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Restricted linear models and their applications

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Abstract—In this paper we have considered the usual Gauss-Marcoff model with some restrictions on parameters. Usually Gauss-Marcoff model has been discussed by many authors. Further some of the applications have been applied to the real life data on kidney infection. Unified theory for both types of models has been developed.

Keywords—Linear models; Gauss-Markov model; Unified theory; kidney infection.

I. INTRODUCTION

A fair size literature on estimation of Gauss-Marcoff linear model is available. [see C. R. Rao [1], John P Hoffman [2]] and Harville [3] has used Gauss-Marcoff model for the estimation of random effects and Rao C.R. [4] obtained the estimators of the mixed effect model. Yau and McGilchirst [5] have used generalized linear mixed models for cluster survival data. Miller SJ [6] has utilized the method of least squares in the linear model. Cankaya et al. [7] have studied the estimation methods for parameters in multiple regression models. Cankaya [8] and Cankaya et al. [9] made comparative study of some estimation methods with and without outliers. Also Alvin C. Rencher [10] has discussed general idea about Linear models.
In section II we have obtained the estimators of Gauss-Marcoff model with linear restrictions on the parameters. Section III devoted for the Unified theory of estimating the parameters of the Gauss-Marcoff model under linear constraints. Section IV deals with the Application to real data of kidney infection.

II. LINEAR ESTIMATION WITH CONSTRAINTS ON $\underline{\beta}$

In Gauss-Marcoff model associated with frailty variable θ^*, the estimation of β, θ^* have been obtained under linear constraint on β.
Considering Gauss-Markoff linear model

$$\underline{Y} = X\underline{\beta} + \underline{e} \qquad (2.1)$$

where $\underline{Y}$ is nx1 vector, $(Y_1, Y_2, \dots Y_n)$, X is nxp matrix $(X_{ij}, i = 1,2,\dots,n; j = 1,2,\dots p)$ of known constants, $\underline{\beta}$ is px1 vector of parameters $\beta_1, \beta_2, \dots \beta_p$ and $\underline{e} = (e_1, e_2, \dots, e_p)'$ is error vector responsible for frailty model. $E(\underline{\epsilon}) = \underline{1}\theta^*$ and $E(\underline{\epsilon}\,\underline{\epsilon}') = h(\theta^*)I$, where $\underline{1}$ is the n-vector of one's and $h(\theta^*) = a_0 + a_1\theta^* + a_2\theta^{*2}$ with linear constraints on $\underline{\beta}$ as

$$H_0 : \Psi\underline{\beta} = \underline{\xi}$$

where Ψ is kxp matrix of known constants and $\underline{\xi}$ is k-vector of known constants.

Now in the set up $(\underline{Y}, X\underline{\beta}, h(\theta^*))$ we have to minimize

$$\underline{\epsilon}'\underline{\epsilon} = \left(\underline{Y} - X\underline{\beta} - \underline{1}\theta^*\right)' [h(\theta^*)I]^{-1} \left(\underline{Y} - X\underline{\beta} - \underline{1}\theta^*\right)$$

subject to

$$\Psi\underline{\beta} = \underline{\xi}$$

equivalently we have to minimize unconditionally

$$W(\underline{\beta}, \theta^*, \underline{\lambda}) = \frac{\left(\underline{Y} - X\underline{\beta} - \underline{1}\theta^*\right)'\left(\underline{Y} - X\underline{\beta} - \underline{1}\theta^*\right)}{h(\theta^*)} + 2(\underline{\beta}'\underline{\Psi}' - \underline{\xi}')\underline{\lambda}$$

where $\underline{\lambda}$ is k-vector of Lagrange undetermined multipliers.
The partial derivatives of $W(\underline{\beta}, \theta^*, \underline{\lambda})$ w.r.t. $\underline{\beta}$, gives the normal equations as

$$X'X\underline{\beta} + X'\underline{1}\theta^* + \Psi'\underline{\lambda} = X'\underline{Y} \qquad (2.2)$$

