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FIXED POINT THEOREMS IN C-FUZZY METRIC SET IN FUZZY METRIC SPACES

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Abstract: In this paper the fixed point theorems for commute c-fuzzy metric set satisfying the generalized Lipschitzconditions are obtained, without appealing to continuity formappings in the setting of fuzzy metric spaces over the Banach algebra. Furthermore, we notonly get the existence of the fixed point but also get the uniqueness.These results greatly improve and generalize several well-knowncomparable results in the literature.

Key-words**:** Fixed point, c-fuzzy metric set, Fuzzy metric spaces, commuting, invertible,Lipschitzconditions.

I. INTRODUCTION

Fuzzy set theory was introduced by the Electrical Engineer L . A. Zadeh in 1965. In 2002, Lii, introduced the concept of converse commuting functions and proved the fixed point theorems for converse commuting functions. Various authors have proved generalized fixed point theorems for multi valued converse commuting mappings in the setting of metric spaces. Some related examples are also discussed. In this paper the fixed point theorems for commute c-fuzzy metric set satisfying the generalized Lipschitz conditions are proved, in the setting of fuzzy metric spaces over the Banach algebra. Furthermore, the uniqueness of the fixed point also proved . Our main results improve and generalize some importantknown results in the literature. In addition, we introduced c-fuzzy metric set and also proved the existence of the fixed point it,the main results are indeedreal improvements and generalizations of the correspondingresults in the literature.

II. PRELIMINARIES

Definition 1: A binary operation$*: [0,1] \times [0.1] \rightarrow [0,1]$ is continuous $t - norm$ if $*$ satisfies the following conditions:

i. $a * b = b * a$, $a * (b * c) = (a * b) * c$ for all $a, b, c \in [0,1]$
ii. $*$ is continuous
iii. $a * 1 = a \, for \, all \, a \in [0,1]$
iv. $a * b \leq c * d \, , whenever \; a \leq c \, and \, b \leq d, \; a, b, c, d \in [0,1].$

Definition 2: The triplet $(X, M,*)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous $t - norm$and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following:
(M1). $M(x, y, t) > 0$
(M2). $M(x, y, t) = 1 \; if \; and \; only \; if \; x = y.$
(M3). $M(x, y, t) = M(y, x, t).$
(M4). $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s).$
(M5).$M(x, y,\cdot): [0, \infty) \rightarrow [0,1]$is left continuous for all $x, y, z \in X$ and $s, t > 0$.

Example 1.(**Induced fuzzy metric space**) Let (X, d)be a metric space and $a * b = ab$ for all $a, b \in [0,1] and \; let \; M_d$ be fuzzy set on $X^2 \times [0, \infty)$defined as follows:$M_d(x, y, t) = \frac{t}{t+d(x,y)}$ then $(x, M_d,*)$ is a fuzzy metric space. We call this fuzzy metric induced by a metric d.
Example 2: Let $X = N. \; Define \; a * b = \max\{0, a + b - 1\} \; for \; all \; a, b \in [0,1]$and let M be a fuzzy set on $X^2 \times [0, \infty)$ as follows:

$$M(x,y,t) = \begin{cases} \frac{x}{y}, & if\ x \le y \\ \frac{y}{x}, & if\ y \le x \end{cases}$$

for all $x, y \in X$. Then $(X, M,*)$is a fuzzy metric space.

Definition 3: Let $(X, M,*)$ be a fuzzy metric space. A sequence $\{x_n\}$in X is said to be convergent to x if for each $\varepsilon > 0$ and each $t > 0$ then there exists$n_0 \in N\ such\ that\ \ M(x_n, x, t) > 1 - \varepsilon$ for all $n > n_0$.

Definition 4: A sequence $\{x_n\}$ in X is said to be Cauchy if for each $\varepsilon > 0$ and $t > 0$ then there exists$n_0 \in N\ such\ that\ \ M(x_n, x_m, t) > 1 - \varepsilon$ for all $n, m > n_0$.

Definition 5:A fuzzy metric space is said to be complete if in which every Cauchy sequence is convergent.
Preposition1: In a fuzzy metric space$(X, M,*)$ if$a * a \ge a\ \ for\ all\ a \in [0,1]$, then$a * b = \min\{a, b\}\ for\ all\ a, b \in [0,1]$.Let$(X, M,*)$be a fuzzy metric space with the condition $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$.

Definition 6: Let $A, B : X \to X$ and $ABx = BAx$ then $x \in X$ is called a commuting point of A, B.

Definition7: Let $(X, M,*)$be a fuzzy metric space. Functions $A, B : X \to X$are said to be converse commuting if $ABx = BAx$implies $Ax = Bx$.

