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On Concomitants of Upper Record Statistics and Survival Analysis for a Morgenstern Type Bivariate Transmuted Exponentiated Gumbel Distribution

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Abstract- This paper presents upper record statistics and their concomitants for a Morgenstern Type Bivariate Transmuted Exponentiated Gumbel Distribution. Survival Functions of the Concomitants of Upper Record Statistics are derived for the distribution. The numerical values of the survival functions are displayed for some selected values of the parameters.

Keywords- *Record values, Concomitants, Morgenstern Family, Survival function, Numerical Study.*

I. INTRODUCTION

In the theory of statistical sciences, the record values and record statistics have been widely used. Record values was first discussed by Candler [7]. Record on a random variable X are realized as a sequence of observation that are larger or smaller than all the previously recorded ones in the sequence. So, the record values are distinct elements in the successive maxima or minima about a sequence of variable. Every change in the maximum during the observation process for X means that a record is observed. In this regard, records are dealt with in the scope of the extremal processes as discussed by Resnick [15]. Many theoretical or applied areas of science use the theory and methodology about the record values in the empirical investigations about the timing and magnitude of record type extremities.

The concomitants in case of record statistics are relatively new one in the field of ordered random variables. This branch of record concomitants was initiated by Houchens [12]. Concomitants of record values assume its importance as record values hold for the univariate case. Considering a sequence of pair wise r.v.'s $\{(Xi, Yi), i \geq 1\}$, when the experimenter is interested in studying just the sequence of records of the first component, the second component associated with a record value of the first one is termed as the concomitants of that record value (Amini & Ahmadi, [5]). For a sequence $\{(X_i, Y_i); i = 1,2,3, ...\}$ of iid random variables if $\{R_r, r \geq 1\}$ be the sequence of upper record values in the sequence of $X's$ then the r^{th} value of Y variate associated with X values will be called the concomitants of r^{th} upper record and is denoted by$R_{[r]}$.

The statistical features and potential applications of record values were introduced Candler [7]. Since then, a rich literature has grown on the statistical theory and methodology for the data analysis and inference about the record values, which have been done by (Resnick, [5]; Ahsanullah, [5]; Arnold *et al.*, [6]; David and Nagaraja, [10]). Ahsanullah and Nevzorov [4] studied the theory of concomitants of record values. Chacko [8], Chacko and Thomas [9] studied the use of concomitants of record values in estimation problem. The records have been used to characterize many probability distributions, see Ahsanullah, [1]; Kirmani and Beg, [13]; Ahsanullah and Kirmani, [2].

As concomitants of ordered random variables assume its importance in the bivariate setup, so it can be applied to characterize bivariate probability distributions. Veena and Thomas [18] and Thomas and Veena [16] have attempted to characterize some bivariate distributions using the properties of concomitants of record statistics. In the theory of concomitants of records, Thomas and Veena [17] have studied some results on characterizing a class of bivariate distributions by properties of concomitants of record values. Science the record value holds its importance in distribution theory that is why an attempt has been made to study the upper record statistics and concomitants of upper record statistics for Morgenstern Type Bivariate Transmuted Exponentiated Gumbel Distribution.

The following sections of the paper begins with the definitions and distributions of the upper record values and their concomitants. Then, a Bivariate Transmuted Exponentiated Gumbel Distribution is introduced, and the distributional properties of the upper record values and their concomitants are derived for the new distribution. Thereafter, the survival analysis models are presented and some interpretations and implications about these models are provided. And, a conclusion is drawn accordingly.

II. CONCOMITANTS OF UPPER RECORD STATISTICS

For a random variable *X* distribution of r^{th}, upper record, R_r, has been given by Ahsanullah, [3], as

$$f_{R_r}(x) = \frac{1}{\Gamma(r)} f(x)[H(x)]^{r-1} \quad (1)$$

where $H(x) = -ln[1 - F(x)]$.

The joint probability density function of the r^{th} and s^{th} upper records R_r and $R_s, r < s$, is presented by Ahsanullah, [3], using the following general expression

$$f_{R_r R_s}(x_1, x_2) = \frac{h(x_1) f(x_2)}{\Gamma(r)\,\Gamma(s-r)} [H(x_1)]^{r-1} [H(x_2) - H(x_1)]^{s-r-1} \quad (2)$$

where $h(x_r) = \frac{d}{dx_r} H(x_r)$ and $-\infty < x_1 < x_2 < +\infty$.

The probability density function for the concomitant of the r^{th} upper record is also given by Ahsanullah, [3], as

$$f_{R_{[r]}}(y) = \int_{-\infty}^{+\infty} f(y|x) f_{R_r}(x)\, dx, \quad (3)$$

And the corresponding *cdf* is

$$F_{R_{[r]}}(y) = \int_{-\infty}^{\infty} F_{Y|X}(y|x)\, f_{R_r}(x) dx \quad (4)$$

Ahsanullah, [3], has also given joint distribution of concomitant of r^{th} and s^{th} upper record values as

$$f\left(y_{[r]}, y_{[s]}\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(y_r|x_r) f(y_s|x_s) f(x_r, x_s) dx_r dx_s \quad (5)$$

Where $f(x_r, x_s)$ is given in equation (2).