From (2.2) we have $\underline{\beta}$ in terms of θ^* and $\underline{\lambda}$ as

$$\underline{\beta} = \begin{cases} (X'X)^{-1}X'\underline{Y} - (X'X)^{-1}X'\underline{1}\theta^* - (X'X)^{-1}\Psi'\underline{\lambda}, & \text{if } |X'X| \neq 0 \\ (X'X)^{-}X'\underline{Y} - (X'X)^{-}X'\underline{1}\theta^* - (X'X)^{-}\Psi'\underline{\lambda}, & \text{if } |X'X| = 0 \end{cases} \qquad (2.3)$$

where $(X'X)^-$ is generalized inverse of $(X'X)$.

Also the partial derivative of $W(\underline{\beta}, \theta^*, \underline{\lambda})$ w.r.t. θ^* gives the normal equation as

$$(a_1 + 2a_2\theta^*)\left(\underline{Y} - X\underline{\beta} - \underline{1}\theta^*\right)'\left(\underline{Y} - X\underline{\beta} - \underline{1}\theta^*\right) + 2\underline{1}'\left(\underline{Y} - X\underline{\beta} - \underline{1}\theta^*\right)\left(a_0 + a_1\theta^* + a_2\theta^{*2}\right) = 0 \quad (2.4)$$

As generalized inverse is not unique, we substitute the value of $\underline{\beta}$ when $|X'X| \neq 0$, obtained in (2.3) and use the same everywhere.

Using $\underline{Y} - X\underline{\beta} - \underline{1}\theta^* = A\underline{Y} - A\underline{1}\theta^* + C'\underline{\lambda}$

and

$$\left(\underline{Y} - X\underline{\beta} - \underline{1}\theta^*\right)'\left(\underline{Y} - X\underline{\beta} - \underline{1}\theta^*\right) = (\underline{1}'A\underline{1})\theta^{*2} - 2\underline{1}'A\left(\underline{Y} + C'\underline{\lambda}\right)\theta^* + 2\underline{Y}'AC'\underline{\lambda} + \underline{\lambda}'R\underline{\lambda} + \underline{Y}'A\underline{Y}$$

where

$$C = \Psi(X'X)^{-1}X'$$
$$A = I - X(X'X)^{-1}X'$$
$$R = \Psi(X'X)^{-1}\Psi'$$

with

$$\underline{\lambda} = R^{-1}C\underline{Y} - R^{-1}C\underline{1}\theta^* - R^{-1}\underline{\xi}$$

(2.4) will be

$$L^*\theta^{*2} + M^*\theta^* + N^* = 0, \text{ assuming that } |R| \neq 0 \quad (2.5)$$

where

$$L^* = 2a_2\underline{1}'\left\{C'R^{-1}\underline{\xi} - (Q + A)\underline{Y}\right\} - a_1\underline{1}'(Q + A)\underline{1} \quad (2.6)$$

$$M^* = 2a_2\underline{Y}'(Q + A)\underline{Y} + 2a_2\underline{\xi}'R^{-1}\underline{\xi} - 4a_2\underline{\xi}'R^{-1}C\underline{Y} - 2a_0\underline{1}'(Q + A)\underline{1} \quad (2.7)$$

$$N^* = a_1\underline{\xi}'R^{-1}\underline{\xi} - 2a_1\underline{\xi}'R^{-1}C\underline{Y} + a_1\underline{Y}'(Q + A)\underline{Y} + 2a_0\underline{1}'(Q + A)\underline{Y} - 2a_0\underline{1}'C'R^{-1}\underline{\xi} \quad (2.8)$$

$Q = C'R^{-1}C$

Solution of quadratic equation (2.5) yields

$$\widehat{\theta^*} = \frac{-M^* \pm \sqrt{\Delta}}{2L^*} \quad (2.9)$$

where $\Delta = M^{*2} - 4L^*N^*$.

