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Weighted Ratio-cum-Product Estimator for Finite Population Mean

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Abstract– We have, in this paper, proposed a new ratio-cum-product estimator of finite population mean using information on two auxiliary variates. The bias and mean squared error of the proposed estimator, up to the first order of approximation have been derived. The proposed estimator, under optimal weights, is shown to be superior to the competing estimators. Empirical investigations have been carried out in support of the theoretical findings.

***Keywords*:** Auxiliary variable, Ratio-cum-product estimator

I. INTRODUCTION

In survey sampling, a considerable attention is given for improving upon the usual unbiased estimator by the use of supplementary variable(s) in sampling theory and practice for estimation of population characteristics. The literature on survey sampling describes agreement of various techniques for utilizing information on auxiliary variate by ratio, product and regression methods of estimation to estimate the population parameters, out of which ratio and product are being easily obtainable and are more prevalent in practice.

Over the years, various estimators have been developed in simple random sampling to estimate the population characters using auxiliary information. Some noteworthy contributions in this area have been made by several authors including Hansen(1953), Olikin(1958), Goodman(1960), Koop(1964), Murthy(1964), Singh(1965), Singh(1966), Singh (1967) and many others.

In the present paper, we have studied some of the existing estimators for the population mean of a study variate by utilizing information on two auxiliary variates of which one is positively correlated with the study variate, while the other is negatively correlated. The proposed weighted ratio-cum-product estimator performs better than Singh's estimator, usual ratio and product estimators and simple unbiased estimatorunder practical conditions.

We consider a finite population of size N, arbitrarily labelled 1, 2....N. Let Y and (X_1, X_2) be the study and auxiliary variates, respectively, where X_1 is positively correlated with Y, while X_2 is negatively correlated with Y. Assuming that the population means $\overline{X}_1$ and $\overline{X}_2$ are known, a sample of size n (with n < N) is drawn from the population size N using simple random sampling without replacement (SRSWOR) scheme to estimate the population mean $\overline{Y}=\frac{\sum_{i=1}^{N} Y_i}{N}$ of the study variate.

The whole paper is composed of six sections followed by references and authors profile. Section-I contains noteworthy contributions by several authors over the years to estimate population characters using auxiliary information. Section-II describes briefly some existing estimators by different authors. In section-III, the expressions of bias and mean square error of the proposed estimator have been derived, and also expression for optimum weight has been arrived at. Section-IV discusses efficiency comparisons of the proposed estimator with respect to competing estimators. Section-V deals with the empirical study to justify the supremacy of theoretical findings. Section-VI presents the summary and future directions.

II. REVIEW OF LITERATURE OF EXISTING ESTIMATORS

The variance of simple unbiased estimator is given by

$$V(\bar{y}) = \theta \bar{Y}^2 C_y^2. \quad (1)$$

The usual ratio and product estimators of $\bar{Y}$ are, respectively,

$$\bar{y}_R = \frac{\bar{y}}{\bar{x}} \bar{X} \quad (2)$$

and $$\bar{y}_P = \frac{\bar{y}}{\bar{X}} \bar{x}, \quad (3)$$

whose biases and mean square errors, up to first degree of approximation, are, respectively,

$$B(\bar{y}_R) = \theta \bar{Y}(1 - C) C_x^2, \quad (4)$$

$$B(\bar{y}_P) = \theta \bar{Y} C C_x^2, \quad (5)$$

$$MSE(\bar{y}_R) = \theta \bar{Y}^2 \{C_y^2 + (1 - 2C) C_x^2\} \quad (6)$$

and

$$MSE(\bar{y}_P) = \theta \bar{Y}^2 \{C_y^2 + (1 + 2C) C_x^2\}, \quad (7)$$

where C_x and C_y are the coefficients of variations of x and y, respectively, $C = \rho \frac{C_y}{C_x}$ and $f = \frac{n}{N}$.