Under the assumptions of the model, the expressions given above hold for any bivariate distribution $F\,(x, y)$.

III. Concomitants of Record Values for Morgenstern Type Bivariate Transmuted Exponentiated Gumbel Distribution (MTBTEGD)

Morgenstern [14] introduced a family of bivariate distribution functions having a representation of the form

$$F_{XY}(x, y) = F_X(x) F_Y(y)\{1 + \rho[1 - F_X(x)][1 - F_Y(y)]\} \quad (6)$$

where $F_X(x)$ and $F_Y(y)$ are two univariate distribution functions and the association parameter ρ is constrained to lie in the interval [−1, 1].

The corresponding *pdf* is

$$f_{X,Y}(x, y) = f_X(x) f_Y(y)[1 + \rho\{1 - 2F_X(x)\}\{1 - 2F_Y(y)\}];\ -1 \le \rho \le 1 \quad (7)$$

Deka *et al.*, [11], have studied the Transmuted Exponentiated Gumbel Distribution (TEGD) along with several statistical properties and applied it to model water quality parameters data set. The *cdf* of the TEGD is

$$F(x) = 1 - \left\{1 - exp\left(-exp\left(-\frac{x-\mu}{\sigma}\right)\right)\right\}^{\alpha}\left[1 - \lambda + \lambda\left\{1 - exp\left(-exp\left(-\frac{x-\mu}{\sigma}\right)\right)\right\}^{\alpha}\right], -\infty < (x, \mu) < \infty,$$
$$(\sigma, \alpha) > 0, \quad |\lambda| \le 1 \quad (8)$$

And its corresponding *pdf* is

$$f(x)=\frac{\alpha}{\sigma}\left\{1-exp\left(-exp\left(-\frac{x-\mu}{\sigma}\right)\right)\right\}^{\alpha-1}\left\{exp\left(-exp\left(-\frac{x-\mu}{\sigma}\right)\right)\right\}\left(exp\left(-\frac{x-\mu}{\sigma}\right)\right)\left[1-\lambda+2\lambda\left\{1-exp\left(-exp\left(-\frac{x-\mu}{\sigma}\right)\right)\right\}^{\alpha}\right], -\infty<(x,\mu)<\infty,\quad (\sigma,\alpha)>0, \quad |\lambda|\le 1 \tag{9}$$

Using the marginal Transmuted Exponentiated Gumbel density functions for X and Y in equation (6) we get the *cdf* for MTBTEGD as

$$F_{XY}(x,y)=\left\{1-\left\{exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]\right\}\left\{1-\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}\left[1+\rho\left\{\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}\right] \tag{10}$$

And the corresponding pdf is obtained by using (7) as

$$f_{XY}(x,y)=\frac{\alpha_1\alpha_2}{\sigma_1\sigma_2}\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1-1}\left\{exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\left(exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\left[1-\lambda_1+2\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2-1}\left\{exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}\left(exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\left[1-\lambda_2+2\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\left[1+\rho\left\{\left[\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]-1\right]\left[\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]-1\right]\right\}\right] \tag{11}$$

It can be shown that

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{XY}(x,y)\,dxdy=1$$

Using *cdf* of TEGD from (8) we have

$H(x)=-\ln[1-F(x)]$

$$=-ln\left[\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]\right]$$

$$=-\alpha_1 ln\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}-ln\left\{1-\left\{\lambda_1-\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right\}\right\}$$

$$=> H(x)=\alpha_1\sum_{m=1}^{\infty}\frac{\left(exp\left(-mexp\left(-\left(\frac{x-\mu_1}{\sigma_1}\right)\right)\right)\right)}{m}+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}$$

$$\left(exp\left(-bexp\left(-\left(\frac{x-\mu_1}{\sigma_1}\right)\right)\right)\right);\ -\infty<(x,\mu_1)<\infty,(\alpha_1,\sigma_1)>0\ ,|\lambda|\le 1 \qquad (12)$$

Using equation (1) and (12),we get the expression for the *pdf* of the r^{th} upper record R_r of TEGD as

$$f_{R_r}(x)=\frac{1}{\Gamma r}\left(\frac{\alpha_1}{\sigma_1}\right)\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1-1}\left\{exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}$$
$$\left(exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\left[1-\lambda_1+2\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]$$
$$\left[\alpha_1\sum_{m=1}^{\infty}\frac{\left\{exp\left(-mexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}}{m}+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\right.$$
$$\left.\left\{exp\left(-bexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\right]^{r-1} \qquad (13)$$

