International Journal of Scientific Research in Mathematical and Statistical Sciences
Research Paper
Vol.6, Issue.2, pp.252-257, April (2019)
E-ISSN: 2348-4519
DOI: https://doi.org/10.26438/ijsrmss/v6i2.252257

Survival Modeling and Analysis for Time to Failures of Aircraft Glass

J. Vijay Anto[1*], Lilly George[2]

[1,2]Dept. of Statistics, St. Joseph's College (Autonomous), Tiruchirappalli, Tamil Nadu, India

[*]Corresponding Author: antovijayja@gmail.com, Tel.: +91-8220162713

Available online at: www.isroset.org

Received: 19/Apr/2019, Accepted: 30/Apr/2019, Online: 30/Apr/2019

***Abstract*—** Assuring the reliability of components of an aircraft is very essential, as even a minor problem in the aircraft leads to major risk to the passengers. In this study, the lifetime of aircraft glass is assessed through survival modeling. The aircraft glass failure data has been taken from the National Institute of Standards and Technology (NIST), and it is assumed to follow the lifetime distributions. Kolmogorov-Smirnov test statistic clearly reveals that the empirical data is fitted with the following theoretical distributions: gamma, Weibull and lognormal. The Maximum Likelihood Estimation method is used to estimate the parameters of the theoretical distributions. With the assistance of AIC and BIC statistics, the best model has been chosen among the three distributions. Lognormal distributed empirical data has the lowest AIC and BIC statistic compared with Weibull model and gamma model. Therefore, time to failure of the aircraft glass data best fitted with the lognormal survival model. Once the best model had been identified, the reliability measures like cumulative hazard rate, reliability function and mean time to failure are estimated in the paper. The hazard rate of aircraft glass is maximum during the period of 18.83-23.83 months. The aircraft glass has a decreasing failure rate (DFR) over time. The expected lifetime of aircraft glass is 31 months based on MTTF. The total amount of risk to aircraft glass failure until 48.83 months is 65 percent.

Keywords *— Lifetime distributions, KS test, AIC, BIC, cumulative hazard rate, reliability function, MTTF*

I. INTRODUCTION

Reliability analysis is important for ascertaining the performance of any product or component of a product. Normally, the reliability assessment information is required in order to prevent the failure of the components if the components are maintainable and repairable. The process industries should be aware that, with larger plants involving higher inventories of hazardous material, the practice of learning by mistakes is no longer acceptable. Methods should be developed for identifying hazards and for quantifying the consequences of failures [1]. The primary objective of the study is to find out the appropriate survival model for the time to failure of aircraft glass. The secondary objective of the study is to estimate the reliability rate, cumulative hazard rate and mean time to failure of aircraft glass. Kolmogorov–Smirnov test (KS) test is one of the goodness-of-fit statistics, that is used to check whether the data follows a given theoretical probability distribution or not. Maximum likelihood estimation (MLE) is one of the parameter estimation methods. It is a good estimation method as it satisfies all properties of good estimator such as efficiency, sufficiency, consistency, and unbiasedness. Mean time to failure is the best reliability measurement as aircraft glass is a non-repairable product.

Section I contains the introduction of the study which includes the objective of the research. Section II contains the previous articles which are related to our study. Section III contains the materials and methods which include the mathematical formula and derivation of the MLE, AIC, and BIC. Section IV contains the results and discussion of our study. Section V contains the conclusion of our research. Section VI contains the future directions of this study.

II. RELATED WORK

R.A. Bakoban and Hanaa H. Abu-Zinadah (2015) studied the beta generalized inverted exponential distribution. They used various probability distribution plots in order to ascertain the appropriate theoretical distribution model. The Maximum Likelihood Estimation method is applied to estimate the parameter of the model [2].

Bianca Schroeder and Garth A. Gibson (2006) did the analysis of failure data regarding high-performance computing system. They collected the failure data over the past 9 years and considered the empirical cumulative distribution function and how well it was fit with theoretical distribution. They used MLE estimation to parameterize the distributions and evaluate the goodness-of-fit by visual inspection and the negative log-likelihood [3].

Maydeu-Olivares and C.G. Forero (2010) assessed the inconsistency between the empirical data and theoretical model and summarized through Goodness-of-Fit (GOF) indices. The well-known two GOF indices are AIC and BIC. A researcher selects either AIC or BIC and calculates it for all theoretical models under consideration. Then the model with the lowest index is considered as the best model [4].