M.P. Singh (1967) has suggested ratio-cum-product estimator, which is given by

$$\bar{y}_{RP} = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right)\left(\frac{\bar{x}_2}{\bar{X}_2}\right) \quad (8)$$

The bias and mean square error up to first degree of approximation are, respectively,

$$B(\bar{y}_{RP}) = \theta \bar{Y}\,(C_1^2 + C_{02} - C_{01} - C_{12}) \quad (9)$$ and

$$MSE(\bar{y}_{RP}) = \theta \bar{Y}^2\,(C_0^2 + C_1^2 + C_2^2 - 2C_{01} - 2C_{12} + 2C_{02}). \quad (10)$$

III. PROPOSED WEIGHTED RATIO-CUM-PRODUCT ESTIMATOR

We propose a weighted ratio-cum-product estimator for estimating the population mean $\bar{Y}$, which is given by

$$\bar{y}_{RP}^* = \bar{y}\left(W_1 \frac{\bar{X}_1}{\bar{x}_1} + W_2 \frac{\bar{x}_2}{\bar{X}_2}\right) \quad (11)$$

Let $\varepsilon_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$

$\Rightarrow \bar{y} = \bar{Y}(1+\varepsilon_0)$,

$$\varepsilon_1 = \frac{\bar{x}_1 - \bar{X}_1}{\bar{X}_1}$$

$\Rightarrow \bar{x}_1 = \bar{X}_1(1 + \varepsilon_1)$ and

$$\varepsilon_2 = \frac{\bar{x}_2 - \bar{X}_2}{\bar{X}_2}$$

$\Rightarrow \bar{x}_2 = \bar{X}_2(1 + \varepsilon_2)$,

Now, $E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = 0$,

$E(\varepsilon_0^2) = \theta C_y^2$, $E(\varepsilon_1^2) = \theta C_{x_1}^2$, $E(\varepsilon_2^2) = \theta C_{x_2}^2$

$E(\varepsilon_0 \varepsilon_1) = \theta \rho C_y C_{x_1}$, $E(\varepsilon_1 \varepsilon_2) = \theta \rho C_{x_1} C_{x_2}$,

$E(\varepsilon_0 \varepsilon_2) = \theta \rho C_y C_{x_2}$, where $\theta = \frac{1-f}{n}$

Putting the values of $\bar{y}$, $\bar{x}_1$ and $\bar{x}_2$ in the expression-(11), we get

$$\bar{y}_{RP}^* = \bar{Y}(1+\varepsilon_0)\left(W_1 \frac{\bar{X}_1}{\bar{X}_1(1+\varepsilon_1)} + W_2 \frac{\bar{X}_2(1+\varepsilon_2)}{\bar{X}_2}\right)$$

Upon simplification, we find that

$$\bar{y}_{RP}^* = \bar{Y}(1 - W_1\varepsilon_1 + W_2\varepsilon_2 + W_1\varepsilon_1^2 + \varepsilon_0 - W_1\varepsilon_0\varepsilon_1 + W_2\varepsilon_0\varepsilon_2) \quad (12)$$

The bias of the proposed estimator, up to the first degree of approximation, is

$$B(\bar{y}_{RP}^*) = E(\bar{y}_{RP}^*) - \bar{Y}$$

$$= \bar{Y} E(-W_1\varepsilon_1 + W_2\varepsilon_2 + W_1\varepsilon_1^2 + \varepsilon_0 - W_1\varepsilon_0\varepsilon_1 + W_2\varepsilon_0\varepsilon_2)$$

$$= \theta \bar{Y}\,(W_1 C_1^2 - W_1 C_{01} + W_2 C_{02}). \quad (13)$$

Similarly, the mean square error, to the first degree of approximation, is

$$M(\bar{y}_{RP}^*) = E[\bar{y}_{RP}^* - \bar{Y}]^2$$

$$= \bar{Y} E[-W_1\varepsilon_1 + W_2\varepsilon_2 + W_1\varepsilon_1^2 + \varepsilon_0 - W_1\varepsilon_0\varepsilon_1 + W_2\varepsilon_0\varepsilon_2]^2$$

