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On the Zagreb polynomials of transformation graphs

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Abstract— In the year $\mathbf{2009}$, Fath-Tabar introduced the concept of Zagreb polynomials. The novel topological indices called Zagreb indices can be derived from these polynomials. In order to find the Zagreb polynomials of transformation graphs we introduce the concept of Zagreb co-polynomials, from which one can derive the Zagreb coindices. Further, we establish the relations connecting the Zagreb polynomials of the graph $\boldsymbol{G}$ to those of the transformation graphs.

Keywords— *Zagreb index, Zagreb polynomial, $\boldsymbol{xyz}$-Point-Line transformation graph.*

I. INTRODUCTION

In the present work, a graph $G=(V,E)$ we mean a nontrivial, finite, simple, undirected graph with n vertices and m edges. The *degree* $d_G(v)$ of a vertex v in G is the number of edges incident to it in G. The *degree* $d_G(e)$ of an edge $e=uv$ of G in $L(G)$, is given by $d_G(e)=d_G(u)+d_G(v)-2$. The *complement* $\overline{G}$ of a graph G is a graph whose vertex set is $V(G)$ and two vertices of $\overline{G}$ are adjacent if and only if they are nonadjacent in G. Therefore, $\overline{G}$ has n vertices and $\binom{n}{2}-m$ edges. The *line graph* [15] $L(G)$ of a graph G is a graph with vertex set which is one to one correspondence with the edge set of G and two vertices of $L(G)$ are adjacent whenever the corresponding edges in G have a vertex incident in common. The *subdivision graph* [15] $S(G)$ of a graph G is a graph with the vertex set $V(S(G))=V(G)\cup E(G)$ and two vertices of $S(G)$ are adjacent whenever they are incident in G. The *partial complement of subdivision graph* [18] $\overline{S}(G)$ of a graph G is a graph with the vertex set $V(\overline{S}(G))=V(G)\cup E(G)$ and two vertices of $\overline{S}(G)$ are adjacent whenever they are nonincident in G.

In this paper, we denote $u\sim v$ ($u\nsim v$) for vertices u and v are adjacent (resp., nonadjacent), $e\sim f$ ($e\nsim f$) means that the edges e and f are adjacent (resp., nonadjacent) and $u\sim e$ ($u\nsim e$) means that the vertex u and an edge e are incident (resp., nonincident) in G. The vertices of $T^{xyz}(G)$ representing the vertices of G are referred to as point-vertices while the vertices of $T^{xyz}(G)$ representing the edges of G are referred to as line-vertices. For undefined notations and terminologies one can refer [15, 19].

A topological index is a numerical parameter mathematically derived from the graph structure. It is a graph invariant. The topological indices have their applications in various disciplines of science and technology. The first and second Zagreb indices are amongst the oldest and best known topological indices defined in 1972 by Gutman [12] as follows:

$$M_1(G)=\sum_{v\in V(G)} d_G(v)^2$$

and

$$M_2(G)=\sum_{uv\in E(G)} d_G(u)\cdot d_G(v),$$

respectively. These are widely studied degree based topological indices due to their applications in chemistry, for details see [3,8,11,13,16,24]. The first Zagreb index [21] can also be expressed as

$$M_1(G)=\sum_{uv\in E(G)} [d_G(u)+d_G(v)].$$

Ashrafi et al. [1] defined the first and second Zagreb coindices as

$$\overline{M_1}(G)=\sum_{uv\notin E(G)} [d_G(u)+d_G(v)]$$

and

$$\overline{M_2}(G)=\sum_{uv\notin E(G)} d_G(u)\cdot d_G(v),$$

respectively.

In 2004, Milic$'$evic$'$ et al. [20] reformulated the Zagreb indices in terms of edge-degrees instead of vertex-degrees.

The first and second reformulated Zagreb indices are defined, respectively, as

$$EM_1(G) = \sum_{e\in E(G)} d_G(e)^2$$

and

$$EM_2(G) = \sum_{e\sim f} d_G(e)\cdot d_G(f).$$

In [17], Hosamani et al. defined the first and second reformulated Zagreb coindices respectively as

$$\overline{EM_1}(G) = \sum_{e\nsim f} [d_G(e) + d_G(f)]$$

and

$$\overline{EM_2}(G) = \sum_{e\nsim f} d_G(e)\cdot d_G(f).$$

Considering the Zagreb indices, Fath-Tabar [10] defined first and the second *Zagreb polynomials* as

$$M_1(G,x) = \sum_{v_i v_j\in E(G)} x^{d_G(v_i)+d_G(v_j)}, \quad (1.1)$$

$$M_2(G,x) = \sum_{v_i v_j\in E(G)} x^{d_G(v_i)\cdot d_G(v_j)}, \quad (1.2)$$

and

$$M_3(G,x) = \sum_{v_i v_j\in E(G)} x^{|d_G(v_i)-d_G(v_j)|}, \quad (1.3)$$

respectively, where x is a variable. In addition, Shuxian [23] defined two polynomials related to the first Zagreb index as in the form

$$M_1^*(G,x) = \sum_{v_i\in V(G)} d_G(v_i)x^{d_G(v_i)} \text{ and } M_0 = \sum_{v_i\in V(G)} x^{d_G(v_i)}.$$

In [7], A. R. Bindusree et al. defined the following polynomials

$$M_4(G,x) = \sum_{v_i v_j\in E(G)} x^{d_G(v_i)((d_G(v_i)+d_G(v_j))},$$

$$M_5(G,x) = \sum_{v_i v_j\in E(G)} x^{d_G(v_j)((d_G(v_i)+d_G(v_j))},$$

$$M_{a,b}(G,x) = \sum_{v_i v_j\in E(G)} x^{ad_G(v_i)+bd_G(v_j)},$$

$$M'_{a,b}(G,x) = \sum_{v_i v_j\in E(G)} x^{(d_G(v_i)+a)(d_G(v_j)+b)}.$$

In the following section, we define new graph polynomials called Zagreb co-polynomials of a graph

The rest of the paper is organized as follows: In section II, we define and study the new graph polynomials called Zgreb co-polynomials. The section III discuss the generalized xyz-point line transformation graphs. In section IV, we obtaine the Zagreb polynomials of the generalized xyz-point line transformation graphs when $z = +$.