Definition 8
Let A be a real Banach algebra; that is, Ais a real Banach space in which an operation of multiplication is defined, subject to the following properties: for all x, y, z∈A, a∈ R,
(1) x(yz) = (xy)z;
(2) x(y + z) = xy + xz and (x + y)z = xz + yz;
(3) a(xy) = (ax)y = x(ay);
(4) $\|xy\| \le \|x\|\|y\|$.

In this paper, we shall assume that the Banach algebra A has a unit such that $ex = xe = x$ for all x∈A. An element x∈A is said to be invertible ifthere is an inverse element y∈A such that
$xy = yx = e$. The inverse of x is denoted by x^{-1}.
Proposition 2.*Let* A be a real Banach algebra with a unit e and $x \in A$.If the spectral radius $r(x)$ of x is less than 1, that is $r(x) = \log_{n\to\infty} \|x^n\|^{1/n} = \inf\|x^n\|^{1/n} < 1$, then $e - x$ is invertible. Actually $(e - x)^{-1} = \sum_{i=0}^{\infty} x^i$.

Definition9. Let (X, M, *) be a fuzzy metric space. Then M is called c- fuzzy set on $X^2 \times [0,\infty)$ satisfying the following:
(1)$\left(\frac{1}{M(x,y,t)} - 1\right) \ge 0$for all x, y∈X.
(2)$\left(\frac{1}{M(x,y,t)} - 1\right) \le \left(\frac{1}{M(x,z,t)} - 1\right) + \left(\frac{1}{M(z,y,t)} - 1\right)$for all x, y, z∈X.
(3) For each x∈X and $n \ge 1$, if $\left(\frac{1}{M(x,y_n,t)} - 1\right) \le u$ for some$u = u_x \in X$, then
$\left(\frac{1}{M(x,y,t)} - 1\right) \le u$whenever$\{y_n\}$ is a sequence in X converging to a point$y \in X$.
(4) For all c ∈A with$c \ge 0$, there exists e ∈A,such that
$\left(\frac{1}{M(z,x,t)} - 1\right) \le e\ and\ \left(\frac{1}{M(z,y,t)} - 1\right) \le e$imply$\left(\frac{1}{M(x,y,t)} - 1\right) \le c$.
M is also called a c-fuzzy metric on X.

Lemma 1 [9]Let Abea Banach algebra with a unit e. If x, y∈A, and x commutes with y, then
$r\,(x + y) \le r\,(x) + (y)$,$r\,(xy) \le r\,(x)\,r\,(y)$ …………………(4)
Lemma 2[11]. Let A be a Banach algebra with a unit e and let k be a vector in A.If $0 \le r(k) < 1$, then we have $r((e - k)^{-1}) \le (1 - r(k))^{-1}$.
Lemma 3 [4].Let (X, M, *) be a fuzzy metric space and Mbe a c-fuzzy metric on X. Let $\{x_n\}$ be a sequence in X. Suppose that $\{\alpha_n\}$ and $\{\beta_n\}$ are two sequences in X converging to 0.
If $\left(\frac{1}{M(x_n,y,t)} - 1\right) \le \alpha_n$ and $\left(\frac{1}{M(x_n,z,t)}\right) \le \beta_n$, then $y = z$.

Lemma 4[6].Let A be a Banach algebra with a unit e and X be a c-fuzzy metric space in A. $Let\ u, \alpha, \beta \in X\ hold\ \alpha \leq \beta\ and\ u \leq \alpha u.\ If\ r(\beta) < 1, then\ u = 0.$

III. MAIN RESULTS

Theorem 1. Let $(X$, M, *) be a complete fuzzy metric space over Banach algebra A. Let M be a c-fuzzy metric on X. Suppose the mapping M satisfies generalized Lipschitz conditions:

$$\left(\frac{1}{M(fx,fy,t)}-1\right) \leq \alpha\left(\frac{1}{M(x,y,t)}-1\right)+\beta\left(\frac{1}{M(x,fx,t)}-1\right)+\gamma\left(\frac{1}{M(x,fy,t)}-1\right)+\delta\left(\frac{1}{M(fx,y,t)}-1\right) \quad \ldots\ldots\ldots(1)$$

$$\left(\frac{1}{M(fy,fx,t)}-1\right) \leq \alpha\left(\frac{1}{M(y,x,t)}-1\right)+\beta\left(\frac{1}{M(fx,x,t)}-1\right)+\gamma\left(\frac{1}{M(fy,x,t)}-1\right)+\delta\left(\frac{1}{M(y,fx,t)}-1\right) \quad \ldots\ldots(2)$$

for all $x, y \in X$, where $\alpha, \beta, \gamma, \delta \in X$ are generalized Lipschitz constants with $r(\alpha+\beta+\gamma+2\delta) < 1 - r(\gamma)$. If γ commutes with $\alpha+\beta+\gamma+2\delta$, then f has a unique fixed point.