$=\theta\overline{Y}^2$
$(W_1^2C_1^2 + W_2^2C_2^2 + C_0^2 - 2W_1C_{01} + 2W_2C_{02} - 2W_1W_2C_{12})$. (14)

With a view to determining the most suitable value of W_1, (and thusW_2), we proceed to minimize the mean square error subject to the variation inW_1, implying thereby that

$$\frac{\partial MSE(\overline{y}_{RP}^*)}{\partial W_1}=0$$

$$\Rightarrow\theta\overline{Y}^2\,(2W_1C_1^2 - 2(1-W_1)C_2^2 - 2C_{01} - 2C_{02} - 2C_{12} + 4W_1C_{12}) = 0$$

$$\Rightarrow W_1=\frac{C_2^2+C_{01}+C_{02}+C_{12}}{C_1^2+C_2^2+2C_{12}}=W_{1opt}=1-W_{2opt}, \text{ say.} \quad (15)$$

IV. EFFICIENCY COMPARISON

On comparison of (14) under optimum weights, with (10), we get

$M(\overline{y}_{RP}^*) - M(\overline{y}_{RP}) < 0$

$\Rightarrow \theta\overline{Y}^2(W_1^2C_1^2 + W_2^2C_2^2 + C_0^2 - 2W_1C_{01} + 2W_2C_{02} - 2W_1W_2C_{12}) - \theta\overline{Y}^2(C_0^2 + C_1^2 + C_2^2 - 2C_{01} - 2C_{12} + 2C_{02}) < 0$

$\Rightarrow \theta\overline{Y}^2(W_1^2C_1^2 + W_2^2C_2^2 + C_0^2 - 2W_1C_{01} + 2W_2C_{02} - 2W_1W_2C_{12} - C_0^2 - C_1^2 - C_2^2 + 2C_{01} + 2C_{12} - 2C_{02}) < 0$

$\Rightarrow W_1^2C_1^2 + W_2^2C_2^2 - 2W_1C_{01} + 2W_2C_{02} - 2W_1W_2C_{12} - C_1^2 - C_2^2 + 2C_{01} + 2C_{12} - 2C_{02} < 0$

$\Rightarrow W_1^2C_1^2 + (1-W_1)^2C_2^2 - 2W_1C_{01} + 2(1-W_1)C_{02} - 2W_1(1-W_1)C_{12} - C_1^2 - C_2^2 + 2C_{01} + 2C_{12} - 2C_{02} < 0$

$\Rightarrow W_1^2C_1^2 + C_2^2 + W_1^2C_2^2 - 2W_1C_2^2 - 2W_1C_{01} + 2C_{02} - 2W_1C_{02} - 2W_1C_{12} + 2W_1^2C_{12} - C_1^2 - C_2^2 + 2C_{01} + 2C_{12} - 2C_{02} < 0$

$\Rightarrow W_1^2C_1^2 + W_1^2C_2^2 - 2W_1C_2^2 - 2W_1C_{01} - 2W_1C_{02} - 2W_1C_{12} + 2W_1^2C_{12} - C_1^2 + 2C_{01} + 2C_{12} < 0$

$\Rightarrow W_1^2C_1^2 + W_1^2C_2^2 - 2W_1C_2^2 - 2W_1C_{01} - 2W_1C_{02} - 2W_1C_{12} + 2W_1^2C_{12} < C_1^2 - 2C_{01} - 2C_{12}$

$\Rightarrow W_1^2(C_1^2 + C_2^2 + 2C_{12}) - 2W_1(C_2^2 + C_{01} + C_{02} + C_{12}) < C_1^2 - 2C_{01} - 2C_{12}$

$\Rightarrow \frac{(C_2^2+C_{01}+C_{02}+C_{12})^2}{(C_1^2+C_2^2+2C_{12})^2}(C_1^2 + C_2^2 + 2C_{12}) - 2(\frac{C_2^2+C_{01}+C_{02}+C_{12}}{C_1^2+C_2^2+2C_{12}})(C_2^2 + C_{01} + C_{02} + C_{12})$