Then

$\widehat{\theta^*} > 0 \; if \; \sqrt{\Delta} > M^*, L^* > 0, a_0 > 0, a_1 > 0,$

$a_2 > 0 \;\; or \;\; -\sqrt{\Delta} - M^* < 0, L^* < 0$

Then the hazard rate, survival time and p.d.f. of frailty variables will be estimated by

$$\hat{h}(t) = a_0 + a_1\widehat{\theta^*} + a_2\widehat{\theta^{*2}},$$
$$\hat{S}(t) = \exp\left[-\int_0^t \hat{h}(x)\,dx\right],$$
$$f(t) = \hat{h}(t)\exp\left[-\int_0^t \hat{h}(x)\,dx\right].$$

Also by using (2.9) the regression coefficient vector $\underline{\hat{\beta}}$ is estimated by

$$\underline{\hat{\beta}} = (X'X)^{-1}X'\underline{Y} - (X'X)^{-1}X'\underline{1}\widehat{\theta^*} - (X'X)^{-1}\Psi'\underline{\hat{\lambda}}, \;\; if \; |X'X| \neq 0 \quad (2.10)$$

where

$$\underline{\hat{\lambda}} = R^{-1}C\underline{Y} - R^{-1}C\,\underline{1}\widehat{\theta^*} - R^{-1}\underline{\xi} \quad (2.11)$$

III. UNIFIED THEORY OF ESTIMATING $\underline{\beta}$ AND θ^* UNDER LINEAR CONSTRAINT.

Here in this section effort has been made to unify the theory of estimation of $\underline{\beta}$ and θ^* with linear constraint on $\underline{\beta}$ and θ^* simultaneously. Comparison has been made with constraint and without constraint.

Reconstructing (2.1) as

$$\underline{Y} = X\underline{\beta} + \underline{1}\theta^* + \underline{\epsilon}^* \quad (3.1)$$

where,

$\underline{\epsilon}^* = \underline{\epsilon} - \underline{1}\theta^*$, such that $E(\underline{\epsilon}^*) = \underline{1}\theta^*$and $E(\underline{\epsilon}^*\underline{\epsilon}^{*\prime}) = h(\theta^*)I$.

One may also be interested in estimating the regressors $\beta_1, \beta_2, \ldots, \beta_p(\underline{\beta})$ and the coefficient associated with shared frailty variate θ^* simultaneously under some linear constraints such as

$$H_0: \underline{\eta}\underline{\delta} = \underline{\zeta}$$

where $\underline{\eta} = [K, B]$ of order k x $(p + 1)$, K is k x p matrix of known constants and B is k-vector of known constants.

$\underline{\delta} = \begin{bmatrix}\beta \\ \theta^*\end{bmatrix}$ is $(p + 1)$ vector of parameters

$\underline{\zeta}$ is k-vector of known constants

Then the usual model $Y = X\beta + \epsilon$ turns out as

$$\underline{Y} = Z\underline{\delta} + \underline{\epsilon}$$

where $\underline{Y}$ is observed n-vector ,

$Z = [X \;\; \underline{1}]$ is n x $(p + 1)$ matrix and X is n x p matrix of known constants, $\underline{1}$ is

n-vector of ones. $\underline{\epsilon}$ is frailty n-vector with

$$E(\underline{\epsilon}) = 0 \,,\, E(\underline{\epsilon}\,\underline{\epsilon}') = I$$

Now for the set up $(\underline{Y}, Z\underline{\delta}\,, I)$ we have to minimize

$$\underline{\epsilon}'\underline{\epsilon} = \left(\underline{Y} - Z\underline{\delta}\right)'\left(\underline{Y} - Z\underline{\delta}\right)$$

subject to

$$\underline{\eta}\underline{\delta} = \underline{\zeta}$$

equivalently one has to minimize unconditionally the following function

$$W(\underline{\delta}, \theta^*, \lambda) = \left(\underline{Y} - Z\underline{\delta}\right)'\left(\underline{Y} - Z\underline{\delta}\right) + 2(\underline{\delta}'\underline{\eta}' - \underline{\zeta}')\underline{\lambda}$$

where λ is n vector of Langrange undetermined multipliers.