II. ZAGREB CO-POLYNOMIALS OF A GRAPH

In this section, we define new graph polynomials called *Zagreb co-polynomials*. The first, second, and third *Zagreb co-polynomials* of a graph G are denoted and defined as,

$$\overline{M_1}(G,x) = \sum_{v_i v_j\notin E(G)} x^{d_G(v_i)+d_G(v_j)},$$

$$\overline{M_2}(G,x) = \sum_{v_i v_j\notin E(G)} x^{d_G(v_i)\cdot d_G(v_j)}$$

and

$$\overline{M_3}(G,x) = \sum_{v_i v_j\notin E(G)} x^{|d_G(v_i)-d_G(v_j)|},$$

respectively, where x is a variable.
In addition, we define

$$\overline{M_4}(G,x) = \sum_{v_i v_j\notin E(G)} x^{d_G(v_i)((d_G(v_i)+d_G(v_j))},$$

$$\overline{M_5}(G,x) = \sum_{v_i v_j\notin E(G)} x^{d_G(v_j)((d_G(v_i)+d_G(v_j))},$$

$$\overline{M}_{a,b}(G,x) = \sum_{v_i v_j\notin E(G)} x^{ad_G(v_i)+bd_G(v_j)},$$

$$\overline{M'}_{a,b}(G,x) = \sum_{v_i v_j\notin E(G)} x^{(d_G(v_i)+a)(d_G(v_j)+b)}.$$

Example 2.1. *If $G = K_2\cdot K_3$ is a graph, then the Zagreb co-polynomials of G are as follows:*

$$\overline{M_1}(G,x) = 2x^3, \quad \overline{M_2}(G,x) = 2x^2, \quad \overline{M_3}(G,x) = 2x.$$

Example 2.2. *If C_4 is a cycle of order 4, then the Zagreb co-polynomials of C_4 are as follows:*

$$\overline{M_1}(C_4,x) = 2x^4, \quad \overline{M_2}(C_4,x) = 2x^4, \quad \overline{M_3}(C_4,x) = 2.$$

Remark 2.3. *For the self-complementary graphs the Zagreb polynomials and the Zagreb co-polynomials are always same.*

Proposition 2.4. *The Zagreb co-polynomials of a path, a cycle, a complete graph, a complete bipartite graph and a wheel are as follows:*

(i) For a path P_n of order n, we have

$$\overline{M_1}(P_n,x) = \frac{(n^2-7n+12)}{2}x^4 + 2(n-3)x^3 + x^2,$$

$$\overline{M_2}(P_n,x) = \frac{(n^2-7n+12)}{2}x^4 + 2(n-3)x^2 + x,$$

$$\overline{M_3}(P_n,x) = 2(n-3)x + \frac{(n^2-7n+14)}{2}.$$

(ii) For a cycle C_n of order n, we have

$$\overline{M_1}(C_n,x) = \frac{n(n-3)}{2}x^4, \quad \overline{M_2}(C_n,x) = \frac{n(n-3)}{2}x^4 \quad \text{and} \quad \overline{M_3}(C_n,x) = \frac{n(n-3)}{2}.$$

(iii) For a complete graph of order n, we have

$$\overline{M_i}(K_n,x) = 0, \quad for \quad i = 1,2,3.$$

(iv) For a complete bipartite graph $K_{a,b}$ of order $a+b$, we have

$$\overline{M_1}(K_{a,b},x) = \frac{a(a-1)}{2}x^{2b} + \frac{b(b-1)}{2}x^{2a},$$
$$\overline{M_2}(K_{a,b},x) = \frac{a(a-1)}{2}x^{b^2} + \frac{b(b-1)}{2}x^{a^2}$$
and $\overline{M_3}(K_{a,b},x) = \frac{a(a-1)}{2} + \frac{b(b-1)}{2}$.

(v) For a wheel W_n of order n, we have

$$\overline{M_1}(W_n,x) = \frac{(n^2-5n+4)}{2}x^6,$$
$$\overline{M_2}(W_n,x) = \frac{(n^2-5n+4)}{2}x^9,$$
$$\overline{M_3}(W_n,x) = \frac{(n^2-5n+4)}{2}.$$

Theorem 2.5. *For a graph G, the Zagreb polynomials of its complement $\overline{G}$ are given by*

$M_1(\bar{G},x) = x^{2(n-1)}\,\overline{M}_{-1,-1}(G,x);$
$M_2(\bar{G},x) = \overline{M'}_{(1-n),(1-n)}(G,x);$
$M_3(\bar{G},x) = \overline{M_3}(G,x).$

Proof. The proof follows from the definitions of Zagreb polynomials and the degree of a vertex in the complement of a graph.

III. Generalized xyz-Point-Line Transformation Graph $T^{xyz}(G)$

The procedure of obtaining a new graph from a given graph using adjacency (or nonadjacency) and incidence (or nonincidence) relationship between elements of a graph is known as *Graph Transformation* and the graph obtained by doing so is called a *Transformation graph*. In [2], Wu Bayoindureng et al. introduced the total transformation graphs and studied their basic properties. For a graph $G=(V,E)$, let G^0 be the graph with $V(G^0)=V(G)$ and with no edges, G^1 the complete graph with $V(G^1)=V(G)$, $G^+=G$, and $G^-=\overline{G}$. Let $\mathcal{G}$ denotes the set of simple graphs. The following graph operations depending on $x,y,z\in\{0,1,+,-\}$ induce functions $T^{xyz}:\mathcal{G}\to\mathcal{G}$. These operations are introduced by Deng et al. in [9]. They referred resulting graphs as xyz-transformations of G, denoted by $T^{xyz}(G)=G^{xyz}$ and studied the Laplacian characteristic polynomials and some other Laplacian parameters of xyz-transformations of an r-regular graph G. Motivated by this, Basavanagoud [4] established the basic properties of the xyz-transformation graphs by calling them as *xyz-point-line transformation graphs* by changing the notion of xyz-transformations of a graph G as $T^{xyz}(G)$ to avoid confusion between different transformations of graphs.