$< C_1^2 - 2C_{01} - 2C_{12}$

$\Rightarrow \frac{(C_2^2+C_{01}+C_{02}+C_{12})^2}{(C_1^2+C_2^2+2C_{12})} - 2\frac{(C_2^2+C_{01}+C_{02}+C_{12})}{(C_1^2+C_2^2+2C_{12})}(C_2^2 + C_{01} + C_{02} + C_{12}) < C_1^2 - 2C_{01} - 2C_{12}$

$\Rightarrow \frac{(C_2^2+C_{01}+C_{02}+C_{12})^2 - 2(C_2^2+C_{01}+C_{02}+C_{12})(C_2^2+C_{01}+C_{02}+C_{12})}{C_1^2+C_2^2+2C_{12}}$

$< C_1^2 - 2C_{01} - 2C_{12}$

$\Rightarrow (C_2^2 + C_{01} + C_{02} + C_{12})^2 - 2(C_2^2 + C_{01} + C_{02} + C_{12})(C_2^2 + C_{01} + C_{02} + C_{12}) < (C_1^2 + C_2^2 + 2C_{12})(C_1^2 - 2C_{01} - 2C_{12})$

$\Rightarrow C_2^4 + C_{01}^2 + C_{02}^2 + C_{12}^2 + 2C_2^2C_{01} + 2C_2^202 + 2C_2^2C_{12} + 2C_{01}C_{02} + 2C_{01}C_{12} + 2C_{02}C_{12} - 2C_2^4 - 2C_2^2C_{01} - 2C_2^2C_{02} - 2C_2^2C_{12} - 2C_{01}C_2^2 - 2C_{01}^2 - 2C_{01}C_{02} - 2C_{01}C_{12} - 2C_{02}C_2^2 - 2C_{02}C_{01} - 2C_{02}^2 - 2C_{02}C_{12} - 2C_{12}C_2^2 - 2C_{12}C_{01} - 2C_{12}C_{02} - 2C_{12}^2 < C_1^4 - 2C_1^2C_{01} - 2C_1^2C_{12} + C_1^2C_2^2 - 2C_{01}C_2^2 - 2C_2^2C_{12} + 2C_1^2C_{12} - 4C_{01}C_{12} - 4C_{12}^2$

$\Rightarrow -C_2^4 - C_{01}^2 - C_{02}^2 - C_{12}^2 - 2C_{02}C_2^2 - 2C_{02}C_{01} - 2C_{12}C_2^2 - 2C_{12}C_{01} - 2C_{12}C_{02} < C_1^4 - 2C_1^2C_{01} - 2C_1^2C_{12} + C_1^2C_2^2 - 2C_2^2C_{12} + 2C_1^2C_{12} - 4C_{01}C_{12} - 4C_{12}^2$

$\Rightarrow -C_2^4 - C_{01}^2 - C_{02}^2 - C_{12}^2 - 2C_{02}C_2^2 - 2C_{02}C_{01} - 2C_{12}C_{01} - 2C_{12}C_{02} < C_1^4 - 2C_1^2C_{01} - 2C_1^2C_{12} + C_1^2C_2^2 + 2C_1^2C_{12} - 4C_{01}C_{12} - 4C_{12}^2$

$\Rightarrow C_2^4 + C_{01}^2 + C_{02}^2 + C_{12}^2 + 2C_{02}C_2^2 + 2C_{02}C_{01} + 2C_{12}C_{01} + 2C_{12}C_{01} > -C_1^4 + 2C_1^2C_{01} + 2C_1^2C_{12} - C_1^2C_2^2 - 2C_1^2C_{12} + 4C_{01}C_{12} + 4C_{12}^2$

$\Rightarrow C_2^4 + C_{01}^2 + C_{02}^2 + C_{12}^2 + 2C_{02}C_2^2 + 2C_{02}C_{01} + 2C_{12}C_{01} + 2C_{12}C_{01} + C_1^4 + C_1^2C_2^2 + 2C_1^2C_{12} > 2C_1^2C_{01} + 2C_1^2C_{12} + 4C_{01}C_{12} + 4C_{12}^2$

