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On τ* -Generalized b-closed sets in Topological Spaces

C. Aruna[1*], R. Selvi[2]

[1]Research Centre of Mathematics, Sri Parasakthi College for Women, Courtallam-627802, India
[2] Sri Parasakthi College for Women, Courtallam-627802, India
(Affiliated to Manonmaniam Sundaranar University, Abishekapati,Tirunelveli-627012,India)

**Corresponding Author: arunarobert89@gmail.com (Research Scholar* Reg.No.11504*)*

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Abstract- In this paper we introduce a new class of sets called τ* generalized b closed sets in topological spaces (briefly τ* gb closed set). Also we investigate the characteristics of τ* generalized b closed sets and studied some of their properties. Further we introduced τ* generalized b neighbourhoods in topological spaces by using the notion of τ* generalized b open sets and study some of their properties.

Keywords- Topological Spaces and their properties

I. INTRODUCTION

In 1970, Levine introduced the concept of generalized closed set and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms. Later in 1996 D. Andrijevic[1] gave a new type of generalized closed sets in topological space called b closed sets. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, generalization of continuity and covering properties. A.A.Omar and M.S.M. Noorani[5] made an analytical study and result the concepts of generalized b-closed sets in topological spaces. M.Caldas and S.Jafari[4] discussed some applications of b-open sets in topological spaces in 2007. Using generalized closed sets, Dunham[2] introduced the concept of the closure operator cl* and a new topology τ* and studied some of their properties. A.Pushpalatha, S.Eswaran and P.RajaRubi[6] introduced a new class of sets called τ* generalized closed sets and studied some of their properties. The aim of this paper is to continue the study of τ* Generalized b-closed sets. The notion of τ* Generalized b-closed sets and its different characterization are given in this paper. Throughout this paper (X, $\tau*_1$) and (Y, $\tau*_2$) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let A$\subseteq$ X, the closure of A and interior of A will be denoted by cl(A) and int(A) respectively, union of all b-open sets of X contained in A is called b-interior of

A and it is denoted by b int(A), the intersection of all b-closed sets of X containing A is called b-closure of A and it is denoted by bcl(A).

In this paper, the works are split into 5 sections. Section 1 contains Introduction of τ* generalized b closed sets, Section 2 contains the basic definitions needed for τ* generalized b closed sets, section 3 contains τ* generalized b closed sets and their examples and theorems, section 4 contains characteristics of τ* generalized b closed sets, section 5 contains τ* generalized b open sets and τ* generalized b neighbourhoods and section 6 contains conclusion.

II. PRELIMINARIES

In this section, we recall the following definitions.

Definition 2.1: Let a subset A of a topological space (X, τ) is called a pre-open set [7] if A$\subseteq$ int(cl(A)).

Definition 2.2: Let a subset A of a topological space (X, τ) is called a semi-open set [8] if A$\subseteq$ cl(int (A)).

Definition 2.3: Let a subset A of a topological space (X, τ) is called a α-open set [9] if A$\subseteq$ intl(cl(int (A))).

Definition 2.4: Let a subset A of a topological space (X, τ) is called a b-open set [1] if A$\subseteq$ cl(int (A)) ∪ int(cl(A)).

Definition 2.5: Let a subset A of a topological space (X, τ) is called a generalized closed set (briefly g-closed) [3] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

Definition 2.6: Let a subset A of a topological space (X, τ) is called a generalized α closed set (briefly gα-closed) [10] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α open in X.

Definition 2.7: Let a subset A of a topological space (X, τ) is called a generalized b closed set (briefly gb-closed) [5] if bcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

Definition 2.8 [2] For the subset A of a topological X, the generalized closure operator cl^* is defined by the intersection of all g-closed sets containing A.

Definition 2.9 [2] For the subset A of a topological X, the topology τ^* is defined by $\tau^* = \{G: cl^*(G^c) = G^c\}$.

III. τ*- GENERALIZED B-CLOSED SETS

In this section, we introduce τ* generalized b closed set and investigate some of their properties.

Definition 3.1: Let a subset A of a topological space (X, τ) is called a τ* generalized b closed set (briefly τ* gb closed) if bcl*(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ* open in X.

Definition 3.2: Let A be a subset of a space (X,τ*).

(i) The set ∩{F ⊂ X : A ⊂ F, F is τ* *gb* closed} is called the τ* *gb* closure of A and it is

denoted by τ* *gb* cl(A).