Definition 1 *[8] Given a graph G with vertex set $V(G)$ and edge set $E(G)$ and three variables $x,y,z\in\{0,1,+,-\}$, the graphs obtained by the operator $T^{xyz}(G)$ (xyz-point-line transformation graph $T^{xyz}(G)$) on G is the graph with vertex set $V(T^{xyz}(G))=V(G)\cup E(G)$ and the edge set $E(T^{xyz}(G))=E((G)^x)\cup E((L(G))^y)\cup E(W)$, where $W=S(G)$ if $z=+$, $W=\overline{S}(G)$ if $z=-$, W is the graph with $V(W)=V(G)\cup E(G)$ and with no edges if $z=0$ and W is the complete bipartite graph with parts $V(G)$ and $E(G)$ if $z=1$.*

Since there are 64 distinct 3 - permutations of $\{0,1,+,-\}$. Thus obtained 64 kinds of generalized *xyz-point-line transformation graphs*. There are 16 different graphs for each case when $z=0$, $z=1$, $z=+$, $z=-$. In this paper, we consider the *xyz-point-line transformation graphs* $T^{xyz}(G)$ when $z=+$. In which,

- Subdivision graph $S(G)=T^{00+}(G)$, for details see [15],
- Semitotal-point graph $T_2(G)=T^{+0+}(G)$, for details see [22],
- Semitotal-line graph $T_1(G)=T^{0++}(G)$, for details see [14],
- Total graph $T(G)=T^{+++}(G)$, for details see [15].

The Zagreb polynomials of the graph operators $S(G)$, $T_2(G)$ and $T_1(G)$ can be found in [7].

For instance, the total graph $T(G)$ is a graph with vertex set $V(G)\cup E(G)$ and two vertices of $T(G)$ are adjacent whenever they are adjacent or incident in G. The xyz-point-line transformation graph $T^{--+}(G)$ is a graph with vertex set $V(G)\cup E(G)$ and two vertices of $T^{--+}(G)$ are adjacent whenever they are nonadjacent or incident in G.

Example 3.2. *If $G=K_2\cdot K_3$ is a graph, then G^0 be the graph with $V(G^0)=V(G)$ and with no edges, G^1 the complete graph with $V(G^1)=V(G)$, $G^+=G$, and $G^-=\overline{G}$ which are depicted in the following Figure 1:*

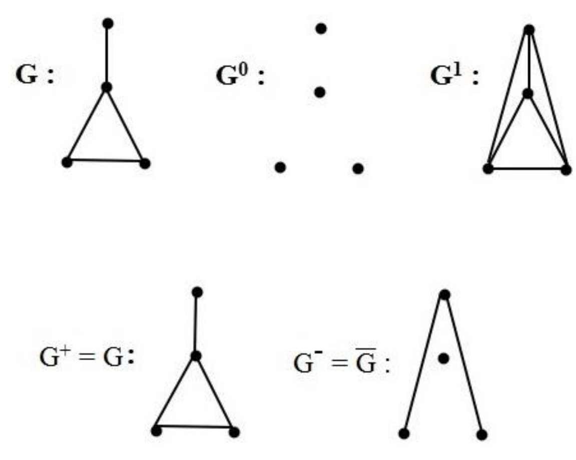

Figure 1.

The self-explanatory examples of the path P_4 and its *xyz-point-line transformation graphs* $T^{xy+}(P_4)$ are depicted in Figure 2.

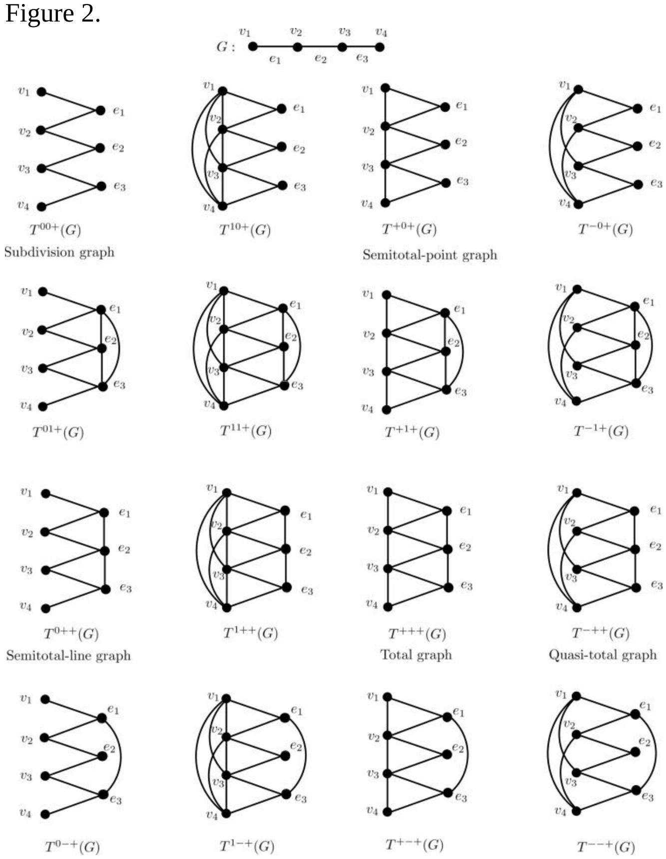

Figure 2.

IV. RESULTS ON THE ZAGREB POLYNOMIALS OF $\mathbf{T^{xy+}(G)}$

In this section, we obtain relations connecting the the *Zagreb polynomials* of a graph G and the *Zagreb polynomials* of *xyz-point-line transformation graphs* $T^{xy+}(G)$.

Let n vertices of the graph G be $v_1, v_2, \ldots, v_n$, and let $e_1, e_2, \ldots, e_m$ be the edges in G, where $n \geq 2$ and $m \geq 1$.