$\Rightarrow C_2^4 + C_{01}^2 + C_{02}^2 + C_{12}^2 + 2C_{02}C_2^2 + 2C_{02}C_{01} + C_1^4 + C_1^2C_2^2 > 2C_1^2C_{01} + 4C_{12}^2$

$\Rightarrow (C_{01} + C_{02})^2 + C_1^2 + C_2^4 + C_{12}^2 + C_1^2C_2^2 + 2C_{02}C_2^2 > 2C_1^2C_{01} + 4C_{12}^2$

$\Rightarrow (C_{01} + C_{02})^2 + C_1^2 + C_2^4 + C_1^2C_2^2 + 2C_{02}C_2^2 > 2C_1^2C_{01} + 3C_{12}^2$

$$\Rightarrow (C_{01}+C_{02})^2 + C_1^2(1+C_2^2) + C_2^2(1+2C_{02}) > 2C_1^2C_{01} + 3C_{12}^2 \quad (16)$$

Thus, the proposed weighted ratio-cum-product estimator fares better than the usual ratio-cum-product estimator if the above inequality holds good.

V. EMPIRICAL STUDY

Example-1

Source: Singh (1969)

Y: Number of females employed

X_1: Number of females in service

X_2:Number of educated females

Table 5.1: Parameters of populations

Sl. No	Parameters	Values of the Parameters
1	N	61
2	$\overline{Y}$	7.46
3	$\overline{X}_1$	5.31
4	$\overline{X}_2$	179.00
5	ρ_{YX_1}	0.7737
6	ρ_{YX_2}	−0.2070
7	$\rho_{X_1X_2}$	−0.0033
8	C_y	0.7103
9	C_{X_1}	0.7574
10	C_{X_2}	0.2515
11	C_y^2	0.5046
12	$C_{X_1}^2$	0.5737
13	$C_{X_2}^2$	0.0633
14	C_{01}	0.4162
15	C_{02}	0.0369
16	C_{12}	-0.0006
Let's further assume that the sample size is 20.		

W_{1opt}=$\frac{C_2^2+C_{01}+C_{02}+C_{12}}{C_1^2+C_2^2+2C_{12}}$

=0.695169954

W_{2opt}=0.3048300646 as $W_{1opt}+W_{2opt}$=1

The biases and MSEs of the competing estimators have been computed and presented in Table 5.2

Table 5.2: Bias and MSE of the competing estimators

$Sl. No.$	$Competing\ Estimator$	$\lvert Bias\rvert/\theta\overline{Y}$	$MSE/\theta\overline{Y}^2$
1	$\overline{y}$	0.0000	0.5045
2	$\overline{y}_R$	0.1574	0.2457
3	$\overline{y}_P$	−0.0369	0.4938
4	$\overline{y}_{RP}$	0.1211	0.2363
5	$\overline{y}_{RP}^*$	0.4131	0.1866

Table 5.3PRE of various estimators w.r.t $\overline{y}$

$Sl. No.$	$Competing Estimator$	Percentage relative efficiency (PRE)
1	$\overline{y}$	100
2	$\overline{y}_R$	205.32
3	$\overline{y}_p$	102.16
4	$\overline{y}_{RP}$	213.49
5	$\overline{y}_{RP}^*$	270.30

From Tables 5.2 & 5.3, it is clearly seen that the newly proposed estimator $\overline{y}_{RP}^*$ comes out to be the most efficient among all the competing estimators.

VI. CONCLUSION

The proposed weighted ratio-cum-product estimator has been shown to fare better than its competing estimators, e.g., $\overline{y}$, $\overline{y}_R$, $\overline{y}_P$, $\overline{y}_{RP}$ under practicalconditions. The theoretical developments have been numerically established. New ratio-cum-product estimators can be designed using different weighting systems such as weighted geometric mean and weighted harmonic mean and their performance can be evaluated vis-a-vis the existing ratio-cum-product estimators.

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