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CASP for Truncated Exponentiated Weibull Distribution Based on CUSUM Schemes and its Optimization

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Abstract: Acceptance Sampling plan is a procedure employed mainly either to accept or reject the lots for finished goods. It is one of the Statistical tool helps in the improvement of the quality of the product but not to control. There are several techniques developed in order to improve and control the quality of a product under the assumption that variable with regard to the quality characteristic is distributed according to certain probability law. In the present paper, we assume that variable under study follows the Truncated Exponentiated Weibull Distribution and its Optimization of CASP-CUSUM Schemes. Critical comparison made based on the obtained numerical results.

***Keywords**: CASP–CUSUM Schemes, O C Curve, Optimal Truncated Point, Truncated Exponentiated Weibull Distribution*

1. INTRODUCTION

For any product to be introduced in the market the manufacturer looks out for a consumer to be willing to buy the product at an affordable price. In the similar lines, a consumer looks out the product being manufactured as intended purpose to satisfy him in all respects. A level or degree of satisfaction of a consumer with a manufactured product, we termed as Quality of a product. The assessment of a quality of a product is subjective, as it is different from person to person. Hence it is difficult to define quality. However the most commonly acceptable definition of quality is “Fit for use”. A quality of a product is also defined as it is being manufactured as intended purpose or conformance to requirements to satisfy consumer expectations.

In this regard several techniques are developed in order to improve and control the quality of product. Quality of a product depends on so many features like color, dimensions, safety, performance, durability etc. Of all the features, Durability is the most important feature to decide the quality of a product. Durability of a product is defined as lifetime or lifespan of a product. Testing the Lifetime of a product is a complex process, as some products are explosive or destructive in nature. Even for testing lifetime of some other products like Bulbs, Batteries etc., the manufacturer has to wait till the product stop working. In the above both cases 100% inspection is not possible. To overcome the above situation, Acceptance sampling plans were first applied in the US Military for testing the bullets during the World War II. It is the middle path between 100% inspection and no inspection. They used to decide whether to accept or reject a lot of finished goods. These techniques may not have a direct impact in the controlling the quality but it involves indirect effect in improving the quality of the products. Continuous Accepting Sampling Plans – Cumulative Sum Schemes (CASP-CUSUM Schemes) are widely used in the Industries in life testing of the Products. It is a powerful, versatile and diagnostic tool in Statistical Quality Control as it reduces time, cost, and machinery.

Many researchers worked in these directions for the optimization CASP-CUSUM Schemes. Mohammed Akhtar. P and Sarma K.L.A.P [1], Narayana Murthy, B.R. and Mohammed Akhtar.P[2], Sainath.B and Mohammed Akhtar .P [3], Venkatesulu.G and Mohammed Akhtar.P[4] applied CASP-CUSUM Schemes for various truncated distributions and its optimization.

In the present section, the importance of the Quality and a brief literature review is presented on Sampling plans and CUSUM Schemes. In Section-2, we provide the definition of the Exponentiated Weibull distribution and its truncation. In Section-3, we provide the description of the Type-C OC Curve. The method for obtaining the different solutions is presented in the section-4. We developed a computer program for obtaining ARL’s and P (A) for different parameters such that the variable under study follows Truncated Exponentiated Weibull distribution. The results obtained are presented in tabular form under section-5. Finally, In Section-6, we made conclusions based on the results obtained in the section-5.

2. EXPONENTIATED WEIBULL DISTRIBUTION

It is well known that the Gamma, Weibull and Exponentiated family of distributions is widely used for analyzing lifetime data in reliability engineering. It has wide applications in the field of Automotives, aerospace, Electronics etc. There are several generalization forms of Weibull distribution. Exponentiated Weibull distribution is a one of such generalization of Weibull distribution has some similar properties of another Weibull family of distributions.
In the present study we use Exponentiated Weibull distribution given by Mudholkar, G.S., Srivastava, D.K., [5].