If we take $r=2$; in equation (13), then the distribution of the second order upper record statistics X_{U_2} for bivariate TEGD is

$$f_{R_2}(x)=\left(\frac{\alpha_1}{\sigma_1}\right)\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1-1}\left\{exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}$$
$$\left(exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\left[1-\lambda_1+2\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]$$
$$\left[\alpha_1\sum_{m=1}^{\infty}\frac{\left\{exp\left(-mexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}}{m}+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\right.$$
$$\left.\left\{exp\left(-bexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\right] \qquad (14)$$

Also we have

$$h(x)=\frac{d}{dx}[H(x)] \qquad (15)$$

Using (12) in (15), and after differentiation we get

$$h(x)=\alpha_1\sum_{m=1}^{\infty}\frac{1}{m^2\sigma_1}\left\{exp\left(-mexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right\}+$$
$$\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n\sigma_1 b}\left\{exp\left(-bexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right\} \qquad (16)$$

Thus we have

$$h(x_1)=\alpha_1\sum_{m=1}^{\infty}\frac{1}{m^2\sigma_1}\left\{exp\left(-mexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right\}+$$
$$\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n\sigma_1 b}\left\{exp\left(-bexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right\} \qquad (17)$$

$$h(x_2)=\alpha_1\sum_{m=1}^{\infty}\frac{1}{m^2\sigma_1}\left\{exp\left(-mexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right\}+$$
$$\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n\sigma_1 b}\left\{exp\left(-bexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right\} \qquad (18)$$

$$H(x_1) = \alpha_1 \sum_{m=1}^{\infty} \frac{\left(exp\left(-mexp\left(-\left(\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right)\right)}{m} + \sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}$$
$$\left(exp\left(-bexp\left(-\left(\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right)\right) \quad (19)$$

And

$$H(x_2) = \alpha_1 \sum_{m=1}^{\infty} \frac{\left(exp\left(-mexp\left(-\left(\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right)\right)}{m} + \sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}$$
$$\left(exp\left(-bexp\left(-\left(\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right)\right) \quad (20)$$

Using equations (17), (18), (19) and (20) in equation (2); we get the expression for the joint *pdf* of r^{th} and s^{th} upper record R_r and R_s for TEGD is obtained as

$$f_{R_r,R_s}(x_1,x_2) = \frac{1}{\Gamma(r)\Gamma(s-r)}\left[\alpha_1 \sum_{m=1}^{\infty}\frac{1}{m^2\sigma_1}\left\{exp\left(-mexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right\} + \right.$$
$$\left.\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n\sigma_1 b}\left\{exp\left(-bexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right\}\right]$$
$$\left[\alpha_1 \sum_{m=1}^{\infty}\frac{1}{m^2\sigma_1}\left\{exp\left(-mexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right\}\right.$$
$$\left.+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\left\{exp\left(-bexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right\}\right]$$
$$\left[\alpha_1 \sum_{m=1}^{\infty}\frac{\left\{exp\left(-mexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}}{m} + \sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\left\{exp\left(-bexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\right]^{r-1}$$
$$\left[\alpha_1 \sum_{m=1}^{\infty}\frac{1}{m}\left\{\left\{exp\left(-mexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\} - \left\{exp\left(-mexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\right\} + \right.$$
$$\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\left\{\left\{exp\left(-bexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\right.$$
$$\left.\left.-\left\{exp\left(-bexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\right\}\right]^{s-r-1} \quad (21)$$

Putting $r = 1$ and $s = 2,$ in equation (21); we get the joint *pdf* of 1st and 2nd order upper record R_1 and $R_2,$ for TEGD is obtained as

$$f_{R_1,R_2}(x_1,x_2) = \left[\alpha_1 \sum_{m=1}^{\infty}\frac{1}{m^2\sigma_1}\left\{exp\left(-mexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right\} + \right.$$
$$\left.\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n\sigma_1 b}\left\{exp\left(-bexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right\}\right]$$
$$\left[\alpha_1 \sum_{m=1}^{\infty}\frac{1}{m^2\sigma_1}\left\{exp\left(-mexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right\} + \right.$$

$$\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n\sigma_1 b}\left\{exp\left(-bexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right\}\Bigg] \quad (22)$$