Khaled Haddad and Ataur Rahman (2010) discussed that when the theoretical model is three-parameter distribution, Anderson Darling Criterion (ADC) is the best goodness-of-fit index. Contradictorily, AIC and BIC are better GOF indices when the theoretical model is a two-parameter distribution. However, ADC gives similar results to the AIC and BIC for small samples especially for asymmetrical distributions [5].

J. G. Elerath (2000) discussed that the failure rate is one of the simple methods for determining product reliability. All products involve some sort of "failure rate" averaging over some period of time. Even though the failure rate is not constant over time, estimating an average can easily generate meaningless results [6].

Milena Krasich (2009) studied that the most common reliability of the product can be estimated through Mean time to failure, Mean time between failure, and Mean time to the first failure. Mean Time To Failure (MTTF) is understood to be the universal attribute of a non-repairable item [7].

Dong Shang Chang (2000) discussed the unimodal failure rate function of lifetime distributions. Critical time segregates the product's operating life into two phases namely increasing failure rate and decreasing failure rate. He used lognormal distribution to illustrate the effective procedure for screening out failures in the early operating period of the product [8].

III. METHODOLOGY

The aircraft glass failure data have been taken from the National Institute of Standards and Technology (NIST). In the study, the lifetime of the glass has been studied. The nature of the skewness of data has been verified by a histogram. Through the histogram, we can know whether the data follows skewed distribution or not. Once the nature of distribution was found out, we can list some of the most essential lifetime distributions. Kolmogorov–Smirnov test (KS) test can be used to find out the Goodness-of-fit between theoretical probability distribution and empirical probability distribution [9]. Thus, we can construct more than one survival modeling for our data.

Once we build survival models, Maximum likelihood estimation (MLE) method has been used to estimate the parameters of the models. Perhaps our empirical data are fitted with more than one survival models, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) statistics have been calculated to choose the best-fitted survival model among them for our data. On the basis of the best-fitted survival model, we calculate the hazard function, mean time to failures and hazard rate. The statistical calculations have been computed through R software.

Maximum Likelihood Estimation

Let $S_1, S_2, S_3, \ldots, S_n$ be the identically independent random samples of random variable S. The random variable S follows probability distribution with unknown parameter θ. Here, the objective is to estimate the parameter θ, using the empirical data $s_1, s_2, s_3, \ldots, s_n$ that are obtained from the random samples.

$L(\theta | s)$ is the likelihood function of parameters θ, where $\theta \in \Theta$ parameter space that describes the probability of obtaining the empirical data s. The joint probability density function of $S_1, S_2, S_3, \ldots, S_n$ is L(θ).

$$L(\theta) = f(s_1;\theta).\ f(s_2;\theta).\ f(s_3;\theta) \ldots f(s_n;\theta) = \prod_{i=1}^{n} f(s_i|\theta)$$

Apply natural logarithm on both sides, we will get the log-likelihood function lnL(θ).

$$\ln(L(\theta)) = \ln \prod_{i=1}^{n} f(s_i|\theta) = \sum_{i=1}^{n} \ln\left(f(s_i|\theta)\right)$$

The maximum likelihood estimator can be calculated by using the first order derivative of log-likelihood lnL(θ) with respect to the parameter θ is equal to 0.

$$\text{(i.e)}\ \frac{\partial \ln L(\theta)}{\partial(\theta)} = 0 \quad (1)$$

If $\hat{\theta}$ is the maximum likelihood estimator which is calculated based on the equation 1, the second order derivative of the log-likelihood function at $\theta = \hat{\theta}$ should be less than 0.

$$\text{(i.e)}\ \frac{\partial^2 \ln L(\theta)}{\partial \theta^2} < 0$$

Hence, $\hat{\theta}$ is the maximum likelihood estimator for the probability distribution function f(S; θ).

Weibull Distribution

The probability density function (PDF) of Weibull distribution is given as follows:

$$f(s;\alpha,\beta) = \frac{\beta}{\alpha}\left(\frac{s}{\alpha}\right)^{\beta-1} e^{-\left(\frac{s}{\alpha}\right)^{\beta}} \quad \text{if } s \geq 0 \quad (2)$$

where α is the scale parameter and β is the shape parameter.