Setting the partial derivatives $W(\underline{\delta}, \theta^*, \lambda)$ w.r.t. $\underline{\delta}$ to zero it yields the following normal equation

$$(Z'Z)\underline{\delta} + \underline{\eta}'\underline{\lambda} = Z'\underline{Y} \quad (3.2)$$

and expressing $\underline{\delta}$ in terms of

$$\underline{\delta} = \begin{cases} (Z'Z)^{-1}Z'\underline{Y} - (Z'Z)^{-1}\underline{\eta}'\underline{\lambda}, & \text{if } |Z'Z| \neq 0 \\ (Z'Z)^{-}Z'\underline{Y} - (Z'Z)^{-}\underline{\eta}'\underline{\lambda}, & \text{if } |Z'Z| = 0 \end{cases} \quad (3.3)$$

where $(Z'Z)^{-}$ is generalized inverse of $Z'Z$.

Taking $|Z'Z| \neq 0$ and pre multiplying (3.3) with $\underline{\eta}$ we get

$$\underline{\eta}\underline{\delta} = \underline{\eta}(Z'Z)^{-1}Z'\underline{Y} - \underline{\eta}(Z'Z)^{-1}\underline{\eta}'\underline{\lambda} = \underline{\zeta}$$

i.e. $$D\underline{Y} - G\underline{\lambda} = \underline{\zeta} \quad (3.4)$$

where $D = \underline{\eta}(Z'Z)^{-1}Z'$

$G = \underline{\eta}(Z'Z)^{-1}\underline{\eta}'.$

From (3.4) we get

$$\underline{\lambda} = G^{-1}D\underline{Y} - G^{-1}\underline{\zeta} \quad \text{if } |G| \neq 0 \qquad (3.5)$$

Substituting $\underline{\lambda}$ from (3.5) in (3.3) we get

$$\underline{\delta} = (Z'Z)^{-1}Z'\underline{Y} - (Z'Z)^{-1}\underline{\eta}'G^{-1}\left(D\underline{Y} - \underline{\zeta}\right), \text{if } |Z'Z| \neq 0 \qquad (3.6)$$

With some simplification, one gets

$$G = \underline{\eta}(Z'Z)^{-1}\underline{\eta}'$$

$$= \frac{1}{\underline{1}'A\underline{1}}\left(\left(\underline{1}'A\underline{1}\right)K(X'X)^{-1}K' + (V-B)(V-B)'\right)$$

where $V = KU$ and $U = (X'X)^{-1}X'\underline{1}$,

$$G^{-1} = J^{-1} - \frac{J^{-1}(U-B)(U-B)'J^{-1}}{(\underline{1}'A\underline{1}) + (U-B)'J^{-1}(U-B)}$$

where $J = K(X'X)^{-1}K'$

and

$$D = \frac{1}{\underline{1}'A\underline{1}}\left(\left(\underline{1}'A\underline{1}\right)K(X'X)^{-1}X' + VU'X' - V\underline{1}' - BU'X' + B\underline{1}'\right).$$

