon+1)}\right)$$

$$\Phi(\zeta)=\frac{(2+\varepsilon)(2+2\varepsilon)(\varphi)^{2+\varepsilon+1}\left((1+\omega)(2+2\varepsilon+1)(\nu)^{\varepsilon}-2\omega(2+\varepsilon+1)(\varphi)^{\varepsilon}\right)}{\mu_1'\nu^{2+2\varepsilon}\left(2(1-\omega)+2\varepsilon\right)(2+\varepsilon+1)(2+2\varepsilon+1)}$$

10. Conclusion

In this study, we have undertaken an extensive investigation into a novel extension of the Transmuted Mukherjee-Islam distribution, which we have termed the area biased Transmuted Mukherjee-Islam distribution. This new variant is a specific case within the broader class of weighted distributions. By integrating the area biased framework into the foundational structure of the three-parameter Transmuted Mukherjee-Islam distribution, we have introduced a new distribution that offers additional flexibility and applicability in statistical modeling.

Our analysis delves deeply into the mathematical and statistical properties of this newly proposed area biased distribution. We have provided a thorough examination of its characteristics, demonstrating how it differs from and enhances the existing distribution. Moreover, the study addresses the practical aspect of parameter estimation for this innovative distribution. We have employed maximum likelihood estimation techniques, which are well-suited to handle the complexity of the model, ensuring accurate and reliable estimation of the parameters.

In conclusion, this study contributes a significant new tool to the field of statistical distributions, offering researchers and practitioners a versatile option for modeling data that exhibits area bias. The rigorous exploration of its properties and the effective parameter estimation methods we have presented make this distribution a valuable addition to the statistical literature.

Data Availability: No data has been used in this paper.

Conflict of Interest: The authors confirm that there are no conflicts of interest related to this work.

Funding source: No funding available.

Author's contribution:
Asgar Ali: Writing-original draft, software, Visualization. **Aafaq A. Rather:** Writing-review & editing. **Berihan R. Elemary:** Visualization, Resources.

Acknowledgment: We would like to express our gratitude to all those who contributed to the completion of this work. We appreciate the valuable feedback and insights provided by the Editor and Reviewers, which helped to enhance the quality of this research.

References

[1] R.A. Fisher, “The Effects of Methods of Ascertainment Upon the Estimation of Frequencies,” *Annals of Eugenics*, Issue **6,** pp.**13- 25, 1934**.

[2] C.R. Rao, “On Discrete Distributions Arising Out of Method of Ascertainment, In Classical and Contagious Discrete,” *G.P. Patiled; Pergamum Press and Statistical Publishing Society*, *Calcutta*, pp.**320-332, 1965**.

[3] A.A. Rather, G. Ozel, “The Weighted Power Lindley Distribution with Applications on the Life Time Data,” *Pakistan Journal of Statistics and Operation Research*, Vol.**16**, Issue.**2**, pp. **225-237, 2020**.

[4] A. Ahmad, A.A. Rather, Y.A. Tashkandy, M.E. Bakr, M.M.M. El-Din, A.M. Gemeay, E.M. Almetwally, M. Salem, “Deriving the New Cotangent Frechet Distribution With Real Data Analysis,” *Alexandria Engineering Journal*, Vol.**105**, Issue.**10**, pp.**12-24, 2024**. https://doi.org/10.1016/j.aej.2024.06.038

[5] D. Qayoom, A. A. Rather, “Weighted Transmuted Mukherjee-Islam Distribution with Statistical Properties,” *Reliability: Theory and Applications*, Vol.**78**, Issue.**2**, pp. **124-137, 2024**.

[6] A. Ahmad, A.A. Rather, A.M. Gemeay, M. Nagy, L.P. Sapkota, A.H. Mansi, “Novel Sin-G Class of Distributions with an Illustration of Lomax Distribution: Properties and Data Analysis,” *AIP Advances*, Vol.**14**, pp.**1-17, 2024**.

[7] A.A. Rather, C. Subramanian, “Transmuted Mukherjee-Islam Failure Model,” *Journal of Statistics Applications & Probability*, Vol.**7**, Issue.**2,** pp.**343-347, 2018**.

[8] W.T. Shaw, I.R.C. Buckley, “The Alchemy of Probability Distributions: Beyond Gram-Charlier Expansions and A Skew-Kurtotic-Normal Distribution from A Rank Transmutation Map,” *Research report*, **2007**.

[9] R. Alfred, “On Measures of Information and Entropy,” *Proceedings of the fourth Berkeley Symposium on Mathematics, Statistics and Probability 1960*. pp.**547–561, 1961**.

[10] C. Tsallis, “Possible Generalization of Boltzmann-Gibbs Statistics,”. *Journal of Statistical Physics*, Vol.**52**, Issue.**1-2**, pp.**479–487, 1988**.

[11] C.E. Bonferroni, “*Elementi Di Statistica Generale, Seeber, Firenze*,” **1930.**

[12] M.O. Lorenz, “Methods of measuring the concentration of wealth,” Publications of the American Statistical Association, Vol.**9**, pp.**209–219, 1905**.