A continuous random variable X is said to follow Exponentiated Weibull distribution if it assumes non-negative values with parameters α, K and λ and its probability density function is given by

$$f(x) = \alpha \frac{K}{\lambda}\left(\frac{x}{\lambda}\right)^{K-1} e^{-\left(\frac{x}{\lambda}\right)^{K}} \left[1 - e^{-\left(\frac{x}{\lambda}\right)^{K}}\right]^{\alpha-1} \quad \text{.....2.1}$$

Truncated Exponentiated Weibull distribution

It is defined as the ratio of the probability density function of Exponentiated Weibull distribution to its probability cumulative distribution function at the point B.

A random variable X is said to follow Truncated Exponentiated Weibull distribution if its probability density function is given by

$$f_B(X) = \frac{\alpha \frac{K}{\lambda}\left(\frac{x}{\lambda}\right)^{K-1} e^{-\left(\frac{x}{\lambda}\right)^{K}} \left[1 - e^{-\left(\frac{x}{\lambda}\right)^{K}}\right]^{\alpha-1}}{\left[1 - e^{-\left(\frac{B}{\lambda}\right)^{K}}\right]^{\alpha}} \quad \text{.....2.2}$$

Where 'B' is the upper truncated point of Truncated Exponentiated Weibull distribution

3. DESCRIPTION OF THE TYPE-C OC CURVE

Beattie [6] has suggested the method for constructing the continuous acceptance sampling plans. The procedure suggested by him consists of a chosen decision interval namely "Return interval" with the length h' above the decision line is taken. We plot on the chart, the sum $S_m = \sum (X_i - k)$ $X_i's$ (i= 1, 2, 3………) are distributed independently and k is the reference value. If the sum lies in the area of the normal chart, the product is accepted and if it lies on the return chart, then the product is rejected subject to the following assumptions.

When the recently plotted point on the chart touches the decision line, then the next point is to be plotted at the maximum, i.e., h=h'. When the decision line is reached or crossed from above, the next point on the chart is to plot at the baseline. When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

4. METHOD OF SOLUTION

Using the procedure suggested by Kakoty, S and Chakraborty, A.B. [7], we solve the integral equations for Truncated Exponentiated Weibull distribution (2.1) and we obtained the solutions for P (0), N(0), P′(0) and N′(0). Using these values L(0), L′(0) and P(A) are calculated for various parameters namely K, λ, α, h, h' and B are given in Tables 5.1 to 5.18.

5. COMPUTATION OF ARL's AND P (A)

We developed a computer program to solve the integral equations and get the following results given in the tables (5.1) to (5.18).

TABLE – 5.1

Values of ARL's and Type C-OC Curves when

α = 2 λ = 2 K = 1

k = 1 h = 0 .1 h'= 0.1

B	L(0)	L'(0)	P(A)
1.5	2.08863	1.0083095	0.674417257
1.4	2.36545	1.0090479	0.700978279
1.3	2.83066	1.0099237	0.737039089
1.2	3.76762	1.0109689	0.788437903
1.1	6.59243	1.0122205	0.866894603
1.0	5387.50439	1.0137144	0.999811888

TABLE – 5.2

Values of ARL's and Type C-OC Curves when

α = 2 λ = 2 K = 1

k = 1 h = 0.4 h'= 0.4

B	L(0)	L'(0)	P(A)
1.5	1.69484	1.0262965	0.622842371
1.4	1.88375	1.0275732	0.647042572
1.3	2.20349	1.0287299	0.681726515
1.2	2.85077	1.0295196	0.734679639
1.1	4.79996	1.0294383	0.823405802
1.0	305.12897	1.0274417	0.996644080

TABLE – 5.3

Values of ARL's and Type C-OC Curves when

α = 2 λ = 2 K = 1

k = 2 h = 0.1 h'= 0.1

B	L(0)	L'(0)	P(A)
2.5	4.45873	1.0047059	0.816103697
2.4	5.2675	1.0048982	0.839790404
2.3	6.61916	1.0051129	0.868169308
2.2	9.32824	1.0053537	0.90271014
2.1	17.46684	1.0056251	0.945560873
2	82866.555	1.0059326	0.999987841