Theorem 4.1. *[5] Let G be a graph of order n, size m. Let v be the point-vertex of $T^{xy+}(G)$ and e be the line-vertex of $T^{xy+}(G)$ corresponding to a vertex v of G and to an edge e of G, respectively. Then*

$$d_{T^{xy+}(G)}(v) = \begin{cases} d_G(v) & if \;\; x = 0, y \in \{0,1,+,-\} \\ n-1+d_G(v) & if \;\; x = 1, y \in \{0,1,+,-\} \\ 2d_G(v) & if \;\; x = +, y \in \{0,1,+,-\} \\ n-1 & if \;\; x = -, y \in \{0,1,+,-\}, \end{cases}$$

$$d_{T^{xy+}(G)}(e) = \begin{cases} 2 & if \;\; x = 0, y \in \{0,1,+,-\} \\ m+1 & if \;\; x = 1, y \in \{0,1,+,-\} \\ 2+d_G(e) & if \;\; x = +, y \in \{0,1,+,-\} \\ m+1-d_G(e) & if \;\; x = -, y \in \{0,1,+,-\}. \end{cases}$$

Theorem 4.2. *For the graph $T^{11+}(G)$, the Zagreb polynomials are given by*

$$M_1(T^{11+}(G),x) = x^{2(n-1)}M_1(G,x) + x^{2(n-1)}\overline{M_1}(G,x) + x^{n+m}M_1^*(G,x) + \frac{1}{2}m(m-1)x^{2(m+1)};$$

$$M_2(T^{11+}(G),x) = M'_{n-1,n-1}(G,x) + \overline{M'}_{n-1,n-1}(G,x) + x^{(n-1)(m+1)}M_3(G,x) + \frac{1}{2}m(m-1)x^{(m+1)^2};$$

$$M_3(T^{11+}(G),x) = M_3(G,x) + \overline{M_3}(G,x) + x^{n-m-2}M_1^*(G,x) + \frac{1}{2}m(m-1).$$

Proof. From Theorem 4.1, we have

$$d_{T^{11+}(G)}(v) = \begin{cases} n-1+d_G(v) & if \;\; v \in V(G), \\ m+1 & if \;\; v \in E(G). \end{cases}$$

Therefore, from Eq. (1.1), we get

$$\begin{aligned} M_1(T^{11+}(G),x) &= \sum_{uv \in E(T^{11+}(G))} x^{d_{T^{11+}(G)}(u)+d_{T^{11+}(G)}(v)} \\ &= \sum_{uv \in E(G)} x^{d_{T^{11+}(G)}(u)+d_{T^{11+}(G)}(v)} + \sum_{uv \notin E(G)} x^{d_{T^{11+}(G)}(u)+d_{T^{11+}(G)}(v)} \\ &\quad + \sum_{uv \in E(S(G))} x^{d_{T^{11+}(G)}(u)+d_{T^{11+}(G)}(v)} + \sum_{u \sim v} x^{d_{T^{11+}(G)}(u)+d_{T^{11+}(G)}(v)} \\ &= \sum_{uv \in E(G)} x^{n-1+d_G(u)+n-1+d_G(v)} + \sum_{uv \notin E(G)} x^{n-1+d_G(u)+n-1+d_G(v)} \\ &\quad + \sum_{uv \in E(S(G))} x^{n-1+d_G(u)+m+1} + \sum_{u \sim v} x^{m+1+m+1} \\ &= x^{2(n-1)}M_1(G,x) + x^{2(n-1)}\overline{M_1}(G,x) + x^{n+m}\sum_{v \in V(G)} d_G(v)x^{d_G(v)} \\ &\quad + \frac{1}{2}m(m-1)x^{2(m+1)}. \\ &= x^{2(n-1)}M_1(G,x) + x^{2(n-1)}\overline{M_1}(G,x) + x^{n+m}M_1^*(G,x) + \frac{1}{2}m(m-1)x^{2(m+1)}. \end{aligned}$$

Now, from Eq. (1.2), we get

$$\begin{aligned} M_2(T^{11+}(G),x) &= \sum_{uv \in E(T^{11+}(G))} x^{d_{T^{11+}(G)}(u) \cdot d_{T^{11+}(G)}(v)} \\ &= \sum_{uv \in E(G)} x^{d_{T^{11+}(G)}(u) \cdot d_{T^{11+}(G)}(v)} + \sum_{uv \notin E(G)} x^{d_{T^{11+}(G)}(u) \cdot d_{T^{11+}(G)}(v)} \\ &\quad + \sum_{uv \in E(S(G))} x^{d_{T^{11+}(G)}(u) \cdot d_{T^{11+}(G)}(v)} + \sum_{u \sim v} x^{d_{T^{11+}(G)}(u) \cdot d_{T^{11+}(G)}(v)} \\ &= \sum_{uv \in E(G)} x^{(n-1+d_G(u))(n-1+d_G(v))} + \sum_{uv \notin E(G)} x^{(n-1+d_G(u))(n-1+d_G(v))} \\ &\quad + \sum_{uv \in E(S(G))} x^{(n-1+d_G(u))(m+1)} + \sum_{u \sim v} x^{(m+1)(m+1)} \\ &= M'_{n-1,n-1}(G,x) + \overline{M'}_{n-1,n-1}(G,x) \\ &\quad + x^{(n-1)(m+1)}M_3(G,x^{m+1}) \\ &\quad + \frac{1}{2}m(m-1)x^{(m+1)^2} \end{aligned}$$

