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Some Improved Exponential Ratio Type Estimators in Presence of Auxiliary Attributes

B. Mahanty[1*], G. Mishra[2]

[1,2]Department of Statistics, Utkal University, Bhubaneswar, -751004, Odisha, India

[*]*Corresponding Author: bulu.mahanty@gmail.com*

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Abstract- In this paper some improved exponential ratio type estimators of finite population mean have been suggested in presence of auxiliary attributes. For construction of estimator for population mean, we have used the technique of using a priori and a posteriori information on coefficient of variation and auxiliary attributes .The efficiencies of these estimators are compared with the existing estimators and among themselves with regard to biases and mean square errors both theoretically and numerically**.**

***Keywords*:** Auxiliary attributes, Population proportion, Simple random sampling, Exponential ratio type estimators, Bias, Mean square error, Percent relative bias, Efficiency.

I. INTRODUCTION

Use of auxiliary information to increase the efficiency of estimator is a common technique in survey sampling methods. Cochran first suggested a ratio estimator for estimating population mean using the known population mean of auxiliary variable [1]. In survey sampling in many situations auxiliary information is available in form of attributes. The judicious utilization of this auxiliary attributes, several researchers have suggested different improved efficient estimators to estimate finite population mean. Some contributions for estimation population mean using auxiliary attributes are due to Naik and Gupta [2], Jhajj et.al [3], Sabir and Gupta [4, 5], Singh et. al. [6], Koyuncu [7], Singh and Solanki [8], Malik and Singh [9], Singh and Kumar [10, 11], Sharma et.al. [12], Khare et al. [13].

In this paper we proposed some exponential ratio type estimators to estimate population mean using auxiliary attributes and a priori / a posterior information on coefficient variation of study variable.

Seals suggested an estimator to estimate population mean $\overline{Y}$ using known population coefficient variation of study variable [14]. i.e. $C_y = \frac{S_y}{\overline{Y}}$,

Let there be a finite population U consisting of N unit $U = (U_1, U_2, U_3 .... U_i .... U_N)$. The i[th] unit is indexed by a pair of real value (y_i, a_i) where y_i is the study variable and a_i is the auxiliary attribute. It is assumed that y_i and a_i are positively correlated and the correlation coefficient between them is denoted by ρ .

II. PROPOSED ESTIMATORS

From the finite population U, a sample of size 'n' is selected using simple random sampling without replacement (SRSWOR). We denote the sample mean of study variable $\overline{y}$ and sample proportion $(\frac{a}{n})$ = p respectively.

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \overline{Y})^2$. The suggested estimator is given by $\hat{Y}_s = \frac{\overline{y}}{1+\theta_1 C_y^2}$, (2.1)

Where, $\theta_1 = \frac{1}{n} - \frac{1}{N}$.

Following Bhal and Tuteja [15], Singh et.al. [6], have proposed an exponential ratio type estimator of population mean using population proportion (i.e. in presence of auxiliary attribute), is given by

$$t_{ERP1} = \bar{y} \exp\left[\frac{P-p}{P+p}\right] \quad (2.2)$$

Where P and p are the population proportion and sample proportion respectively.

Now we proposed an exponential ratio type estimator of population mean when we have a priori knowledge of coefficient of variation of study variable i.e. C_y and presence of auxiliary attribute.

$$t_{ERP2} = \frac{\bar{y}}{1+\theta C_y^2} \exp\left[\frac{P-p}{P+p}\right] \quad (2.3)$$

Further, if the a priori knowledge of C_y is not known, we can construct an improved estimator by considering the estimate of coefficient of variation of study variable y from the sample. The estimator is given by

$$t_{ERP3} = \frac{\bar{y}}{1+\theta \hat{C}_y^2} \exp\left[\frac{P-p}{P+p}\right] \quad (2.4)$$

Where, $\hat{C}_y^2 = \frac{s_y^2}{\bar{y}^2}$ and

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

Following Upadhyaya and Srivastava [16, 17]. We suggested another estimator

$$t_{ERP4} = \bar{y}(1+\theta \hat{C}_y^2) \exp\left[\frac{P-p}{P+p}\right] \quad (2.5)$$

III. BIAS AND MSE OF DIFFERENT ESTIMATORS

Assuming the validity of Taylor's series expansion of t_{ERP1}, t_{ERP2}, t_{ERP3} and t_{ERP4}, considering the expected value to $O\left(\frac{1}{n}\right)$,the bias of the different estimators are given as.