TABLE – 5.4

Values of ARL's and Type C-OC Curves when

α = 2 λ = 2 K = 1

k = 2 h = 0.4 h'= 0.4

B	L(0)	L'(0)	P(A)
2.5	3.95655	1.0170293	0.795513451
2.4	4.65159	1.0176266	0.820499718
2.3	5.81328	1.0182821	0.850944519
2.2	8.14106	1.0190023	0.888755977
2.1	15.1255	1.0197943	0.936836421
2.0	5742.5405	1.0206649	0.999822319

TABLE – 5.5

Values of ARL's and Type C-OC Curves when

α = 2 λ = 2 K = 1

k = 1 h = 0.1 h'= 0.1

B	L(0)	L'(0)	P(A)
3.5	8.36194	1.003541	0.892846882
3.4	10.03397	1.0036166	0.909072757
3.3	12.82608	1.0036987	0.927424848
3.2	18.41844	1.0037881	0.948317587
3.1	35.2111	1.0038854	0.972279847
3.0	284121.38	1.0039917	0.999996483

TABLE – 5.6

Values of ARL's and Type C-OC Curves when

α = 2 λ = 3 K = 1

k = 3 h = 0.4 h'= 0.4

B	L(0)	L'(0)	P(A)
3.5	7.6384	1.0132147	0.882887185
3.4	9.14787	1.0134718	0.900261939
3.3	11.66819	1.0137496	0.920063496
3.2	16.71466	1.01405	0.942801833
3.1	31.85463	1.0143756	0.969138861
3.0	20182.066	1.0147288	0.999949753

TABLE – 5.7

Values of ARL's and Type C-OC Curves when

α = 2 λ = 2 K = 1

k = 4 h = 0.1 h'= 0.1

B	L(0)	L'(0)	P(A)
4.5	14.79527	1.0030314	0.936510146
4.4	17.88598	1.0030675	0.946896911
4.3	23.04617	1.0031064	0.958289564
4.2	33.37993	1.0031477	0.970824361
4.1	64.40649	1.0031923	0.98466295
4.0	582305.44	1.0032401	0.999998271

TABLE – 5.8

Values of ARL's and Type C-OC Curves when

α = 2 λ = 2 K = 1

k = 4 h = 0.4 h'= 0.4

B	L(0)	L'(0)	P(A)
4.5	13.68803	1.0114499	0.931191444
4.4	16.53134	1.011577	0.942337036
4.3	21.27767	1.0117128	0.954610109
4.2	30.7794	1.0118579	0.968171835
4.1	59.28225	1.0120132	0.983215451
4.0	38885.227	1.0121795	0.999973953

TABLE – 5.9

Values of ARL's and Type C-OC Curves when

α = 2 λ = 3 K = 1

k = 1 h = 0.1 h'= 0.1

B	L(0)	L'(0)	P(A)
1.5	1.99545	1.009447	0.664065778
1.4	2.26294	1.0104605	0.691311657
1.3	2.7126	1.0116811	0.728355169
1.2	3.61847	1.0131668	0.781250894
1.1	6.35043	1.0149944	0.862194777
1.0	7520.4453	1.0172621	0.999864757

TABLE – 5.10

Values of ARL's and Type C-OC Curves when

α = 2 λ = 3 K = 1

k = 4 h = 0.1 h'= 0.1

B	L(0)	L'(0)	P(A)
4.5	9.75136	1.002494	0.906778216
4.4	11.81695	1.002543	0.921795428
4.3	15.26288	1.0025949	0.938360572
4.2	22.15972	1.0026505	0.956712127
4.1	42.86015	1.0027097	0.97713989
4.0	1507289.8	1.0027732	0.999999344

TABLE – 5.11

Values of ARL's and Type C-OC Curves when

α = 2 λ = 4 K = 1

k = 1 h = 0.1 h'= 0.1

B	L(0)	L'(0)	P(A)
1.5	1.94182	1.0100362	0.657830477
1.4	2.20228	1.0112062	0.685324371
1.3	2.64034	1.0126239	0.722793877
1.2	3.52323	1.0143628	0.776453376
1.1	6.18685	1.0165225	0.858882308
1.0	8877.2969	1.0192381	0.999885201