Finally, from Eq. (1.3), we get

$$\begin{aligned} M_3(T^{11+}(G),x) &= \sum_{uv \in E(T^{11+}(G))} x^{\left|d_{T^{11+}(G)}(u)-d_{T^{11+}(G)}(v)\right|} \\ &= \sum_{uv \in E(G)} x^{|d_{T^{11+}(G)}(u)-d_{T^{11+}(G)}(v)|} + \sum_{uv \notin E(G)} x^{|d_{T^{11+}(G)}(u)-d_{T^{11+}(G)}(v)|} \end{aligned}$$

$$
\begin{aligned}
&+ \sum_{uv\in E(S(G))} x^{|d_{T^{11+}(G)}(u)-d_{T^{11+}(G)}(v)|} + \sum_{u\sim v} x^{|d_{T^{11+}(G)}(u)-d_{T^{11+}(G)}(v)|} \\
&= \sum_{uv\in E(G)} x^{|n-1+d_G(u)-n+1-d_G(v)|} + \sum_{uv\notin E(G)} x^{|n-1+d_G(u)-n+1-d_G(v)|} \\
&\quad + \sum_{uv\in E(S(G))} x^{|n-1+d_G(u)-m-1|} + \sum_{u\sim v} x^{|m+1-m-1|} \\
&= M_3(G,x) + \overline{M_3}(G,x) + x^{(n-1)(m+1)} \sum_{v\in V(G)} d_G(v) x^{d_G(v)+(n-m-2)} + \frac{1}{2}m(m-1) \\
&= M_3(G,x) + \overline{M_3}(G,x) + x^{(n-1)(m+1)(n-m-2)} M_1^*(G,x) + \frac{1}{2}m(m-1).
\end{aligned}
$$

Theorem 4.3. *For the graph $T^{01+}(G)$, the Zagreb polynomials are given by*

$$
\begin{aligned}
M_1(T^{01+}(G),x) &= x^{(m+1)} M_1^*(G,x) + \frac{1}{2}m(m-1)x^{2(m+1)}; \\
M_2(T^{01+}(G),x) &= M_3(G,x^{(m+1)}) + \frac{1}{2}m(m-1)x^{(m+1)^2}; \\
M_3(T^{01+}(G),x) &= \frac{1}{x^{(m+1)}} M_1^*(G,x) + \frac{1}{2}m(m-1).
\end{aligned}
$$

Proof. From Theorem 4.1, we get

$$
d_{T^{01+}(G)}(v) = \begin{cases} d_G(v) & if \;\; v\in V(G), \\ m+1 & if \;\; v\in E(G). \end{cases}
$$

Therefore, from Eq. (1.1), we get

$$
\begin{aligned}
M_1(T^{01+}(G),x) &= \sum_{uv\in E(T^{01+}(G))} x^{d_{T^{01+}(G)}(u)+d_{T^{01+}(G)}(v)} \\
&= \sum_{uv\in E(S(G))} x^{d_{T^{01+}(G)}(u)+d_{T^{01+}(G)}(v)} + \sum_{u,v\in E(G)} x^{d_{T^{01+}(G)}(u)+d_{T^{01+}(G)}(v)} \\
&= \sum_{uv\in E(S(G))} x^{d_G(u)+m+1} + \sum_{u,v\in E(G)} x^{m+1+m+1} \\
&= \textstyle\sum_{v\in V(G)} d_G(v) x^{d_G(v)+(m+1)} + \frac{1}{2}m(m-1)x^{2(m+1)} \\
&= x^{(m+1)} M_1^*(G,x) + \frac{1}{2}m(m-1)x^{2(m+1)}.
\end{aligned}
$$

Now, from Eq. (1.2), we get

$$
\begin{aligned}
M_2(T^{01+}(G),x) &= \sum_{uv\in E(T^{01+}(G))} x^{d_{T^{01+}(G)}(u)\cdot d_{T^{01+}(G)}(v)} \\
&= \sum_{uv\in E(S(G))} x^{d_{T^{01+}(G)}(u)\cdot d_{T^{01+}(G)}(v)} + \sum_{u,v\in E(G)} x^{d_{T^{01+}(G)}(u)\cdot d_{T^{01+}(G)}(v)} \\
&= \sum_{uv\in E(S(G))} x^{d_G(u)(m+1)} + \sum_{u,v\in E(G)} x^{(m+1)(m+1)} \\
&= \sum_{v\in V(G)} d_G(v) x^{d_G(v)(m+1)} + \frac{1}{2}m(m-1)x^{(m+1)^2} \\
&= M_3(G,x^{(m+1)}) + \frac{1}{2}m(m-1)x^{(m+1)^2}.
\end{aligned}
$$

Finally, from Eq. (1.3), we get

$$
\begin{aligned}
M_3(T^{01+}(G),x) &= \sum_{uv\in E(T^{01+}(G))} x^{\left|d_{T^{01+}(G)}(u)-d_{T^{01+}(G)}(v)\right|} \\
&= \sum_{uv\in E(S(G))} x^{|d_{T^{01+}(G)}(u)-d_{T^{01+}(G)}(v)|} \\
&\quad + \sum_{u,v\in E(G)} x^{|d_{T^{01+}(G)}(u)-d_{T^{01+}(G)}(v)|} \\
&= \sum_{uv\in E(S(G))} x^{|d_G(u)-m-1|} + \sum_{u,v\in E(G)} x^{|m+1-m-1|} \\
&= \sum_{v\in V(G)} d_G(v) x^{|d_G(v)-(m+1)|} + \frac{1}{2}m(m-1) \\
&= \frac{1}{x^{(m+1)}} M_1^*(G,x) + \frac{1}{2}m(m-1).
\end{aligned}
$$

Theorem 4.4. *For the graph $T^{-0+}(G)$, the Zagreb polynomials are given by*

$$
\begin{aligned}
M_1(T^{-0+}(G),x) &= \left(\binom{n}{2}-m\right)x^{2(n-1)} + 2mx^{(n+1)}; \\
M_2(T^{-0+}(G),x) &= \left(\binom{n}{2}-m\right)x^{(n-1)^2} + 2mx^{2(n-1)}; \\
M_3(T^{-0+}(G),x) &= \binom{n}{2} - m + 2mx^{|n-3|}.
\end{aligned}
$$

Proof. From Theorem 4.1, we have

$$
d_{T^{-0+}(G)}(v) = \begin{cases} n-1 & if \;\; v\in V(G), \\ 2 & if \;\; v\in E(G). \end{cases}
$$