B (t_{ERP1}) = E ($t_{ERP1} - \bar{Y}$)

$$= \theta_1 \bar{Y}\left[\frac{3}{8}C_{20} - \frac{1}{2}C_{11}\right] \quad (3.1)$$

B (t_{ERP2}) = E ($t_{ERP2} - \bar{Y}$)

$$= \theta_1 \bar{Y}\left[\frac{3}{8}C_{20} - \frac{1}{2}C_{11} - C_{02}\right] \quad (3.2)$$

B (t_{ERP3}) = E ($t_{ERP3} - \bar{Y}$)

$$= \theta_1 \bar{Y}\left[\frac{3}{8}C_{20} - \frac{1}{2}C_{11} - C_{02}\right] \quad (3.3)$$

B (t_{ERP4}) = E ($t_{ERP4} - \bar{Y}$)

$$= \theta_1 \bar{Y}\left[\frac{3}{8}C_{20} - \frac{1}{2}C_{11} + C_{02}\right] \quad (3.4)$$

Where, $C_{rs} = \frac{\mu_{rs}(p,y)}{P^r \bar{Y}^S}$

Where, $\mu_{rs}(p,y)$ in the (r, s) th bivariate moments of p and y.

The Mean Square Error (MSE) of different estimators to $O\left(\frac{1}{n^2}\right)$ are given as

MSE (t_{ERP1}) = E ($t_{ERP1} - \bar{Y}$)^2

$$= \bar{Y}^2\left[\begin{array}{l}\theta_1(C_{02} + \frac{1}{4}C_{20} - C_{11}) + \theta_2(C_{20}C_{02} + 2C_{11}^2 + \frac{5}{4}C_{21} - C_{12} - \frac{3}{8}C_{30} \\ -\frac{31}{8}C_{11}C_{20} + \frac{79}{64}C_{20}^2)\end{array}\right] \quad (3.5)$$

Where, $\theta_2 = \left(\frac{1}{n^2} - \frac{1}{N^2}\right)$

MSE (t_{ERP2}) = E ($t_{ERP2} - \bar{Y}$)^2

$$= \bar{Y}^2\left[\begin{array}{l}\theta_1(C_{02} + \frac{1}{4}C_{20} - C_{11}) + \theta_2(-\frac{1}{4}C_{20}C_{02} + 2C_{11}^2 + \frac{5}{4}C_{21} - C_{12} - \frac{3}{8}C_{30} \\ -\frac{31}{8}C_{11}C_{20} + \frac{79}{64}C_{20}^2 + 3C_{11}C_{02} - C_{02}^2)\end{array}\right]$$

(3.6)

MSE (t_{ERP3}) = E ($t_{ERP3} - \bar{Y}$)^2

$$= \bar{Y}^2 \begin{bmatrix} \theta_1(C_{02} + \frac{1}{4}C_{20} - C_{11}) + \theta_2(-\frac{1}{4}C_{20}C_{02} + 2C_{11}^2 + \frac{5}{4}C_{21} - C_{12} - \frac{3}{8}C_{30} \\ -\frac{31}{8}C_{11}C_{20} + \frac{79}{64}C_{20}^2 + C_{11}C_{02} + C_{02}^2 - 2C_{02}C_{03} + C_{02}C_{12}) \end{bmatrix} \quad (3.7)$$

MSE (t_{ERP4}) = E ($t_{ERP4} - \bar{Y}$)^2

$$= \bar{Y}^2 \begin{bmatrix} \theta_1(C_{02} + \frac{1}{4}C_{20} - C_{11}) + \theta_2(\frac{9}{4}C_{20}C_{02} + 2C_{11}^2 + \frac{5}{4}C_{21} - C_{12} - \frac{3}{8}C_{30} \\ -\frac{31}{8}C_{11}C_{20} + \frac{79}{64}C_{20}^2 - 3C_{11}C_{02} + 3C_{02}^2 + 2C_{02}C_{03} - C_{02}C_{12}) \end{bmatrix}$$

(3.8)

IV. COMPARISON OF BIASES AND MEAN SQUARED ERRORS

The comparisons of efficiencies of different estimators are made under two cases.
Case I: Under general condition
Case II: Under the Bivariate Symmetrical Distribution.