If $r = 2$ and $s = 3$, then we get the joint *pdf* of 2nd and 3rd upper record R_2 and R_3 as

$$f_{R_2,R_3}(x_1,x_2) = \Bigg[\alpha_1\sum_{m=1}^{\infty}\frac{1}{m^2\sigma_1}\left\{exp\left(-mexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right\}+$$
$$\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n\sigma_1 b}\left\{exp\left(-bexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right\}\Bigg]$$
$$\Bigg[\alpha_1\sum_{m=1}^{\infty}\frac{1}{m^2\sigma_1}\left\{exp\left(-mexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right\}+$$
$$\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n\sigma_1 b}\left\{exp\left(-bexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right\}\Bigg]$$
$$\left[\alpha_1\sum_{m=1}^{\infty}\frac{\left\{exp\left(-mexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}}{m}+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\lambda_1^n\frac{\left\{exp\left(-bexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}}{n}\right]$$
$$\Bigg[\alpha_1\sum_{m=1}^{\infty}\frac{1}{m}\left\{\left\{exp\left(-mexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}-\left\{exp\left(-mexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\right\}+$$
$$\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\Bigg\{\left\{exp\left(-bexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}$$
$$-\left\{exp\left(-bexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\Bigg\}\Bigg] \quad (23)$$

Similarly, for different values of r and s, we can obtained the joint *pdf* for different upper record statistics. Also the conditional distribution of Y given X is obtained as

$$f(y|x)\frac{f_{XY}(x,y)}{f_X(x)}$$
$$\therefore f(y|x) = \frac{\alpha_2}{\sigma_2}\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2-1}\left(exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right)\left(exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)$$
$$\left[1-\lambda_2+2\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\Bigg[1+\rho\Bigg\{\Bigg[\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}$$
$$\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]-1\Bigg]\Bigg[\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}$$
$$\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]-1\Bigg]\Bigg\}\Bigg] \quad (24)$$

Using (13) and (24) in equation (3), the *pdf* of the concomitants of the r^{th} order upper record value for TEGD is obtained as

$$f_{R_{[r]}}(y) = \frac{1}{\Gamma r}\left(\frac{\alpha_1\alpha_2}{\sigma_1\sigma_2}\right)\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2-1}\left\{exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}$$
$$\left\{exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right\}\left[1-\lambda_2+2\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]$$
$$\int_{-\infty}^{\infty}\Bigg[1+\rho\Bigg\{\Bigg[\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]-1\Bigg]$$
$$\Bigg[\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]-1\Bigg]\Bigg\}\Bigg]$$

$$\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1-1}\left\{exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right\}$$
$$\left[1-\lambda_1+2\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]\left[\alpha_1\sum_{m=1}^{\infty}\frac{\left\{exp\left(-mexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}}{m}+\right.$$
$$\left.\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\left\{exp\left(-bexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\right]^{r-1}dx \qquad (25)$$

Putting $r=1$, in (25), we get the concomitants of 1st order upper record for TEGD as

$$f_{R_{[1]}}(y)=\left(\frac{\alpha_1\alpha_2}{\sigma_1\sigma_2}\right)\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2-1}\left\{exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}\left\{exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right\}$$
$$\left[1-\lambda_2+2\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\int_{-\infty}^{\infty}\left[1+\rho\left\{\left[\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right.\right.\right.$$
$$\left.\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]-1\right]\left[\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}$$
$$\left.\left.\left.\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]-1\right]\right\}\right]\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1-1}$$
$$\left\{exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right\}\left[1-\lambda_1+2\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]dx$$

After integration in the above equation we get the expression for the concomitants of 1st order upper record value for TEGD as

$$f_{R_{[1]}}(y)=\left(\frac{\alpha_2}{\sigma_2}\right)\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2-1}\left\{exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}$$
$$\left\{exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right\}\left[1-\lambda_2+2\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\left[1+\frac{\rho}{2}-\right.$$
$$\left.\frac{\rho}{2}\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right] \qquad (26)$$

Putting $r=2$, in (25), we can obtained the concomitants of 2nd order upper record value for TEGD as

$$f_{R_{[2]}}(y)=\left(\frac{\alpha_1\alpha_2}{\sigma_1\sigma_2}\right)\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2-1}\left\{exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}\left\{exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right\}$$
$$\left[1-\lambda_2+2\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\int_{-\infty}^{\infty}\left[1+\rho\left\{\left[\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right.\right.\right.$$
$$\left.\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]-1\right]\left[\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}$$
$$\left.\left.\left.\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]-1\right]\right\}\right]\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1-1}$$
$$\left\{exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right\}\left[1-\lambda_1+2\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]$$

$$\left[\alpha_1 \sum_{m=1}^{\infty} \frac{\left\{exp\left(-mexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}}{m} + \sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\left\{exp\left(-bexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\right]dx \quad (27)$$