Therefore, from Eq. (1.1), we get

$$
\begin{aligned}
M_1(T^{-0+}(G),x) &= \sum_{uv\in E(T^{-0+}(G))} x^{d_{T^{-0+}(G)}(u)+d_{T^{-0+}(G)}(v)} \\
&= \sum_{uv\notin E(G)} x^{d_{T^{-0+}(G)}(u)+d_{T^{-0+}(G)}(v)} + \sum_{uv\in E(S(G))} x^{d_{T^{-0+}(G)}(u)+d_{T^{-0+}(G)}(v)} \\
&= \sum_{uv\notin E(G)} x^{n-1+n-1} + \sum_{uv\in E(S(G))} x^{n-1+2} \\
&= \left(\binom{n}{2}-m\right)x^{2(n-1)} + 2mx^{(n+1)}.
\end{aligned}
$$

Now, from Eq. (1.2), we get

$$
\begin{aligned}
M_2(T^{-0+}(G),x) &= \sum_{uv\in E(T^{-0+}(G))} x^{d_{T^{-0+}(G)}(u)\cdot d_{T^{-0+}(G)}(v)} \\
&= \sum_{uv\notin E(G)} x^{d_{T^{-0+}(G)}(u)\cdot d_{T^{-0+}(G)}(v)} + \sum_{uv\in E(S(G))} x^{d_{T^{-0+}(G)}(u)\cdot d_{T^{-0+}(G)}(v)} \\
&= \sum_{uv\notin E(G)} x^{(n-1)^2} + \sum_{uv\in E(S(G))} x^{2(n-1)} \\
&= \left(\binom{n}{2}-m\right)x^{(n-1)^2} + 2mx^{2(n-1)}.
\end{aligned}
$$

Finally from Eq. (1.3), we get

$$
\begin{aligned}
M_3(T^{-0+}(G),x) &= \sum_{uv\in E(T^{-0+}(G))} x^{|d_{T^{-0+}(G)}(u)-d_{T^{-0+}(G)}(v)|} \\
&= \sum_{uv\notin E(G)} x^{|d_{T^{-0+}(G)}(u)-d_{T^{-0+}(G)}(v)|} + \sum_{uv\in E(S(G))} x^{|d_{T^{-0+}(G)}(u)-d_{T^{-0+}(G)}(v)|} \\
&= \sum_{uv\notin E(G)} x^{|n-1-n+1|} + \sum_{uv\in E(S(G))} x^{|n-1-2|} \\
&= \binom{n}{2} - m + 2mx^{|n-3|}.
\end{aligned}
$$

Theorem 4.5. *For the graph $T^{-1+}(G)$, the Zagreb polynomials are given by*

$$M_1(T^{-1+}(G),x) = \left(\binom{n}{2}-m\right)x^{2(n-1)} + 2mx^{(n+m)} + \binom{m}{2}x^{2(m+1)};$$
$$M_2(T^{-1+}(G),x) = \left(\binom{n}{2}-m\right)x^{(n-1)^2} + 2mx^{(n-1)(m+1)} + \binom{m}{2}x^{(m+1)^2};$$
$$M_3(T^{-1+}(G),x) = \frac{1}{2}(n(n-1)-m(m+1)) + 2mx^{|n-m-2|}.$$

Proof. From Theorem 4.1, we have
$$d_{T^{-1+}(G)}(v) = \begin{cases} n-1 & if\ \ v \in V(G), \\ m+1 & if\ \ v \in E(G). \end{cases}$$
Therefore, from Eq. (1.1), we get
$$\begin{aligned} M_1(T^{-1+}(G),x) &= \sum_{uv\in E(T^{-1+}(G))} x^{d_{T^{-1+}(G)}(u)+d_{T^{-1+}(G)}(v)} \\ &= \sum_{uv\notin E(G)} x^{d_{T^{-1+}(G)}(u)+d_{T^{-1+}(G)}(v)} + \sum_{uv\in E(S(G))} x^{d_{T^{-1+}(G)}(u)+d_{T^{-1+}(G)}(v)} \\ &\quad + \sum_{u,v\in E(G)} x^{d_{T^{-1+}(G)}(u)+d_{T^{-1+}(G)}(v)} \\ &= \sum_{uv\notin E(G)} x^{2(n-1)} + \sum_{uv\in E(S(G))} x^{(n+m)} + \sum_{u,v\in E(G)} x^{2(m+1)} \\ &= \left(\binom{n}{2}-m\right)x^{2(n-1)} + 2mx^{(n+m)} + \binom{m}{2}x^{2(m+1)}. \end{aligned}$$
Now, from Eq. (1.2), we get
$$\begin{aligned} M_2(T^{-1+}(G),x) &= \sum_{uv\in E(T^{-1+}(G))} x^{d_{T^{-1+}(G)}(u)\cdot d_{T^{-1+}(G)}(v)} \\ &= \sum_{uv\notin E(G)} x^{d_{T^{-1+}(G)}(u)\cdot d_{T^{-1+}(G)}(v)} + \sum_{uv\in E(S(G))} x^{d_{T^{-1+}(G)}(u)\cdot d_{T^{-1+}(G)}(v)} \\ &\quad + \sum_{u,v\in E(G)} x^{d_{T^{-1+}(G)}(u)\cdot d_{T^{-1+}(G)}(v)} \\ &= \sum_{uv\notin E(G)} x^{(n-1)(n-1)} + \sum_{uv\in E(S(G))} x^{(n-1)(m+1)} + \sum_{u,v\in E(G)} x^{(m+1)(m+1)} \\ &= \left(\binom{n}{2}-m\right)x^{(n-1)^2} + 2mx^{(n-1)(m+1)} + \binom{m}{2}x^{(m+1)^2}. \end{aligned}$$
Finally, from Eq. (1.3), we get
$$\begin{aligned} M_3(T^{-1+}(G),x) &= \sum_{uv\in E(T^{-1+}(G))} x^{|d_{T^{-1+}(G)}(u)-d_{T^{-1+}(G)}(v)|} \\ &= \sum_{uv\notin E(G)} x^{|d_{T^{-1+}(G)}(u)-d_{T^{-1+}(G)}(v)|} + \sum_{uv\in E(S(G))} x^{|d_{T^{-1+}(G)}(u)-d_{T^{-1+}(G)}(v)|} \\ &\quad + \sum_{u:v} x^{|d_{T^{-1+}(G)}(u)-d_{T^{-1+}(G)}(v)|} \\ &= \sum_{uv\notin E(G)} x^0 + \sum_{uv\in E(S(G))} x^{|n-m-2|} + \sum_{u,v\in E(G)} x^0 \\ &= \frac{1}{2}(n(n-1)-m(m+1)) + 2mx^{|n-m-2|}. \end{aligned}$$