Thus

$$\hat{\underline{\delta}} = \frac{1}{\underline{1}'A\underline{1}}\begin{pmatrix} \left(\underline{1}'A\underline{1}\right)\underline{b} + UU'X'\underline{Y} - U\underline{1}'\underline{Y} - [\left(\underline{1}'A\underline{1}\right)(X'X)^{-1}K' + \\ UV' - UB']G^{-1}(D\underline{Y} - \underline{\zeta})\underline{1}'A\underline{Y} + (V' - B')G^{-1}(D\underline{Y} - \underline{\zeta}) \end{pmatrix}$$

where $\underline{b} = (X'X)^{-1}X'\underline{Y}$

$\therefore \hat{\beta}$

$$= \frac{1}{\underline{1}'A\underline{1}}\begin{pmatrix} \left(\underline{1}'A\underline{1}\right)\underline{b} + UU'X'\underline{Y} - U\underline{1}'\underline{Y} - [\left(\underline{1}'A\underline{1}\right)(X'X)^{-1}K' + \\ UV' - UB']G^{-1}(D\underline{Y} - \underline{\zeta}) \end{pmatrix} \qquad (3.7)$$

$$= \underline{b} + \frac{1}{\underline{1}'A\underline{1}}\left\{UU'X'\underline{Y} - U\underline{1}'\underline{Y} - [\left(\underline{1}'A\underline{1}\right)(X'X)^{-1}K' + UV' - UB']G^{-1}(D\underline{Y} - \underline{\zeta})\right\}$$

and

$$\widehat{\theta^*} = \frac{1}{\underline{1}'A\underline{1}}\left(\underline{1}'A\underline{Y} + (V' - B')G^{-1}(D\underline{Y} - \underline{\zeta})\right). \qquad (3.8)$$

Thus the estimator of frailty variable θ^* is obtained in (3.8) and thereby one can get the estimation of survival function and p.d.f.

The estimator of regression coefficient vector of $\underline{\beta}$ is obtained in (3.7) and it is observed that it has been improved from usual linear estimator $\underline{b}$ of $\underline{\beta}$. provided

$$\|\underline{b} + \underline{w}\| \geq \|\underline{b}\|$$

where

$$\underline{w} = \frac{1}{\underline{1}'A\underline{1}}\left\{UU'X'\underline{Y} - U\underline{1}'\underline{Y} - [\left(\underline{1}'A\underline{1}\right)(X'X)^{-1}K' + UV' - UB']G^{-1}(D\underline{Y} - \underline{\zeta})\right\}$$

IV. APPLICATION TO KIDNEY INFECTION PATIENTS.

For illustration we use the kidney infection data (see Appendix A) given by McGilchrist and Aisbett [11] of 38 kidney Patients using portable dialysis. A catheter is inserted to each admitted kidney Patient and keeps it until infection is removed, that is, infection is cleared. After some time again catheter is inserted and removed when infection occurs due to kidney or some other reasons. Time (in number of days) recorded when catheter is reinserted. The purpose is to note the incidence rate of infection to risk variables (Age, Sex and disease type of kidney disorder, Glomerulo Neptiritis(GN),Acute Neptiritis(AN) and Polycyatic Neptiritis(PKD). The time interval recorded is considered as random variable and according to Cox PH model we have illustrated the estimation by MLE, using the best linear unbiased prediction (BLUP). Estimation of the problem is described in McGilchrist and Aisbett [11]. The estimates of regression coefficients $\beta_1,\beta_2,\beta_3,\beta_4,\beta_5$ and frailty parameter $\boldsymbol{\theta}^*$ are calculated by using (3.7) and (3.8) respectively.

4.1. Further for the same data given by McGilchrist and Aisbett [11] the calculated value of L^*, M^* and N^* given in (2.6), (2.7) and (2.8) respectively in terms of a_0, a_1, a_2 are as under.