After integration we get the expression for the concomitants of 2nd order upper record value for TEGD as

$$f_{R_{[2]}}(y) = \frac{\alpha_1\alpha_2}{\sigma_2}\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2-1}\left\{exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}\left\{exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right\}$$
$$\left[1-\lambda_2+2\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\left[\alpha_1\sum_{m=1}^{\infty}\frac{1}{m}\{(1-\lambda_1)(1+\rho)\beta(\alpha_1,m+1)+2\lambda_1(1+\rho)\right.$$
$$\beta(2\alpha_1,m+1)-\rho\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]$$
$$\{(1-\lambda_1)\beta(\alpha_1,m+1)+2\lambda_1\beta(2\alpha_1,m+1)\}+\rho\left\{\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right.$$
$$\left.\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]-1\right\}[(1-\lambda_1)^2\beta(2\alpha_1,m+1)+3\lambda_1(1-\lambda_1)\beta(3\alpha_1,m+1)+$$
$$2\lambda_1^2\beta(4\alpha_1,m+1)]\}+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\{(1-\lambda_1)(1+\rho)\beta(\alpha_1,b+1)+$$
$$2\lambda_1(1+\rho)\beta(2\alpha_1,b+1)-\rho\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]$$
$$\{(1-\lambda_1)\beta(\alpha_1,b+1)+2\lambda_1\beta(2\alpha_1,b+1)\}+\rho\left\{\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right.$$
$$\left.\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]-1\right\}[(1-\lambda_1)^2\beta(2\alpha_1,b+1)+3\lambda_1(1-\lambda_1)\beta(3\alpha_1,b+1)+$$
$$\left.2\lambda_1^2\beta(4\alpha_1,b+1)]\}\right] \quad (28)$$

Also we have,

$$F_{Y|X}(y|x) = \frac{F_{XY}(x,y)}{F_X(x)}$$

Thus we get,

$$F_{Y|X}(y|x) = \left\{1-\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}$$
$$\left[1+\rho\left\{\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]\right.\right.$$
$$\left.\left.\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}\right] \quad (29)$$

Using equations (13) and (29) in (4); we get *cdf* of the concomitants of r^{th} upper record as

$$F_{R_{[r]}}(y) = \frac{1}{\Gamma r}\left(\frac{\alpha_1}{\sigma_1}\right)\left\{1-\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}$$
$$\int_{-\infty}^{\infty}\left[1+\rho\left\{\left(1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right)^{\alpha_1}\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]\right.\right.$$
$$\left.\left.\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}\right]$$

$$\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1-1}\left\{exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right\}$$
$$\left[1-\lambda_1+2\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]\left[\alpha_1\sum_{m=1}^{\infty}\frac{\left\{exp\left(-mexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}}{m}\right.$$
$$\left.+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\left\{exp\left(-bexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\right]^{r-1}dx \tag{30}$$

Putting $r=1$ in the equation (30) we can obtained the *cdf* of the concomitants of 1st order upper record statistics $X_{U_{[1]}}$ as

$$F_{R_{[1]}}(y)=\left(\frac{\alpha_1}{\sigma_1}\right)\left\{1-\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}$$
$$\int_{-\infty}^{\infty}\left[1+\rho\left\{\left(1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right)^{\alpha_1}\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]\right.\right.$$
$$\left.\left.\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}\right]$$
$$\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1-1}\left\{exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right\}$$
$$\left[1-\lambda_1+2\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]dx \tag{31}$$

After integration we get the expression for the *cdf* of the concomitants of 1st order upper record statistics $X_{U_{[1]}}$ as

$$F_{R_{[1]}}(y)=\left\{1-\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}$$
$$\left[1+\frac{\rho}{2}\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right] \tag{32}$$

Putting $r=2$, in equation (30), we can obtained the *cdf* of the concomitants of 2nd order upper record statistics $X_{U_{[2]}}$ as

$$F_{R_{[2]}}(Y)=\left(\frac{\alpha_1}{\sigma_1}\right)\left\{1-\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}$$
$$\int_{-\infty}^{\infty}\left[1+\rho\left\{\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]\right.\right.$$
$$\left.\left.\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}\right]$$
$$\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1-1}\left\{exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right\}$$
$$\left[1-\lambda_1+2\lambda_1\left\{1-exp\left(-exp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]\left[\alpha_1\sum_{m=1}^{\infty}\frac{\left\{exp\left(-mexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}}{m}+\right.$$
$$\left.\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\left\{exp\left(-bexp\left(-\frac{x-\mu_1}{\sigma_1}\right)\right)\right\}\right]dx \tag{33}$$

After integration we get the expression for $F_{R_{[2]}}(y)$ as

$$F_{R_{[2]}}(y)=\alpha_1\left\{1-\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}$$

$$\left[\alpha_1\sum_{m=1}^{\infty}\frac{1}{m}\left\{(1-\lambda_1)\beta(\alpha_1,m+1)+2\lambda_1\beta(2\alpha_1,m+1)+\rho\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right.\right.$$
$$\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\{(1-\lambda_1)^2\beta(2\alpha_1,m+1)+3\lambda_1(1-\lambda_1)\beta(3\alpha_1,m+1)$$
$$+2\lambda_1^2\beta(4\alpha_1,m+1)\}\}+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\{(1-\lambda_1)\beta(\alpha_1,b+1)+2\lambda_1\beta(2\alpha_1,b+1)+$$
$$\rho\left(1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right)^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\{(1-\lambda_1)^2\beta(2\alpha_1,b+1)$$
$$\left.+3\lambda_1(1-\lambda_1)\beta(3\lambda_1,b+1)+2\lambda_1^2\beta(4\alpha_1,b+1)\}\}\right] \quad (34)$$