Gamma Distribution

The probability density function of gamma distribution with shape and rate parameters α and β respectively is given below.

$$f(s;\alpha,\beta) = \frac{1}{\Gamma\alpha}\beta^{\alpha}s^{\alpha-1}e^{-\beta s} \qquad (3)$$

Lognormal Distribution

If random variable S follows a lognormal distribution, then ln(S) is normally distributed. The probability density function of lognormal distribution with location (μ) and scale parameter (σ) is given below

$f(s) = \frac{1}{s}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left[\frac{\ln(s)-\mu}{\sigma}\right]^2}$ where μ is the mean of the ln(S) and σ is the standard deviation of ln(S).

Akaike Information Criterion

Akaike Information Criterion (AIC) is used in statistics when the theoretical model is fitted with empirical data. The theoretical model may not exactly represent empirical data. Therefore, some information regarding our empirical data can be missed when represented by the theoretical model. AIC calculates the amount of information missing from the data by a given theoretical model. Akaike Information Criterion (AIC) is a fined technique based on in-sample fit to estimate the likelihood of a model to predict/estimate the future values [10]. A good model is the one that has minimum AIC among all the other models.

AIC can be calculated through log-likelihood and the number of parameters in the proposed theoretical model:

AIC = -2ln(L) + 2k where 'k' is the number of parameters and ln(L) is the value of log-likelihood [11].

Bayesian Information Criterion

Bayesian Information Criterion (BIC) is another criterion for model selection that measures the trade-off between model fit and complexity of the model [12]. A lower BIC value indicates a better fit.

BIC = -2ln(L) + 2kln(N) where 'k' is the number of parameters in the proposed theoretical model, ln(L) is the value of log-likelihood and 'N' is the number of observations [11].

Mean Time To Failure

Mean time to failure (MTTF) is the best reliability measurement for the non-repairable component. It measures the average time to first failure [13]. It can be calculated from the mean of the probability density of the time to failure f(t).

$$\text{MTTF} = \int_0^{\infty} tf(t)dt$$

Cumulative hazard rate

The cumulative hazard rate (H(x)) assesses the total amount of accumulated risk the component has been facing from the starting of time (t_0) to the present time (t_P) [14].

$$H(x) = \int_0^{t_P} h(x)dx$$

IV. Results and Discussion

A total of 31 data observations regarding the failures of aircraft glass were included in the study. A frequency distribution table is formulated for the observed data that is displayed in table 1. The Sturge's rule is applied for ascertaining the number of class interval required to make a frequency distribution table. As per the rule, six class intervals are made with a width of five.

Table 1: Frequency Distribution for Time to Failures of glass in the aircraft

Time to Failures	No. of Failures
18.83-23.83	5
23.83-28.83	10
28.83-33.83	5
33.83-38.83	7
38.83-43.83	1
43.83-48.83	3

From table 1, it is observed that the majority of time to failures of aircraft glass i.e., 10 occurred during the time interval of 23.83-28.83 months which is followed by, seven failures occurred during the period of 33.83-38.83 months.

The probability density curve clearly reveals that the time to failures data follows positively skewed distribution (Figure 1). Hence, we expect that the data follows one of the lifetime distributions such as exponential, gamma, Weibull and lognormal distribution.

Initially, the empirical data has been tested with exponential survival model. The probability plot clearly depicts that the observed data does not fit with the exponential distribution (Figure 2). Hence, the data have been further verified with other lifetime distributions viz. gamma, Weibull and lognormal distribution. Kolmogorov-Smirnov (KS) test clearly unveils that the empirical data fits with Weibull, gamma and lognormal distribution (Table 2). Since the P-values are more than 0.05 at 5 percent level of significance, the null hypothesis is accepted (H0: The empirical data follows the theoretical distribution). In addition, KS-D statistic reveals that the maximal absolute difference between empirical cumulative relative frequency and theoretical cumulative frequency is very less. The maximal absolute differences for Weibull, Gamma and Lognormal distributions are 0.152, 0.143 and 0.124 respectively.

Table 2. Kolmogorov Smirnov Test

Proposed Model	KS –Test D-statistic	P-value
Weibull	0.152	0.427
Gamma	0.143	0.506
Lognormal	0.124	0.684

The Maximum likelihood estimation method is used to estimate the parameters of Weibull, gamma, and lognormal distribution. The estimated parameters are displayed in table 3. The estimated values are substituted in equation 2, 3 and 4 in order to construct the Weibull, gamma, and lognormal survival models respectively for time to failure of aircraft glass.