Theorem 4.6. *For the graph $T^{10+}(G)$, the Zagreb polynomials are given by*
$$M_1(T^{10+}(G),x) = x^{2(n-1)}M_1(G,x) + x^{2(n-1)}\overline{M_1}(G,x) + x^{n+1}M_1^*(G,x);$$
$$M_2(T^{10+}(G),x) = M'_{n-1,n-1}(G,x) + \overline{M'}_{n-1,n-1}(G,x) + x^{2(n-1)}M_3(G,x^2);$$
$$M_3(T^{10+}(G),x) = M_3(G,x) + \overline{M_3}(G,x) + x^{|n-3|}M_1^*(G,x).$$

Proof. From Theorem 4.1, we have
$$d_{T^{10+}(G)}(v) = \begin{cases} n-1+d_G(v) & if\ \ v \in V(G), \\ 2 & if\ \ v \in E(G). \end{cases}$$
Therefore, from Eq. (1.1), we get
$$\begin{aligned} M_1(T^{10+}(G),x) &= \sum_{uv\in E(T^{10+}(G))} x^{d_{T^{10+}(G)}(u)+d_{T^{10+}(G)}(v)} \\ &= \sum_{uv\in E(G)} x^{d_{T^{10+}(G)}(u)+d_{T^{10+}(G)}(v)} + \sum_{uv\notin E(G)} x^{d_{T^{10+}(G)}(u)+d_{T^{10+}(G)}(v)} \\ &\quad + \sum_{uv\in E(S(G))} x^{d_{T^{10+}(G)}(u)+d_{T^{10+}(G)}(v)} \\ &= \sum_{uv\in E(G)} x^{n-1+d_G(u)+n-1+d_G(v)} + \sum_{uv\notin E(G)} x^{n-1+d_G(u)+n-1+d_G(v)} \\ &\quad + \sum_{uv\in E(S(G))} x^{n-1+d_G(u)+2} \\ &= x^{2(n-1)}M_1(G,x) + x^{2(n-1)}\overline{M_1}(G,x) + x^{(n+1)}\sum_{v\in V(G)} d_G(v)x^{d_G(v)} \\ &= x^{2(n-1)}M_1(G,x) + x^{2(n-1)}\overline{M_1}(G,x) + x^{(n+1)}M_1^*(G,x). \end{aligned}$$
Now, from Eq. (1.2), we get
$$\begin{aligned} M_2(T^{10+}(G),x) &= \sum_{uv\in E(T^{10+}(G))} x^{d_{T^{10+}(G)}(u)\cdot d_{T^{10+}(G)}(v)} \\ &= \sum_{uv\in E(G)} x^{d_{T^{10+}(G)}(u)\cdot d_{T^{10+}(G)}(v)} + \sum_{uv\notin E(G)} x^{d_{T^{10+}(G)}(u)\cdot d_{T^{10+}(G)}(v)} \\ &\quad + \sum_{uv\in E(S(G))} x^{d_{T^{10+}(G)}(u)\cdot d_{T^{10+}(G)}(v)} \\ &= \sum_{uv\in E(G)} x^{(n-1+d_G(u))(n-1+d_G(v))} + \sum_{uv\notin E(G)} x^{(n-1+d_G(u))(n-1+d_G(v))} \\ &\quad + \sum_{uv\in E(S(G))} x^{(n-1+d_G(u))(2)} \\ &= M'_{n-1,n-1}(G,x) + \overline{M'}_{n-1,n-1}(G,x) + x^{2(n-1)}\sum_{v\in V(G)} d_G(v)x^{2d_G(v)} \\ &= M'_{n-1,n-1}(G,x) + \overline{M'}_{n-1,n-1}(G,x) + x^{2(n-1)}M_3(G,x^2). \end{aligned}$$
Finally, from Eq. (1.3), we get
$$\begin{aligned} M_3(T^{10+}(G),x) &= \sum_{uv\in E(T^{10+}(G))} x^{|d_{T^{10+}(G)}(u)-d_{T^{10+}(G)}(v)|} \\ &= \sum_{uv\in E(G)} x^{|d_{T^{10+}(G)}(u)-d_{T^{10+}(G)}(v)|} + \sum_{uv\notin E(G)} x^{|d_{T^{10+}(G)}(u)-d_{T^{10+}(G)}(v)|} \\ &\quad + \sum_{uv\in E(S(G))} x^{|d_{T^{10+}(G)}(u)-d_{T^{10+}(G)}(v)|} \\ &= \sum_{uv\in E(G)} x^{|n-1+d_G(u)-n+1-d_G(v)|} + \sum_{uv\notin E(G)} x^{|n-1+d_G(u)-n+1-d_G(v)|} \\ &\quad + \sum_{uv\in E(S(G))} x^{|n-1+d_G(u)-2|} \\ &= M_3(G,x) + \overline{M_3}(G,x) + x^{|n-3|}M_1^*(G,x). \end{aligned}$$

The proof of following theorems are analogous to that of the above theorems.