IV. JOINT DISTRIBUTION OF TWO CONCOMITANTS OF RECORD STATISTICS

In this section an attempt has been made to derive the joint distribution of concomitants of record statistics for Bivariate Transmuted Exponentiated Gumbel Distribution. The expression (5) has been used to derive the expression for joint distribution of two concomitant of record statistics. To derive the joint distribution of the concomitants we need the conditional distribution, which are obtained using equation (24) as

$$f(y_1|x_1)=\left(\frac{\alpha_2}{\sigma_2}\right)\left\{1-exp\left(-exp\left(-\frac{y_1-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2-1}\left\{exp\left(-exp\left(-\frac{y_1-\mu_2}{\sigma_2}\right)\right)\right\}\left(exp\left(\frac{y_1-\mu_2}{\sigma_2}\right)\right)$$
$$\left[1-\lambda_2+2\lambda_2\left\{1-exp\left(-exp\left(-\frac{y_1-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\left[1+\rho\left\{\left[\left\{1-exp\left(-exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right.\right.\right.$$
$$\left.\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]-1\right]\left[\left\{1-exp\left(-exp\left(-\frac{y_1-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right.$$
$$\left.\left.\left.\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y_1-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]-1\right\}\right] \quad (35)$$

and

$$f(y_2|x_2)=\left(\frac{\alpha_2}{\sigma_2}\right)\left\{1-exp\left(-exp\left(-\frac{y_2-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2-1}\left\{exp\left(-exp\left(-\frac{y_2-\mu_2}{\sigma_2}\right)\right)\right\}\left(exp\left(\frac{y_2-\mu_2}{\sigma_2}\right)\right)$$
$$\left[1-\lambda_2+2\lambda_2\left\{1-exp\left(-exp\left(-\frac{y_2-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\left[1+\rho\left\{\left[\left\{1-exp\left(-exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right.\right.\right.$$
$$\left.\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]-1\right]\left[\left\{1-exp\left(-exp\left(-\frac{y_2-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right.$$
$$\left.\left.\left.\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y_2-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]-1\right\}\right] \quad (36)$$

Now using equation (35), (36) and (21), in equation (5), we get the joint *pdf* for the concomitants of r^{th} and s^{th} upper record values for BTEGD as

$$f_{R_{[r]},R_{[s]}}(y_1,y_2)=\frac{1}{\Gamma(r)\Gamma(s-r)}\left(\frac{\alpha_2}{\sigma_2}\right)^2\left\{1-exp\left(-exp\left(-\frac{y_1-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2-1}\left\{exp\left(-exp\left(-\frac{y_1-\mu_2}{\sigma_2}\right)\right)\right\}$$
$$\left\{exp\left(-\frac{y_1-\mu_2}{\sigma_2}\right)\right\}\left[1-\lambda_2+2\lambda_2\left\{1-exp\left(-exp\left(-\frac{y_1-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\left\{1-exp\left(-exp\left(-\frac{y_2-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2-1}$$
$$\left\{exp\left(-exp\left(-\frac{y_2-\mu_2}{\sigma_2}\right)\right)\right\}\left\{exp\left(-\frac{y_2-\mu_2}{\sigma_2}\right)\right\}\left[1-\lambda_2+2\lambda_2\left\{1-exp\left(-exp\left(-\frac{y_2-\mu_2}{2}\right)\right)\right\}^{\alpha_2}\right]$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{x_s}\left[1+\rho\left\{\left[\left\{1-exp\left(-exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x_1-\mu_1}{1}\right)\right)\right\}^{\alpha_1}\right]-1\right]\right.\right.$$
$$\left.\left.\left[\left\{1-exp\left(-exp\left(-\frac{y_1-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(\frac{y_1-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]-1\right]\right\}\right]$$