Table 3. Parameters Estimation by MLE

Distribution	Parameters – Point estimate (SE)	Loglikelihood
Weibull	Shape = 4.63 (0.63) & Scale = 33.67 (1.38)	-105.49
Gamma	Shape = 18.9 (4.76) & Rate = 0.61(0.16)	-104.12
Lognormal	Meanlog = 3.40 (0.04) & SD log = 0.23 (0.03)	-104.01

Therefore, the Weibull survival model for the scale parameter 33.67 and shape parameter 4.63 is as follows:

$$f(s, 33.67, 4.63) = \frac{4.63}{33.67}\left(\frac{s}{33.67}\right)^{3.63} e^{-\left(\frac{s}{33.67}\right)^{4.63}}$$

The gamma survival model (shape=18.9, rate=0.61) for the time to failure of aircraft glass is as given below

$$f(s;18.90,0.61) = \frac{1}{\Gamma 18.9} 0.61^{18.9} s^{17.9} e^{-0.61s}.$$

Lognormal survival model for the time to failure of aircraft glass is as follows:

$$f(s) = \frac{1}{s}\frac{1}{(0.23)\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\ln(s)-3.40}{0.23}\right]^2}$$

The Goodness-of-fit of gamma, Weibull and lognormal distribution is given in table 4. AIC and BIC values for the lognormal distribution are less than the values of Weibull and gamma distribution. The relative distance between the likelihood function of the empirical data and the likelihood function of the lognormal model is very less. Hence, the lognormal survival model is the best-fitted model for the time to failure of aircraft glass (Figure 3).

Table 4. Goodness-of- Fit

Distribution	AIC	BIC
Weibull	214.98	217.85
Gamma	212.23	215.09
Lognormal	212.02	214.89

i.e., $f(s) = \frac{1}{s}\frac{1}{(0.23)\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\ln(s)-3.40}{0.23}\right]^2}$ is the best-fitted survival model for our time to failure data. The cumulative distribution function of lognormal model F(x) is as follows,

$F(x) = P(X \le x) = \frac{1}{2} + \frac{1}{2} erf\left[\frac{\sqrt{2}(\ln(s)-\mu)}{2\sigma}\right]$ where $x \ge 0$ [15].

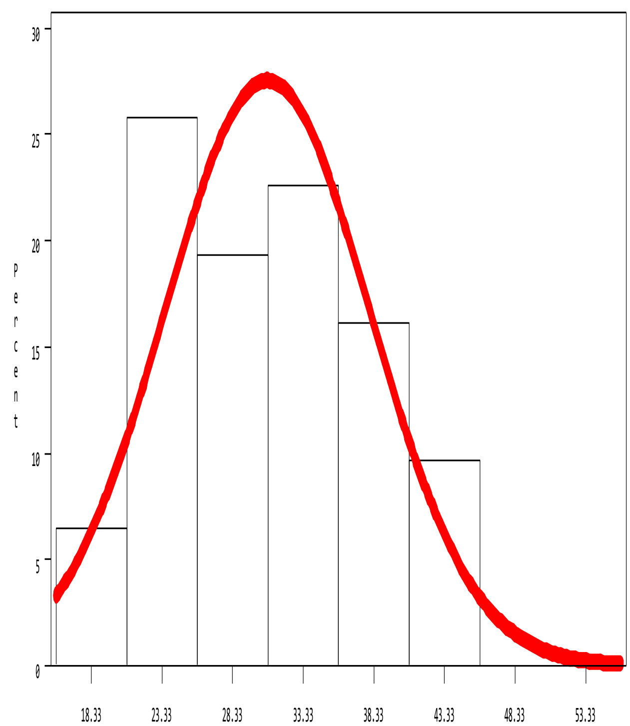

Figure 1. Nature of distribution

The survival function can be denoted by S(x), where S(x) = 1- F(x). Therefore

$S(x) = \frac{1}{2} - \frac{1}{2} erf\left[\frac{\sqrt{2}(\ln(s)-\mu)}{2\sigma}\right]$ and the hazard rate can be denoted by h(x), where h(x) = f(x)/R(x).