I. t_{ERP2} is more efficient than t_{ERP1} if

Case I : $C_{11} < \frac{1}{3}(\frac{5}{4}C_{20} + 4C_{02})$ (4.1)

Case II : $\rho < \frac{1}{12W}(5W^2 + 4)$ (4.2)

Where, W = $\left(\frac{C_{20}}{C_{02}}\right)^{\frac{1}{2}}$

II. t_{ERP3} is more efficient than t_{ERP1} if

Case I: $C_{11} < \frac{1}{3C_{02}}(\frac{5}{4}C_{20}C_{02} - C_{02}^2 - 2C_{03} + C_{12})$ (4.3)

Case II: $\rho < \frac{1}{12W}(5W^2 - 4)$ (4.4)

III. t_{ERP4} is more efficient than t_{ERP1} if

Case I: $C_{11} > \frac{1}{3C_{02}}(\frac{5}{4}C_{20}C_{02} + 3C_{02}^2 + 2C_{03} - C_{12})$ (4.5)

Case II: $\rho > \frac{1}{12W}(5W^2 + 4)$ (4.6)

When the sample is large enough the biases of the estimators t_{ERP1}, t_{ERP2}, t_{ERP3} and t_{ERP4} of $O\left(\frac{1}{n}\right)$ are negligible .
From the above equation (3.2) and (3.3) both the estimator to t_{ERP2} and t_{ERP3} to $O\left(\frac{1}{n}\right)$ the biases are same i.e.

B (t_{ERP2}) = B (t_{ERP3})

However, the estimators t_{ERP2}, t_{ERP3} and t_{ERP4} are more biased than t_{ERP1}. The mean square errors t_{ERP1}, t_{ERP2}, t_{ERP3} and t_{ERP4} to $O\left(\frac{1}{n}\right)$ are same. Thus for the purpose of comparison of efficiencies, the MSE of the estimators are considered up to second order of approximations.

IV. t_{ERP3} is more efficient than t_{ERP2} if

Case I: $C_{11} < \frac{1}{2C_{02}}(2C_{02}^2 - 2C_{03} + C_{12})$ (4.7)

Case II: $\rho > \frac{1}{W}$ (4.8)

V. t_{ERP4} is more efficient than t_{ERP2} if

Case I: $C_{11} < \frac{1}{6C_{02}}(\frac{5}{2}C_{20}C_{02} + 4C_{02}^2 + 2C_{03} - C_{12})$ (4.9)

Case II: $\rho > \frac{1}{12W}(5W^2 + 8)$
(4.10)

VI. t_{ERP4} is more efficient than t_{ERP3} if

Case I: $C_{11} > \frac{1}{4C_{02}}(\frac{5}{2}C_{20}C_{02} + 2C_{02}^2 - 4C_{03} + 2C_{12})$
(4.11)

Case II: $\rho > \frac{1}{8W}(5W^2 + 16)$
(4.12)

V. NUMERICAL ILLUSTRATION

For comparison of biases and mean square errors, we consider four natural populations. Two populations are considered from Sukhatme and Sukhatme [18]. And two other natural populations are given by Hossain et.al. [19].

Biases are calculated considering terms up to first order of approximation and mean square errors are calculated considering second order of approximations. For calculations we consider four data sets showing N, n, $\overline{Y}$, P and $C_{rs}(p, y)$

Where, $C_{rs}(p, y) = \dfrac{\mu_{rs}(p, y)}{P^r \overline{Y}^s}$

Data Set-1

The data for the empirical analysis are taken from natural population dataset considered by Sukhatme and Sukhatme [1970, P.256]

y = Number of villages in the circles

p = A circle consists of more than three Villages

$\overline{Y} = 1102$, N = 89, n = 16, $C_{20} = 0.7115$, $C_{02} = 0.4181$, $C_{11} = 0.2116$ $C_{30} = -0.2052$, $C_{03} = 0.3493$, $C_{12} = 0.1281$, $C_{21} = -0.0610$