(ii) The set ∪{G ⊂ X : G ⊂ A, G is τ* *gb* open} is called the τ* *gb* interior of A and it is

denoted by τ* *gb* int(A).

Theorem 4.2.3

Every closed set is τ* gb closed.

Proof

Let A be a closed set and A ⊆ G, where G is τ^*open in X. Since A is closed, Cl(A) = A ⊆ G. But bcl*(A) ⊆ Cl(A) ⊆ G . Then bcl*(A) ⊆ G whenever A ⊆ G and G is τ^*open. Hence A is τ^* generalized b closed.

The converse of above theorem need not be true from the following example.

Example 4.2.4

Let X = {a,b,c}and τ = {X,∅,{a,b}}. The set {a,c} is τ* gb closed but not a closed set.

Theorem 4.2.5

Every b-closed set is τ* gb closed.

Proof

Suppose A is b closed set and A ⊆ G, where G is τ^*open in X. Since A is b closed, bcl(A) = A ⊆ G. But bcl*(A) ⊆ bcl(A) ⊆ G . Thus, we have bcl*(A) ⊆ G whenever A ⊆ G and G is τ^*open. Therefore A is τ^* generalized b closed.

The converse of above theorem need not be true from the following example

Example 4.2.6

Let X = {a,b,c} and τ = {X,∅,{a},{c}{a,c},}. The set {c} is τ^* gb closed but not a b-closed set.

Theorem 4.2.7

Every g-closed set is τ* gb closed set.

Proof

Let A be a generalized closed set. Let A ⊆ G, where G is τ^*open in X. Since A is generalized closed, cl(A) ⊆ G. But bcl*(A) ⊆ cl(A) ⊆ G.Then bcl*(A) ⊆ G whenever A ⊆ G and G is τ^*open. Thus A is τ^* generalized b closed.

The converse of above theorem need not be true from the following example.

Example 4.2.8

Let X = {a,b,c} and τ = {X,∅,{a},{b}{a,b},{b,c}}. The set {a,c} is *t** gb closed but not a g-closed set.

Theorem 4.2.9
Every generalized b closed set in X is τ^* generalized b closed
Proof :
Let A be a generalized b closed set. Let A ⊆ G, where G is τ^*open in X. Since A is generalized b closed, bcl (A) ⊆ G. But bcl*(A) ⊆ bcl(A) ⊆ G. Thus, bcl*(A) ⊆ G whenever A ⊆ G and G is τ^*open. Therefore A is τ^* generalized b closed.

IV. CHARACTERISTICS OF τ*-GB-CLOSED SETS

In this section, we investigate the characterization of τ*-generalized b-closed set.

Theorem 4.1: If A and B are τ*-gb closed sets in X then AUB is τ*-gb closed set in X.

Proof: Let A and B are τ*-gb closed sets in X and U be any τ*-open set containing A and B. Therefore bcl*(A) ⊆U, bcl*(B) ⊆U. Since A⊆U and B⊆U then A∪B⊆U. Hence bcl*(A∪B)= bcl*(A) ∪ bcl*(B) ⊆U. Therefore A∪B is τ*-gb closed sets in X.

Theorem 4.2: If a set A is τ*-gb closed sets iff bcl*(A) – A contains no non empty τ*- closed set.

Proof: Necessary: Let F be τ*-closed set in X such that F ⊆ bcl*(A) – A. Then A⊆ X –F. Since A is τ*-gb closed set and X –F is open then bcl*(A) ⊆ X –F. (i.e) F ⊆ X – bcl*(A). So F ⊆ (X – bcl*(A)) ∩ (bcl*(A) – A). Therefore F = ∅.

Sufficiency: Let us assume that bcl*(A) – A contains no non empty τ*-closed set. Let A⊆ ∪, ∪ is τ*-open. Suppose that bcl*(A) is not contained in U, bcl*(A) ∩U^C is a non empty τ*-closed set of bcl*(A) – A which is a contradiction. Therefore bcl*(A) ⊆ U. Hence A is τ*-gb closed.

Theorem 4.3: The intersection of any two subsets of τ*-gb closed sets in X is τ*-gb closed set in X.

Proof: Let A and B are any two subsets of τ*-gb closed sets. A ⊆ U, U is any τ*-open and B ⊆ U, U is τ*-open. Then bcl*(A) ⊆ U, bcl*(B) ⊆ U, therefore bcl*(A∩B) ⊆ U, U is τ*-open in X. Since A and B are τ*-gb closed set. Hence A∩B is τ*-gb closed set.