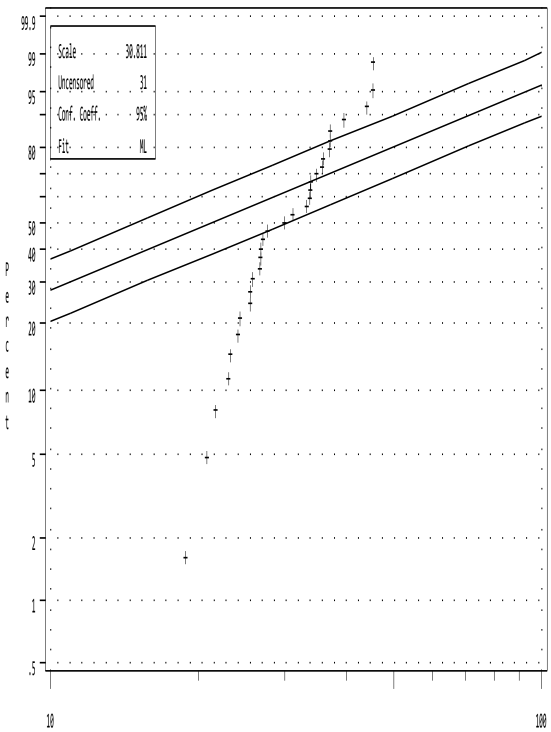

Figure 2. Probability plot of exponential

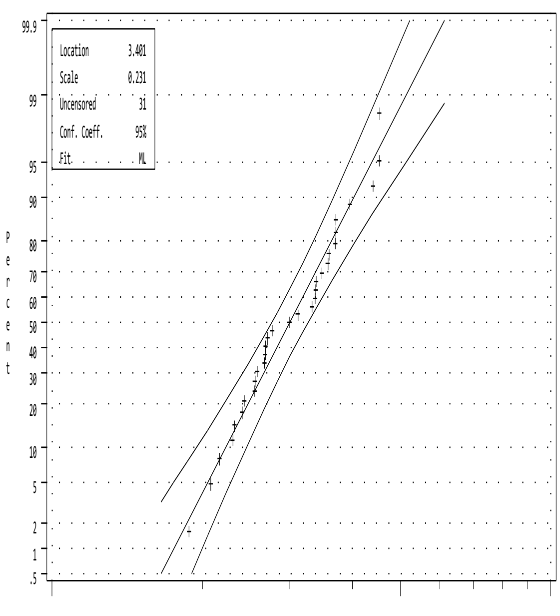

Figure 3. Probability plot of lognormal

It can be observed from the hazard rates such as $h(x_1)$, $h(x_2),\ldots,h(X_6)$ are gradually decreased when the time increases (Table 5). The hazard rate of aircraft glass is maximum during the period of 18.83-23.83 months (i.e., 14% chance of failure) followed by the period 23.83-28.83 months (12% chance of failure). It could conclude that the aircraft glass has a decreasing failure rate (DFR) over time. Thus, the risk to the aircraft due to glass failure can be avoided by taking early preventive measures.

Table 5. Reliability measurements such as reliability rate and hazard rate

Lifetime (S)	f(x)	F(x)	S(x)	h(x)
18.83-23.83	0.00061	0.9958	0.004229	0.1442
23.83-28.83	0.00027	0.9978	0.002158	0.1251
28.83-33.83	0.00013	0.9988	0.001195	0.1088
33.83-38.83	0.000071	0.9993	0.000704	0.1005
38.83-43.83	0.000039	0.9996	0.000436	0.0915
43.83-48.83	0.000024	0.9997	0.000281	0.0844

The aircraft glass is a non-repairable component. Hence, Mean time to failure (MTTF) is the best reliability measures than Mean time between failures (MTBF). The expected lifetime of aircraft glass is 31 months based on MTTF. Cumulative hazard rate H(x) is as follows:

$$H(x) = \int_0^{t_P} h(x)dx =$$

$$\int_0^{18.83} h_0(x)dx + \int_{18.83}^{23.83} h_1(x)dx + \ldots + \int_{43.83}^{48.83} h_6(x)dx$$

It estimates the total amount of risk to aircraft glass failure until 48.83 months is 65 percent.

V. Conclusion

The Survival of aircraft glass was studied based on their time to failures. On the basis of the KS test statistic, the data was fitting with the following theoretical model namely, Weibull, Gamma, and Lognormal distribution. Among the three theoretical models, the aircraft glass failure data were best fitted with the lognormal model. The hazard rate of aircraft glass is decreased when the duration of a lifetime is increased. The expected time to failures of aircraft glass is 31 months. The glass has a 65 percent chance to attain failure after 48.83 months from the date of installation.

VI. Future Work

We can enhance the research towards the applications of stochastic modeling and failure prediction of aircraft glass. We hope to discuss the same in further papers.