Theorem 4.7. *For the graph $T^{1-+}(G)$, the Zagreb polynomials are given by*
$$M_1(T^{1-+}(G),x) = x^{2(n-1)}M_1(G,x) + x^{2(n-1)}\overline{M_1}(G,x) + x^{n+m}\sum_{u\sim e} x^{d_G(u)+d_G(e)} + x^{-2(m+1)}\overline{M_1}(L(G),x);$$

$$M_2(T^{1-+}(G),x) = M'_{n-1,n-1}(G,x) + \overline{M'}_{n-1,n-1}(G,x) + \sum_{u\sim e} x^{(n-1+d_G(u))(m+1-d_G(e))} + \overline{M'}_{-(m+1),-(m+1)}(L(G),x);$$

$$M_3(T^{1-+}(G),x) = M_3(G,x) + \overline{M_3}(G,x) + x^{|n-m-2|}\sum_{u\sim e} x^{|d_G(u)-d_G(e)|} + \overline{M_3}(L(G),x).$$

Theorem 4.8. *For the graph $T^{+1+}(G)$, the Zagreb polynomials are given by*

$$M_1(T^{+1+}(G),x) = M_1(G,x^2) + x^{(m+1)}M_1^*(G,x^2) + \binom{m}{2}x^{2(m+1)};$$

$$M_2(T^{+1+}(G),x) = M_2(G,x^4) + M_1^*(G,x^{2(m+1)}) + \binom{m}{2}x^{(m+1)^2};$$

$$M_3(T^{+1+}(G),x) = M_3(G,x^2) + \frac{1}{x^{(m+1)}}M_1^*(G,x^2) + \binom{m}{2}.$$

Theorem 4.9. *For the graph $T^{1++}(G)$, the Zagreb polynomials are given by*

$$M_1(T^{1++}(G),x) = x^{2(n-1)}M_1(G,x) + x^{2(n-1)}\overline{M_1}(G,x) + x^{n+1}\sum_{u\sim e} x^{d_G(u)+d_G(e)} + x^4 M_1(L(G),x);$$

$$M_2(T^{1++}(G),x) = M'_{n-1,n-1}(G,x) + \overline{M'}_{n-1,n-1}(G,x) + \sum_{u\sim e} x^{(n-1+d_G(u))(2+d_G(e))} + M_2(L(G),x^4);$$

$$M_3(T^{1++}(G),x) = M_3(G,x) + \overline{M_3}(G,x) + x^{|n-3|}\sum_{u\sim e} x^{|d_G(u)-d_G(e)|} + M_3(L(G),x).$$

Theorem 4.10. *For the graph $T^{+++}(G)$, the Zagreb polynomials are given by*

$$M_1(T^{+++}(G),x) = M_1(G,x^2) + M_{3,1}(G,x) + x^4 M_1(L(G),x);$$

$$M_2(T^{+++}(G),x) = M_2(G,x^4) + x^2 M_4(G,x) + M'_{2,2}(L(G),x);$$

$$M_3(T^{+++}(G),x) = M_3(G,x^2) + M_3(G,x) + M_3(L(G),x).$$

Theorem 4.11. *For the graph $T^{-++}(G)$, the Zagreb polynomials are given by*

$$M_1(T^{-++}(G),x) = \left(\binom{n}{2} - m\right)x^{2(n-1)} + x^{(n+m-2)}M_1(G,x) + x^{-2(m+1)}M_1(L(G),x);$$

$$M_2(T^{-++}(G),x) = \left(\binom{n}{2} - m\right)x^{2(n-1)} + x^{(n+m-2)}M_1(G,x) + x^{-2(m+1)}M_1(L(G),x);$$

$$M_3(T^{-++}(G),x) = \left(\binom{n}{2} - m\right) + x^{n-m-4}M_1(G,x) + M_3(L(G),x).$$

Theorem 4.12. *For the graph $T^{0-+}(G)$, the Zagreb polynomials are given by*

$$M_1(T^{0-+}(G),x) = x^{m+1}\sum_{u\sim e} x^{d_G(u)-d_G(e)} + x^{2(m+1)}\overline{M_1}(L(G),x);$$

$$M_2(T^{0-+}(G),x) = \sum_{u\sim e} x^{d_G(u)(m+1-d_G(e))} + \overline{M'}_{-(m+1,-(m+1)}(G,x);$$

$$M_3(T^{0-+}(G),x) = x^{|1-m|}M_1^*(G,x) + \overline{M_3}(L(G),x).$$

Theorem 4.13. *For the graph $T^{+-+}(G)$, the Zagreb polynomials are given by*

$$M_1(T^{+-+}(G),x) = M_1(G,x^2) + x^{(m+1)}\sum_{u\sim e} x^{2d_G(u)-d_G(e)} + x^{-2(m+1)}\overline{M_1}(L(G),x);$$

$$M_2(T^{+-+}(G),x) = M_2(G,x^4) + \sum_{u\sim e} x^{2d_G(u)(m+1-d_G(e))} + \overline{M'}_{-(m+1),-(m+1)}(L(G),x);$$

$$M_3(T^{+-+}(G),x) = M_3(G,x^2) + \sum_{u\sim e} x^{|2d_G(u)-m-1+d_G(e)|} + \overline{M_3}(L(G),x).$$

Theorem 4.14. *For the graph $T^{--+}(G)$, the Zagreb polynomials are given by*

$$M_1(T^{--+}(G),x) = \left(\binom{n}{2} - m\right)x^{2(n-1)} + 2x^{-(n+m)}M_0(L(G),x) + x^{-2(m+1)}\overline{M_1}(L(G),x);$$

$$M_2(T^{--+}(G),x) = \left(\binom{n}{2} - m\right)x^{(n-1)^2} + x^{-(n-1)(m+1)}M_0\big(L(G),x^{(n-1)}\big) + \overline{M}'_{-(m+1),-(m+1)}(L(G),x);$$

$$M_3(T^{--+}(G),x) = \left(\binom{n}{2} - m\right) + x^{(n-m-2)}M_0(L(G),x) + \overline{M_3}(L(G),x).$$

V. CONCLUSION

In this paper, we defined a set of new graph polynomials called Zagreb co-polynomials. Further, the Zagreb polynomials of the generalized xyz-point line transformation graphs (for $z = +$) are obtained. In future, one can study the Zagreb co-polynomials of different transformation graphs.

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