$$\left[1+\rho\left\{\left[\left\{1-exp\left(-exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\left[1-\lambda_1+\lambda_1\left\{1-exp\left(-exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}^{\alpha_1}\right]-1\right]\right.\right.$$
$$\left.\left.\left[\left\{1-exp\left(-exp\left(-\frac{y_2-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y_2-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]-1\right]\right\}\right]$$
$$\left[\alpha_1\sum_{m=1}^{\infty}\frac{1}{m^2\sigma_1}\left\{exp\left(-mexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\left(exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n\sigma_1 b}\right.$$
$$\left.\left\{exp\left(-bexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right\}\right]\left[\alpha_1\sum_{m=1}^{\infty}\frac{1}{m^2\sigma_1}\left\{exp\left(-mexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right\}\right.$$
$$\left.+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n\sigma_1 b}\left\{exp\left(-bexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\left\{exp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right\}\right]\left[\alpha_1\sum_{m=1}^{\infty}\frac{\left\{exp\left(-mexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}}{m}\right.$$
$$\left.+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\left\{exp\left(-bexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\right]^{r-1}\left[\alpha_1\sum_{m=1}^{\infty}\frac{1}{m}\left\{\left\{exp\left(-mexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}\right.\right.$$
$$\left.+\left\{exp\left(-mexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\right\}+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}$$
$$\left.\left\{\left\{exp\left(-bexp\left(-\frac{x_2-\mu_1}{\sigma_1}\right)\right)\right\}-\left\{exp\left(-bexp\left(-\frac{x_1-\mu_1}{\sigma_1}\right)\right)\right\}\right\}\right]^{s-r-1}dx_1dx_2$$

V. SURVIVAL FUNCTIONS FOR MTBTEGD

The survival function for a random variable Y is defined as $S(y)=1-F(y)$, which is actually the probability of the survival of a component or a live after the age of y. Under the bivariate TEG distribution, the survival function for the concomitant of the 1^{st} and 2^{nd} order upper record value is then obtained as

$$S_{R_{[1]}}(y)=1-\left\{1-\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}$$
$$\left[1+\frac{\rho}{2}\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right] \quad (37)$$

and

$$S_{R_{[2]}}(y)=1-\alpha_1\left\{1-\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\right\}$$
$$\left[\alpha_1\sum_{m=1}^{\infty}\frac{1}{m}\left\{(1-\lambda_1)\beta(\alpha_1,m+1)+2\lambda_1\beta(2\alpha_1,m+1)+\rho\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right.\right.$$
$$\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\{(1-\lambda_1)^2\beta(2\alpha_1,m+1)+3\lambda_1(1-\lambda_1)\beta(3\alpha_1,m+1)$$
$$+2\lambda_1^2\beta(4\alpha_1,m+1)\}\}+\sum_{n=1}^{\infty}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}(-1)^{a+b}\binom{n}{a}\binom{\alpha_1 a}{b}\frac{\lambda_1^n}{n}\{(1-\lambda_1)\beta(\alpha_1,b+1)+2\lambda_1\beta(2\alpha_1,b+1)+$$
$$\rho\left(1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right)^{\alpha_2}\left[1-\lambda_2+\lambda_2\left\{1-exp\left(-exp\left(-\frac{y-\mu_2}{\sigma_2}\right)\right)\right\}^{\alpha_2}\right]\{(1-\lambda_1)^2\beta(2\alpha_1,b+1)$$
$$\left.+3\lambda_1(1-\lambda_1)\beta(3\lambda_1,b+1)+2\lambda_1^2\beta(4\alpha_1,b+1)\}\}\right] \quad (38)$$

Record values are the successive maxima values that are matters of interest in many lifetime related events. So, they are usually used in the investigations that focus on survivals and deaths as the core of analysis. The age specific lifetime and remaining lifetime analysis can be precisely based on the order value r and the magnitudes of the record and concomitant values x and y, respectively. The behavior of the survival functions, in case of the bivariate TEG distribution, have some particular features. To study the survivability of concomitants of upper record statistics for Morgenstern Type Bivariate Transmuted Exponentiated Gumbel Distribution (MTBTEGD), numerical values of the survival function of MTBTEGD have been obtained using survival functions of concomitants of first and second order upper record statistics of MTBTEGD with the help of R-Programming. Numerical values of the survival function of MTBEGD for various values of y, and parameter λ_2 are obtained for fixed value of the parameters μ_2, σ_2 and α_2 and are presented in the following tables:

Table 1: Numerical values of the Survival function of the concomitants of 1st order upper record statistics of MTBTEGD

λ_2	-1	-0.5	0	0.5	1
y	$\mu_2 = 0, \sigma_2 = 1, \alpha_2 = 2, \rho = 0.5$				
0.1	0.9140428	0.9103651	0.9063043	0.9043211	0.9030156
0.2	0.9005441	0.895629	0.8898291	0.88714	0.8852839
0.3	0.8856701	0.8791902	0.8709403	0.867334	0.8647062
0.4	0.8695163	0.8611023	0.8494238	0.844646	0.8409434
0.5	0.852266	0.8415249	0.8250859	0.8188408	0.8136526
0.6	0.834208	0.8207537	0.7977667	0.7897241	0.7825003
0.7	0.8157507	0.7992498	0.7673551	0.7571657	0.7471804
0.8	0.7974311	0.7776621	0.7338049	0.7211251	0.7074378
0.9	0.7799148	0.7568366	0.6971525	0.6816798	0.6630963
1.0	0.7639837	0.7378027	0.6575332	0.6390511	0.6140918

Table 2: Numerical values of the Survival function of the concomitants of 2nd order upper record statistics of MTBTEGD