ACKNOWLEDGMENT

Authors would like to thank Dr. Sudhir Ben Nelson and Dr. C. Deiveegan, Faculties of Community Medicine, Velammal Medical College, Madurai for proofreading the content of the research article.

REFERENCES

[1] David J Smith, "*Reliability, Maintainability and Risk*", Butterworth-Heinemann Ltd 1993 publisher, Massachusetts, pp.**4-5**, **1993**.

[2] R.A. Bakoban, Hanaa H. Abu-Zinadah, "*The beta generalized inverted exponential distribution with real data applications*" REVSTAT – Statistical Journal, Vol.**15**, No.**1**, pp.**65–88**, **2017.**

[3] Bianca Schroeder, Garth A. Gibson, "*A large-scale study of failures in high-performance computing systems*" In the Proceedings of the International Conference on Dependable Systems and Networks (DSN2006), Philadelphia, PA, USA, June **25-28, 2006**.

[4] Maydeu-Olivares, C.G. Forero, "*Goodness-of-Fit Testing*", International Encyclopedia of Education, vol.**7**, pp. **190-196**, **2010**.

[5] Khaled Haddad, Ataur Rahman, "*Selection of the best fit flood frequency distribution and parameterestimation procedure: a case study for Tasmania in Australia*" Springer-Verlag, Vol.**25**, pp.**415-428**, **2010**.

[6] J. G. Elerath, "*AFR: problems of definition, calculation and measurement in a commercial environment*" In the Proceedings of the 2000 international symposium on product quality and integrity, Los Angeles, CA, **USA**, pp. **71-76, 2000**.

[7] Milena Krasich, "*How to Estimate and Use MTTF/MTBF Would the Real MTBF Please Stand Up?*", In the Proceedings of the 2009 Annual Reliability and Maintainability Symposium, Fort Worth, TX, **USA**, pp.**353-359**, **2009**.

[8] Dong Shang Chang, "*Optimal burn-in decision for products with an unimodal failure rate function*", European Journal of Operational Research, Vol.**126**, Issue.**3**, pp.**534-540**, **2000**.

[9] Nezih Yigitbasi et al., "*AnalysisandModelingofTime-CorrelatedFailuresinLarge-ScaleDistributed Systems*", In the Proceedings of the 2010 11th IEEE/ACM International Conference on Grid Computing, Brussels, **Belgium**, pp.**65-72, 2010**.

[10] Akaike H, "*A new look at the statistical model identification. Automatic Control 19*" in IEEE Transaction on Automatic control, Vol.**19**, No.**6**, pp.**716-723**, **1974**.

[11] Emad A. Mohammed, Christopher Naugler, Behrouz H. Far "*Emerging Business Intelligence Framework for a Clinical LaboratoryThrough Big DataAnalytics*" Elsevier Publisher, pp.**577-602**, **2015**.

[12] Stone M, "*Comments on model selection criteria of Akaike and Schwarz*", Journal of the Royal Statistical Society. Series B (Methodological), Vol.**41**, No**.2**, pp.**276-278**, **1979**.

[13] Michael Todinov, "*Reliability and risk models: setting reliability requirements*", John Wiley & Sons publisher, pp.**33-34**, **2005**.

[14] Jerald F. Lawless, "*Statistical models and methods for lifetime data*", A John Wiley & Sons, Inc., publication, **USA**, pp.**9-10**, **2003**.

[15] L. E. Wagner, D. Ding, "*Representing aggregate size distributions as modified lognormal distributions*", American society of agricultural and biological engineers, Vol.**37,** No**.3**, pp.**815-821**, **1994**.

AUTHORS PROFILE

Mr. J. Vijay Anto is doing Ph.D. in Statistics under the guidance of Dr. Lilly George at St. Joseph's College, Tiruchirappalli, India. His research areas of interest are reliability theory, Stochastic modeling, acceptance sampling, and Bio-statistics. He is currently working as an Assistant Professor cum Statistician in Community Medicine department at Velammal Medical College Hospital and Research Institute, Madurai. He has published three research papers in the field of statistics and clinical data. He has presented research papers in the national and international conference.

Dr. Lilly George is working as an Assistant professor and head in the department of statistics, St. Joseph's College, Tiruchirappalli. His research areas of interest are acceptance sampling and stochastic modeling. She has published more than 10 research papers in international and national journals. She has presented 13 research papers in the national and international conference. She is a guide for Ph.D. and M.Phil students in Bharathidasan University.