Theorem 4.4: If A is τ*-gb closed set in X and A⊆ B ⊆ bcl*(A), then B is τ*- gb closed set in X.

Proof: Since B ⊆ bcl*(A), we have bcl*(B) ⊆ bcl*(A), then bcl*(B) – B ⊆ bcl*(A) – A. By theorem 4.2, bcl*(A) – A contains no non empty closed set. Hence (bcl*(B) – B contains no non empty closed set. Therefore B is τ*-gb closed set in X.

Theorem 4.5: If A⊆ Y ⊆ X and suppose that A is τ*-gb closed set in X then A is τ*-gb closed set relative to Y.

Proof: Given that A⊆ Y ⊆ X and A is τ*-gb closed set in X. To prove that A is τ*-gb closed set relative to Y, let us assume that A⊆ Y ∩ U, where U is τ*-open in X. Since A is τ*-gb closed set, A⊆ U implies bcl*(A) ⊆ U. It follows that Y ∩ bcl*(A) ⊆ Y∩U. That is A is τ*-gb closed set relative to Y.

Theorem 4.6: If A is both τ*-open and τ*-gb closed set in X, then A is τ* closed set.

Proof: Since A is τ*-open and τ*-gb closed in X, bcl*(A) ⊆ U. But A⊆ bcl*(A). Therefore A = bcl*(A). Hence A is τ*-gb closed set.

V. τ* GENERALIZED B-OPEN SETS AND τ* GENERALIZED B-NEIGHBOURHOODS

In this section, we introduce τ*-generalized b-open sets (briefly τ*-gb open) and τ*-generalized b-neighbourhoods (briefly τ*-gb-nbd) in topological spaces by using the notions of τ*-gb open sets and study some of their properties.

Definition 5.1: A subset A of a topological space (X, τ) is called τ*-generalized b-open set (briefly τ* gb open) if A^C is τ*-gb closed in X. We denote the family of all τ* gb open set in X by τ* gb O(X).

Theorem 5.2: If A and B are τ*-gb open sets in a space X. Then A∩B is also τ*-gb-open set in X.

Proof: If A and B are τ*-gb open sets in a space X. Then A^C and B^C are τ*-gb-closed sets in a space X. By theorem 4.1, A^C U B^C is also τ*-gb-closed set in X (i.e.) A^C U B^C = $(A\cap B)^C$ is τ*-gb closed set in X. Therefore A∩ B is a τ*-gb open set in X.

Theorem 5.3: If int(B) ⊆ B ⊆ A and if A is τ*-gb open in X, then B is τ*-gb open in X.

Proof: Suppose that int(B) ⊆ B ⊆A and A is τ*-gb open in X then $A^C \subseteq B^C \subseteq cl(A^C)$. Since A^C is τ*-gb-closed in X, by theorem 5.2, B is τ*-gb open in X.

Definition 5.4: Let x be a point in a topological space X and let x ∈ X. A subset N of X is said to be a τ*-gb-nbd of x, iff there exists a τ*gb open set G such that x ∈ G ⊂ N.

Definition 5.5: A subset N of Space X is called a τ* gb nbd of A ⊂ X iff there exists a τ*-gb open set G such that A ∈ G ⊂ N.

Theorem 5.6: Every nbd N of x ∈ X is a τ* gb nbd of X.

Proof: Let N be a nbd of point x ∈ X. To prove that N is a τ* gb nbd of x. By definition of nbd, there exists an open set G such that x ∈ G ⊂ N. Hence N is a τ* gb nbd of x.

Theorem 5.7: If a subset N of a space X is τ*-gb open, then N is τ*-gb-nbd of each of its points.

Proof: Suppose N is τ*-gb open. Let x ∈ N. We claim that N is a τ*-gb-nbd of x. For N is a τ*-gb-open set such that x ∈ N ⊂ N. Since x is an arbitrary point of N, it follows that N is a τ*-gb-nbd of each of its points.

Theorem 5.8: Let X be a topological space. If F is τ*-gb-closed subset of X and x ∈ F^C, then prove that there exists a τ*-gb-nbd N of x such that N∩F = ∅.

Proof: Let F be τ*-gb-closed subset of X and x ∈ F^C. Then F^C is τ*-gb-open set of X. So by theorem 5.7, F^C contains a τ*-gb-nbd of each of its points. Hence there exists a τ*-gb-nbd N of x such that N ⊂ F^C (i.e.) N∩F = ∅.