TABLE – 5.12

Values of ARL's and Type C-OC Curves when

α = 2 λ = 4 K = 1

k = 1 h = 0.4 h'= 0.4

B	L(0)	L'(0)	P(A)
1.5	1.77764	1.0367768	0.631619215
1.4	1.99505	1.0404443	0.657241106
1.3	2.36119	1.0446461	0.693278015
1.2	3.09937	1.0493933	0.747058928
1.1	5.31924	1.0545726	0.83454591
1.0	527.58069	1.0597401	0.997995377

TABLE – 5.13

Values of ARL's and Type C-OC Curves when

α = 2 λ = 4 K = 1

k = 4 h = 0.1 h'= 0.1

B	L(0)	L'(0)	P(A)
4.5	8.01747	1.0022	0.888887286
4.4	9.71971	1.0022542	0.906523287
4.3	12.55892	1.0023121	0.926089883
4.2	18.24059	1.0023735	0.947909594
4.1	35.29221	1.0024393	0.972380519
4.0	2081611.6	1.0025094	0.999999523

TABLE – 5.14

Values of ARL's and Type C-OC Curves when

α = 2 λ = 4 K = 1

k = 4 h = 0.4 h'= 0.4

B	L(0)	L'(0)	P(A)
4.5	7.86316	1.0086793	0.886305451
4.4	9.52805	1.0088899	0.904252052
4.3	12.30492	1.009114	0.924206734
4.2	17.86173	1.0093527	0.946513236
4.1	34.53754	1.0096071	0.971598089
4.0	296150.19	1.0098786	0.999996603

TABLE – 5.15

Values of ARL's and Type C-OC Curves when

α = 3 λ = 1 K = 4

k = 3 h = 0.1 h'= 0.1

B	L(0)	L'(0)	P(A)
3.5	3.89817	1.0022122	0.795482755
3.4	4.64855	1.0023379	0.822623074
3.3	5.90226	1.0024769	0.854813278
3.2	8.41456	1.0026312	0.893531799
3.1	15.96171	1.0028032	0.940888166
3.0	850821	1.0029954	0.999998808

TABLE – 5.16

Values of ARL's and Type C-OC Curves when

α = 3 λ = 4 K = 1

k = 3 h = 0.4 h'= 0.4

B	L(0)	L'(0)	P(A)
3.6	3.50385	1.0085928	0.776486516
3.5	4.02469	1.0090717	0.799539387
3.4	4.80827	1.009601	0.826465607
3.3	6.11751	1.0101881	0.85827291
3.2	8.74121	1.0108421	0.896345735
3.1	16.62344	1.0115731	0.942638397

TABLE – 5.17

Values of ARL's and Type C-OC Curves when

α = 3 λ = 4 K = 1

k = 4 h = 0.1 h'= 0.1

B	L(0)	L'(0)	P(A)
4.5	5.59331	1.0014198	0.848148465
4.4	6.74433	1.0014732	0.870707631
4.3	8.66522	1.0015309	0.896394312
4.2	12.51093	1.0015931	0.925876677
4.1	24.05621	1.0016605	0.960026085
4.0	2416513.5	1.0017335	0.999999583

TABLE – 5.18

Values of ARL's and Type C-OC Curves when

α = 3 λ = 4 K = 1

k = 4 h = 0.4 h'= 0.4

B	L(0)	L'(0)	P(A)
4.5	5.70864	1.0057702	0.850207329
4.4	6.88874	1.0059911	0.872574329
4.3	8.85815	1.0062294	0.897993684
4.2	12.80106	1.0064871	0.927106023
4.1	24.6382	1.0067667	0.960742116
4.0	5781059.5	1.0070703	0.999999821

6. CONCLUSIONS

We determined the optimum truncated point B for which the probability of accepting an item i.e, P(A) is maximum and also obtained ARL's which represents the acceptance zone L(0) and rejection zone L' (0) at the hypothetical values α, λ, K, k, h and h' given at the top of each table. The values of truncated point B of random variable X, L (0), L'(0) and the values for Type-C Curve, i.e. P (A) are given in columns I, II, III, and IV respectively.