$$L^* = -41.064426a_2 - 3.9740857a_1$$
$$M^* = -7.948174a_0 + 381.08806a_2$$
$$N^* = 190.54403a_1 + 41.064424a_0 \qquad (4.1)$$

and substituting these values in (2.9),the estimate of $\widehat{\theta^*}$ will be

$$\widehat{\theta^*} = \frac{(7.948174a_0 - 381.08806a_2) \pm \sqrt{\Delta}}{-(7.9481714a_1 + 82.128852a_2)} \qquad (4.2)$$

where

$$\Delta = 63.173469a_0^2 + 145228.1095a_2^2 + 3028.953216a_1^2 + 652.774168a_0a_1 + 31298.32488a_1a_2 + 687.239584a_0a_2 \qquad (4.3)$$

and $\hat{\underline{\lambda}}$ of (2.11) will reduce to

$$\hat{\underline{\lambda}} = \begin{bmatrix} -37.524445 + 3.45717116\hat{\theta} \\ 65.362087 - 6.54138874\hat{\theta} \\ 36.422100 - 3.77935509\hat{\theta} \\ 39.320243 - 3.46094455\hat{\theta} \end{bmatrix} \qquad (4.4)$$

and hence $\hat{\underline{\beta}}$ of (2.10) will reduce to

$$\hat{\underline{\beta}} = \begin{bmatrix} 0.077971 - 0.021083\hat{\theta} \\ -0.299985 - 0.000002\hat{\theta} \\ 0.321971 + 0.021089\hat{\theta} \\ 0.321991 + 0.021083\hat{\theta} \\ 0.078047 - 0.021153\hat{\theta} \end{bmatrix}$$

Considering linear constraints on $\underline{\beta}$ as

$$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 = 0.5$$
$$\beta_1 + \beta_2 + \beta_4 = 0.1$$
$$\beta_3 - \beta_4 = 0$$
$$-\beta_1 + \beta_5 = 0$$

i.e.

$$H_0 : \Psi\underline{\beta} = \underline{\xi}$$

Where

$$\Psi = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \underline{\xi} = \begin{bmatrix} 0.5 \\ 0.1 \\ 0 \\ 0 \end{bmatrix}$$

we obtain estimates of $\underline{\beta}$ under above constraints and with particular value of a_0, a_1, a_2 in the following special cases of different hazard functions and different baseline distributions.

Case (1): $a_0 = a_2 = 0$ *and* $a_1 = 1$ corresponding to hazard function $h(\theta^*) = \theta^*$ and exponential base line distribution the values of L^*, M^*, N^* in (4.1) reduce to

$L^* = -3.9740857,$

$M^* = 0,$

$N^* = 190.54403$

$\Delta = 3028.95321$

and hence after substituting the value of $\hat{\theta}$ and $\underline{\hat{\lambda}}$ respectively as

$\hat{\theta} = 1.935044$

$$\underline{\hat{\lambda}} = \begin{bmatrix} -30.83467 \\ 52.704213 \\ 29.108888 \\ 32.62316 \end{bmatrix}$$

the estimates of $\underline{\hat{\beta}}$ will be

$$\underline{\hat{\beta}} = \begin{bmatrix} 0.037224 \\ -0.3 \\ 0.362776 \\ 0.362776 \\ 0.037224 \end{bmatrix}$$

Remark 4.1: (i) we note that hypotheses corresponding to infection on kidney due to age and PKD and also

(ii) infection on kidney due to GN and AN are significantly not different.

also the total infection on kidney due to age, sex and AN which was assumed to be 10% is justified. However the infection due to sex component being ($e^{-0.3}$) positive it is the smallest.

Case(2): $a_0 = a_1 = 0$ *and* $a_2 = 1$ corresponding to hazard function $h(\theta^*) = \theta^{*2}$ which may be the lognormal distribution among other base line distribution, the value of L^*, M^*, N^* in (4.1) reduces to

L^* = -41.064426, M^* = 381.08806, N^* = 0,

Δ= 145228.1095

and hence after substituting the value of $\hat{\theta}$ and $\underline{\hat{\lambda}}$ respectively as

$\hat{\theta}$ = 2.227888

$$\underline{\hat{\lambda}} = \begin{bmatrix} -29.82225 \\ 50.788609 \\ 28.002129 \\ 31.609644 \end{bmatrix}$$

the estimates of $\underline{\hat{\beta}}$ will be

$$\underline{\hat{\beta}} = \begin{bmatrix} 0.031049 \\ -0.3 \\ 0.368951 \\ 0.368951 \\ 0.031049 \end{bmatrix}$$

Same remark 4.1 also hold in this case.