Definition 5.9: Let x be a point in topological space X. The set of all τ*-gb-nbd of x is called the τ*-gb-nbd system at x and is denoted by τ*-gb N(x).

Theorem 5.10: Let a τ*-gb-nbd N of X be a topological space and each x ∈ X and τ*-gb-N(X, τ) be the collection of all τ*-gb-nbd of x. Then we have the following results.

(i) For all x ∈ X, τ*-gb N(x)≠ ∅
(ii) $N \in$ τ*-gb N(x) ⇒x ∈N

(iii) $N \in \tau$*-gb N(x), M⊃N⇒ $M \in \tau$*-gb N(x)
(iv) $N \in \tau$*-gb N(x), $M \in \tau$*-gb –N(x) ⇒N∩M ∈ τ*-gb N(x)
(v) $N \in \tau$*-gb N(x), ⇒ there exists M ∈ τ*-gb N(x) such that M ⊂ N and M ∈ τ*-gb N(y) for every y ∈M.

Proof: (i) Since X is τ*-gb-open set, it has a τ*-gb-nbd of every x ∈ X. Hence there exists at least one τ*-gb-nbd (namely-X) for each x ∈ X. Therefore τ*-gb –N(x)≠ ∅ for every x ∈ X.

(ii) If $N \in \tau$*-gb N(x), then N is τ*-gb-nbd of x. By definition of τ*-gb-nbd, x ∈N.
(iii) Let $N \in \tau$*-gb N(x) and M⊃N. Then there is a τ*-gb-open set G such that x ∈ G ⊂ N. Since N ⊂ M, x ∈ G ⊂ M and M is τ*-gb nbd of x. Hence M ∈ τ*-gb N(x).
(iv) Let $N \in \tau$*-gb N(x) and M ∈ τ*-gb N(x). Then by definition of τ*-gb-nbd, there exists a τ*-gb- open sets G_1 and G_2 such that x ∈ G_1 ⊂ N and x ∈ G_2 ⊂ M. Hence x ∈ $G_1 \cap G_2 \subset N \cap M$. Since G_1 and G_2 is τ*-gb open set,(being the intersection of two regular open sets) it follows from x ∈ $G_1 \cap G_2 \subset N \cap M$ that N∩M is a τ*-gb-nbd of x. Hence N∩M∈ τ*-gb N(x).
(v) Let $N \in \tau$*-gb N(x). Then there is a τ*-gb open set M such that x ∈ M ⊂ N. Since M is τ*-gb open set, it is τ*-gb-nbd of each of its points. Therefore M ∈ τ*-gb N(y) for every y ∈M.

Theorem 5.11: Let X be a nonempty set, and for each x ∈ X, let τ*-gb –N(x) be a nonempty collections of subsets of X satisfying following conditions.

(i) $N \in \tau$*-gb N(x) ⇒x ∈N
(ii) $N \in \tau$*-gb N(x)M ∈ τ*-gb N(x) ⇒x ∈N, $M \in \tau$*-gb –N(x) ⇒N∩M ∈ τ*-gb N(x)

Let τ consists of the empty set and all those non-empty subsets G of X having the property that x ∈G implies that there exists an $N \in \tau$*-gb N(x) such that x ∈ N ⊂G.
Then τ is a topology for X.

Proof: (i) $\emptyset \in \tau$ By definition. We have to show that x ∈ τ. Let x be any arbitrary element of X. Since τ*-gb –N(x) is non empty, there is an $N \in \tau$*-gb –N(x) and so x ∈N by (i). Since N is a subset of X, we have x ∈ N ⊂ X. Hence x ∈ τ.

(ii) Let $G_1 \in \tau$ and $G_2 \in \tau$. If x ∈ $G_1 \cap G_2$ then x ∈ G_1 and x ∈ G_2. Since $G_1 \in \tau$ and $G_2 \in \tau$. There exists $N \in \tau$*-gb N(x) and, $M \in \tau$*-gb N(x) such that x ∈ N ⊂ G_1 and x ∈ M ⊂ G_2. Then x ∈ N∩M ⊂ $G_1 \cap G_2$. But N∩M∈ τ*-gb N(x) by(2). Hence $G_1 \cap G_2 \in \tau$.

VI. CONCLUSION

My paper titled, 'τ* generalized b closed sets' was introduced and submitted examples and related theorems. Further, in future, I will work on τ* generalized b continuity τ* generalized b compact and connected spaces by using τ* generalized b closed sets and τ* generalized b open sets.

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