Data Set-2

The data for the empirical analysis are taken from natural population dataset considered by Sukhatme and Sukhatme [1970, P.256]

y = Number of villages in the circles

p = A circle consists of more than three Villages

$\overline{Y} = 983.125$, N = 40, n = 10, $C_{20} = 0.4815$, $C_{02} = 0.3657$, $C_{11} = 0.1810$ $C_{30} = -0.2496$, $C_{03} = 0.2363$, $C_{12} = 0.0514$, $C_{21} = -0.0938$

Data Set-3

Due to Hossain et.al. [19]

The data for the empirical analysis are taken from 1981, utter Pradesh District Census Handbook, Aligar. The population consist of 340 villages under Koil police station, with y = Number of agricultural labour in 1981 and p=Area of the villages (in acre) in 1981. The following values are obtained

y = Number of agricultural labour in 1981

p = Area of the Villages (in acres) in 1981

$\overline{Y} = 73.76765$, N = 340, n = 70, $C_{20} = 0.5557$, $C_{02} = 0.7614$, $C_{11} = 0.2667$ $C_{30} = 0.7877$, $C_{03} = 2.6942$, $C_{12} = 0.0747$, $C_{21} = 0.1589$

Data Set-4

Due to Hossain et.al. [19]

The data for the empirical analysis are taken from 1981, utter Pradesh District Census Handbook, Aligar. The population consist of 340 villages under Koil police station, with y = Number of cultivators in the villages in 1981 and p=Area of the villages (in acre) in 1981. The following values are obtained

y = Number of cultivators in the villages in 1981

p = Area of the Villages (in acres) in 1981

$\overline{Y} = 141.1294$, N = 340, n = 70, $C_{20} = 0.5944$, $C_{02} = 0.7614$, $C_{11} = 0.2667$ $C_{30} = 0.7877$, $C_{03} = 2.6942$, $C_{12} = 0.4720$, $C_{21} = 0.4897$

TABLE .I PERCENT RELATIVE BIAS OF ESTIMATORS

(t_{ERP1}, t_{ERP2}, t_{ERP3} and t_{ERP4}) up to $O\left(\frac{1}{n}\right)$

Data set No.	t_{ERP1}	t_{ERP2}	t_{ERP3}	t_{ERP4}
1	0.0588	0.09569	0.09613	0.20297
2	0.04465	0.13935	0.13969	0.20864
3	0.04465	0.13935	0.13969	0.20864
4	0.01189	0.08964	0.09212	0.10827

TABLE. II. MEAN SQUARE ERROR OF ESTIMATORS

($\overline{y}$, t_{ERP1}, t_{ERP2}, t_{ERP3} and t_{ERP4}) up to $O\left(\frac{1}{n^2}\right)$

Data set No.	$t_0 = \overline{y}$	t_{ERP1}	t_{ERP2}	t_{ERP3}	t_{ERP4}
1	2602.92	23929.36	23031.78	22821.75	25979.56
2	26509.63	22114.92	21270.77	21167.78	25941.4
3	47.0036	39.1157	38.7659	36.4601	42.7819
4	50545.88	42705.76	42301.9	40050.08	46514.7

VI. CONCLUSIONS

1. It is observed that the suggested estimators t_{ERP2}, t_{ERP3} and t_{ERP4} are more biased than the estimator t_{ERP1}. However the biases are negligible if sample size is large.
2. Comparing the biases of t_{ERP1}, t_{ERP2}, t_{ERP3} and t_{ERP4} we observed

   $B(t_{ERP1}) < [B(t_{ERP2}) = B(t_{ERP3})] < B(t_{ERP4})$
3. Considering the value of MSE of mean per unit estimator $(\bar{y})$, t_{ERP1}, t_{ERP2}, t_{ERP3} and t_{ERP4} we find the MSE of t_{ERP3} is most efficient and we write the inequality

   $MSE(t_{ERP3}) < MSE(t_{ERP2})$

   $< MSE(t_{ERP1}) < MSE(t_{ERP4}) < V(\bar{y})$

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