From the above tables 5.1 to 5.18 we made the following conclusions:

1. From the tables 5.1 to 5.18 it is observed that the values of P (A) are increased as the value of truncated point decreases. Thus, the truncated point of the random variable and the various parameters for CASP-CUSUM are related.

2. It is also observed that the truncated point B is decreased as the increase in the value of k.

3. From the tables 5.1 and 5.2, it is observed that the truncated point B of the random variable X decreases 1.5 to 1.0 as the value of h changes from 0.01 to 0.04. While the value of L(0) decreases from 5387.50439 to 305.12897 and L'(0) changes from 1.0137144 to 1.0274417 Also, the probability of acceptance P(A) changes from 0.999811888 to 0.996644080. Thus the truncated point B, L(0) and P(A) are inversely related and L'(0) is positively related.

4. In the similar lines, from the tables 5.3 and 5.4, it is observed that the truncated point B of the random variable X decreases 2.5 to 2.0 as the value of h changes from 0.01 to 0.04. While the value of L(0) decreases from 82866.55469 to 5742.54053 and L'(0) changes from 1.0059326 to 1.0206649. Also, the probability of acceptance P(A) changes from 0.999987841 to 0.999822319. Thus the truncated point B, L(0) and P(A) are inversely related and L'(0) is positively related.

5. From the tables 5.1 to 5.18, it is observed that the truncated point B changes from 4.5 to 1.0 as $h \rightarrow 0.04$. The P (A) attains more stability and reaches maximum **i.e. 0.999999821.**

6. From the tables 5.1 to 5.18, it can be observed that the values of P (A) increased as the increase in the values of 'k', where remaining parameters put in constant.

7. From the tables 5.1 to 5.18 it is also observed that there is some increase in the values of probabilities P(A) as increase in the values of the parameter 'λ' where remaining parameters put in constant.

8. The various relations exhibited among the ARL's and Type-C OC curves with the parameters of the CASP-CUSUM based on the above tables 5.1 to 5.18 are observed from the following Table-6.1.

Table-6.1

Consolidated Table

B	α	λ	K	h	h'	k	P(A)
1.0	2	2	1	0.01	0.01	1	0.999811888
1.0	2	2	1	0.04	0.04	1	0.99664408
2.0	2	2	1	0.01	0.01	2	0.999987841
2.0	2	2	1	0.04	0.04	2	0.999822319
3.0	2	2	1	0.01	0.01	3	0.999996483
3.0	2	2	1	0.04	0.04	3	0.999949753
4.0	2	2	1	0.01	0.01	4	0.999998271
4.0	2	2	1	0.04	0.04	4	0.999973953
1.0	2	3	1	0.01	0.01	1	0.999864757
4.0	2	3	1	0.01	0.01	4	0.999999344
1.0	2	4	1	0.01	0.01	1	0.999885201
1.0	2	4	1	0.04	0.04	1	0.997995377
4.0	2	4	1	0.01	0.01	4	0.999999523
4.0	2	4	1	0.04	0.04	4	0.999996603
3.0	3	4	1	0.01	0.01	3	0.999998808
3.1	3	4	1	0.04	0.04	3	0.942638397
4.0	3	4	1	0.01	0.01	4	0.999999583
4.0	3	4	1	0.04	0.04	4	0.999999821

From the above table 6.1, we conclude that the optimum CASP-CUSUM schemes for which the Probability of Acceptance, P (A) reach their maximum (0.999999821) is

$$\begin{bmatrix} B=4 \\ \alpha=3 \\ \lambda=4 \\ K=1 \\ k=4 \\ h=0.04 \\ h'=0.04 \end{bmatrix}$$

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