λ_2	-1	-0.5	0	0.5	1
y	$\mu_2 = 0, \sigma_2 = 1, \alpha_2 = 2, \rho = 0.5, \lambda_1 = 1, \alpha_1 = 1$				
0.1	1.253489	0.8787508	0.5669045	0.3179499	0.131887
0.2	1.105124	0.76915	0.4898927	0.2673522	0.1015285
0.3	0.966955	0.668932	0.4213231	0.2241281	0.07734711
0.4	0.8403028	0.5784283	0.3608113	0.1874516	0.05834928
0.5	0.7258014	0.497589	0.3078201	0.1564948	0.04361289
0.6	0.6235193	0.4260724	0.2617229	0.130471	0.03231658
0.7	0.5330934	0.363331	0.2218534	0.1086604	0.02375216
0.8	0.4538581	0.308688	0.1875425	0.09042157	0.01732518
0.9	0.3849579	0.261399	0.1581444	0.07519401	0.01254794
1.0	0.3254375	0.2206999	0.1330524	0.06249522	0.009028268

From the Tables 1 & 2, it is cleared that that survival probability of the concomitants of first and second order upper record statistics decreases with increase in the value of λ_2 and y holding the fixed values of the parameters $\mu_2, \sigma_2, \alpha_2$ and ρ.

V. CONCLUSION

The record values from a sequence of observations indicate the important features of extremities about the populations that they are observed from. These extremities with certain orders can even be coupled with some other variables which are in the capacity of their concomitants so that some analysis in larger scopes can be conducted. In this paper we introduce the MTBTEG distribution and its upper records that are in the pairwise association with their concomitants. Also the Survivability analysis for the concomitants of upper record statistics for MTBTEGD have been studied.

REFERENCES

[1] Ahsanullah, M., (1982). Characterization of the Exponential Distribution by Some Properties of the Record Values. *Statist. Hefte*, 23, 326-332.

[2] Ahsanullah, M. & Kirmani, S.N.U.A., (1991). Characterization of the Exponential Distribution through a Lower Record. *Comm. Statist. Theory Methods*, 1293-1299.

[3] Ahsanullah, M., (1995). Record Statistics. *Nova Science, USA.*

[4] Ahsanullah, M. & Nevzorov, V.B., (2000). Some Distributions of Induced Record Values. *Biometrical Journal*, 42, 1069-1081.

[5] Amini, M. & Ahmadi, J., (2005). On Moments of Concomitants of Classic Record Values and Nonparametric Upper Bounds for the Mean Under the Farlie-Gumbel-Morgenstern Model. *Journal of Statistical Research of Iran*, 123-125.

[6] Arnold, B.C., Balakrishnan, N. & Nagarajah, H.N., (1998). Records. *Wiley, New York*.

[7] Candler, K.N., (1952). The Distribution and Frequency of Record Values. *Journal of Royal Statistical Society,* Series B14, 220-228.

[8] Chacko, M., (2007). On Concomitants of Order Statistics. *Ph.D. Thesis, University of Kerala, Trivandrum, India.*

[9] Chacko, M. & Thomas, P.Y. (2008). Estimation of Parameters of Binomial Normal Distribution Using Concomitants of Records Values. *Statistical Propers*, 49(2), 263-275.

[10] David, H.A. & Nagaraja, H.N. (2003). Order Statistics, *Third Ed; John Wiley & Sons.*

[11] Deka, D., Das, B. & Baruah, B.H., (2017). Transmuted Exponentiated Gumbel Distribution (TEGD) and its Application to Water Quality Data. *Pak.J.Stat.Oper.Res.* 17(3), 115-126.

[12] Houchens, R.L., (1984). Record value theory and inference. *Ph.D. Dissertation*, University of California, Riverside, California.

[13] Kirmani, S.N.U.A. & Beg, M.I., (1984). On Characterization of Distribution by Expected Records. *Sankhya*, A 46(3), 463-465.

[14] Morgenstern, D., (1956). Einfeche beispiele Zweidimensionder Venteihengen. *Mitteingsblatt für Mathematishe Statistik*, 8, 234-238.

[15] Resnick, S.I., (1987). Extreme Values, Regular Variation and Point Processes. *Springer Verlag.*

[16] Thomas, P.Y. & Veena, T.G., (2011). On an Application of Concomitants of Order Statistics in Characterizing a Family of Bivariate Distributions. *Comm. Stat. Theory Methods*, 40, 1445-1452.

[17] Thomas, P.Y. & Veena, T.G., (2014). Characterization of Bivariate Distributions Using Concomitants of Record Values. *Metrika,* 77, 947-963.

[18] Veena, T.G. & Thomas, P.Y., (2008). Characterization of Bivariate Distributions by Properties of Concomitants of Order Statistics. *Stat. Prob. Lett*. 78, 3350-3354.

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