Proof. Suppose x_0 is an arbitrary point in X and set $x_n = fx_{n-1} = f^n x_0$.

$$\left(\frac{1}{M(x_n, x_{n+1}, t)}-1\right) = \left(\frac{1}{M(fx_{n-1}, fx_n, t)}-1\right)$$

$$\leq \alpha\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\beta\left(\frac{1}{M(x_{n-1},fx_{n-1},t)}-1\right)+\gamma\left(\frac{1}{M(x_{n-1},fx_n,t)}-1\right)+\delta\left(\frac{1}{M(fx_{n-1},x_n,t)}-1\right)$$

$$\leq \alpha\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\beta\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\gamma\left(\frac{1}{M(x_{n-1},x_{n+1},t)}-1\right)+\delta\left(\frac{1}{M(x_n,x_n,t)}-1\right)$$

$$\leq \alpha\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\beta\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\gamma\left\{\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)\right\}$$
$$+\delta\left\{\left(\frac{1}{M(x_n,x_{n-1},t)}-1\right)+\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)\right\}$$

Whichimplies

$$(1-\gamma)\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right) \leq (\alpha+\beta+\gamma+\delta)\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\delta\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right) \quad \ldots\ldots\ldots(3)$$

By (1) and (2) we obtain

$$\left(\frac{1}{M(x_{n+1}, x_n, t)}-1\right) = \left(\frac{1}{M(fx_n, fx_{n-1}, t)}-1\right)$$

$$\leq \alpha\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\beta\left(\frac{1}{M(fx_{n-1},x_{n-1},t)}-1\right)+\gamma\left(\frac{1}{M(fx_n,x_{n-1},t)}-1\right)+\delta\left(\frac{1}{M(x_n,fx_{n-1},t)}-1\right)$$

$$\leq \alpha\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\beta\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\gamma\left(\frac{1}{M(x_{n-1},x_{n+1},t)}-1\right)+\delta\left(\frac{1}{M(x_n,x_n,t)}-1\right)$$

$$\leq \alpha\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\beta\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\gamma\left\{\left(\frac{1}{M(x_n,x_{n-1},t)}-1\right)+\left(\frac{1}{M(x_{n+1},x_n,t)}-1\right)\right\}$$
$$+\delta\left\{\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\left(\frac{1}{M(x_n,x_{n-1},t)}-1\right)\right\}$$

Whichimplies

$$(1-\gamma)\left(\frac{1}{M(x_{n+1},x_n,t)}-1\right) \leq (\alpha+\beta+\gamma+\delta)\left(\frac{1}{M(x_n,x_{n-1},t)}-1\right)+\delta\left(\frac{1}{M(x_n,x_{n-1},t)}-1\right) \quad \ldots\ldots\ldots(4)$$

Thus, the sum of (3) and (4) gives follows

$$(1-\gamma)\left[\left(\frac{1}{M(x_n, x_{n+1}, t)}-1\right)+\left(\frac{1}{M(x_{n+1}, x_n, t)}-1\right)\right]$$
$$\leq (\alpha+\beta+\gamma+2\delta)\left[\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)+\left(\frac{1}{M(x_n,x_{n-1},t)}-1\right)\right]$$

Now, we set $u_n = \left[\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right)+\left(\frac{1}{M(x_{n+1},x_n,t)}-1\right)\right]$.

Then, $(1-\gamma)u_n \geq (\alpha+\beta+\gamma+2\delta)u_{n-1}$ ... ... (5)

Since $r(\gamma) < r(\gamma) + r(\alpha+\beta+\gamma+2\delta) < 1$, then, by Proposition 1, $(1-\gamma)$is invertible. Furthermore, $(1-\gamma)^{-1} = \sum_{i=0}^{\infty}\gamma^i$

Let $R = (1-\gamma)^{-1}(\alpha+\beta+\gamma+2\delta)$. As γcommutes with $\alpha+\beta+\gamma+2\delta$, it follows that$(1-\gamma)^{-1}(\alpha+\beta+\gamma+2\delta) = (\sum_{i=0}^{\infty}\gamma^i)(\alpha+\beta+\gamma+2\delta)$

$$= (\alpha+\beta+\gamma+2\delta)\left(\sum_{i=0}^{\infty}\gamma^i\right)$$

$$= (\alpha+\beta+\gamma+2\delta)(1-\gamma)^{-1} \ldots\ldots\ldots (6)$$

that is to say $(1-\gamma)^{-1}$commutes with $(\alpha+\beta+\gamma+2\delta)$.