4.2. Assuming the linear constraint on $\underline{\beta}$ and θ, we have

$$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \theta = 4.0$$
$$\beta_1 + \beta_2 + \beta_4 + \theta = 3.5$$
$$\beta_3 - \beta_4 + \theta = 0$$
$$-\beta_1 + \beta_5 + \theta = 0$$

i.e.

$$H_0 : \underline{\eta}\underline{\delta} = \underline{\zeta}$$

where

$$\underline{\eta} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \\ -1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \text{ and } \underline{\zeta} = \begin{bmatrix} 4.0 \\ 3.5 \\ 0 \\ 0 \end{bmatrix}$$

we obtain estimates of $\underline{\beta}$ and θ by using (3.7) and (3.8) respectively under above constraints.

$$\underline{\hat{\beta}} = \begin{bmatrix} 0.014709 \\ 1.045056 \\ 0.485291 \\ 0.485291 \\ 0.014709 \end{bmatrix}$$

and

$\hat{\theta} = 1.658959$

Remark 4.2: The hypothesis concerning (β_1, β_5) and (β_3, β_4) are justified.

$\hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3 + \hat{\beta}_4 + \hat{\beta}_5 + \hat{\theta} = 3.7$ is very close to 4.0 and

$\hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_4 + \hat{\theta} = 3.2$ which is also very close to 3.5.

V. CONCLUSION AND FUTURE SCOPE

In this paper we have considered the usual Gauss-Marcoff model with some restrictions on parameters In the presence of some constraint on $\underline{\beta}$ the estimation of regression coefficients has been improved from the usual linear unbiased estimation which is observed in kidney infection data analysis. The constrained theory may be applied to non-linear models.

APPENDIX A

Patient	Time(T)	Y=lnT	Age	Sex	GN	AN	PKD
1	8	2.07944	28	0	0	0	0
2	23	3.13549	48	1	1	0	0
3	22	3.09104	32	0	0	0	0

4	447	6.10256	31.5	1	0	0	0
5	30	3.4012	10	0	0	0	0
6	24	3.17805	16.5	1	0	0	0
7	7	1.94591	51	0	1	0	0
8	511	6.23637	55.5	1	1	0	0
9	53	3.97029	69	1	0	1	0
10	15	2.70805	51.5	0	1	0	0
11	7	1.94591	44	1	0	1	0
12	141	4.94876	34	1	0	0	0
13	96	4.56435	35	1	0	1	0
14	149	5.00395	42	1	0	1	0
15	536	6.28413	17	1	0	0	0
16	17	2.83321	60	0	0	1	0
17	185	5.22036	60	1	0	0	0
18	292	5.67675	43.5	1	0	0	0
19	22	3.09104	53	1	1	0	0
20	15	2.70805	44	1	0	0	0
21	152	5.02388	46.5	0	0	0	1
22	402	5.99645	30	1	0	0	0
23	13	2.56495	62.5	1	0	1	0
24	39	3.66356	42.5	1	0	1	0
25	12	2.48491	43	0	0	1	0
26	113	4.72739	57.5	1	0	1	0
27	132	4.8828	10	1	1	0	0
28	34	3.52636	52	1	0	1	0
29	2	0.69315	53	0	1	0	0
30	130	4.86753	54	1	1	0	0
31	27	3.29584	56	1	0	1	0
32	5	1.60944	50.5	1	0	1	0
33	152	5.02388	27	1	0	0	1
34	190	5.24702	44.5	1	1	0	0
35	119	4.77912	22	1	0	0	0
36	54	3.98898	42	1	0	0	0
37	6	1.79176	52	1	0	0	1
38	63	4.14313	60	0	0	0	1

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