Then by Lemma 1 and Lemma 2, we obtain

$$r(R) = r((1-\gamma)^{-1}(\alpha+\beta+\gamma+2\delta))$$
$$\leq r((1-\gamma)^{-1})\, r(\alpha+\beta+\gamma+2\delta)$$
$$\leq \left(1-r(\gamma)\right)^{-1} r(\alpha+\beta+\gamma+2\delta)$$
$$= \left(\frac{1}{1-r(\gamma)}\right) r(\alpha+\beta+\gamma+2\delta) < 1, \qquad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Which means that $(1-R)^{-1} = (\sum_{i=0}^{\infty} R^i)$and $\|R^n\| \to 0 \; as \; n \to \infty$.

By multiplying in both sides of (5) by $(1-\gamma)^{-1}$, we get

$$\left(\frac{1}{M(x_n,x_{n+1},t)}-1\right) \leq u_n$$
$$\leq (1-\gamma)^{-1}(\alpha+\beta+\gamma+2\delta)u_{n-1}$$

$= Ru_{n-1} \leq \cdots \leq R^n u_0$………………(8)

Let $m>n \geq 1$. We infer

$$\left(\frac{1}{M(x_n,x_m,t)}-1\right) \leq \left(\frac{1}{M(x_n,x_{n+1},t)}-1\right) + \left(\frac{1}{M(x_{n+1},x_{n+2},t)}-1\right) + \cdots + \left(\frac{1}{M(x_{m-1},x_m,t)}-1\right)$$
$$\leq (R^n + R^{n+1} + \cdots + R^{m-1})\, u_0$$
$$\leq (1 + R + \cdots + R^{m-n-1})\, R^n u_0$$

$\leq (\sum_{i=0}^{\infty} R^i) R^n u_0 = (1-R)^{-1} R^n u_0$

Owing to $\|(1-R)^{-1}R^n u_0\| \to 0 \; (n \to \infty)$, it leads to $(1-R)^{-1}R^n u_0 \to 0 \; (n \to \infty)$.

$\Longrightarrow \{x_n\}$ is a Cauchy sequence in (X, M, *).

Since X is complete, there exists $u \in X$ such that $x_n = fx_{n-1} \to u$ as $n \to \infty$.

By Definition, we obtain $\left(\frac{1}{M(x_n,u,t)}-1\right) \leq (1-R)^{-1}R^n u_0$……………(9)

Now, we show that $fu = u$. Substituting $x = x_{n-1}$, $y = u$ in (1), we get

$$\left(\frac{1}{M(x_n,fu,t)}-1\right) = \left(\frac{1}{M(fx_{n-1},fu,t)}-1\right)$$
$$\leq \alpha\left(\frac{1}{M(x_{n-1},u,t)}-1\right) + \beta\left(\frac{1}{M(x_{n-1},fx_{n-1},t)}-1\right) + \gamma\left(\frac{1}{M(x_{n-1},fu,t)}-1\right) + \delta\left(\frac{1}{M(fx_{n-1},u,t)}-1\right)$$
$$\leq \alpha\left(\frac{1}{M(x_{n-1},u,t)}-1\right) + \beta\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right) + \gamma\left(\frac{1}{M(x_{n-1},fu,t)}-1\right) + \delta\left(\frac{1}{M(x_n,u,t)}-1\right)$$
$$\leq \alpha\left[\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right) + \left(\frac{1}{M(x_n,u,t)}-1\right)\right] + \beta\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right)$$
$$+ \gamma\left[\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right) + \left(\frac{1}{M(x_n,fu,t)}-1\right)\right] + \delta\left(\frac{1}{M(x_n,u,t)}-1\right)$$

which implies that

$$(1-\gamma)\left(\frac{1}{M(x_n,fu,t)}-1\right) \leq (\alpha+\beta+\gamma)\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right) + (\alpha+\delta)\left(\frac{1}{M(x_n u,t)}-1\right).$$

Since $r(\gamma) < 1$, $(1-\gamma)$ is invertible. So, it follows immediately from (8) and (9) that

$$\left(\frac{1}{M(x_n,fu,t)}-1\right) \leq (1-\gamma)^{-1}\left[(\alpha+\beta+\gamma)\left(\frac{1}{M(x_{n-1},x_n,t)}-1\right) + (\alpha+\delta)\left(\frac{1}{M(x_n u,t)}-1\right)\right]$$
$$\leq (1-\gamma)^{-1}\left[(\alpha+\beta+\gamma)R^{n-1}u_0 + (\alpha+\delta)(1-R)^{-1}R^n u_0\right]$$

$\leq (1-\gamma)^{-1}\left[(\alpha+\beta+\gamma) + (\alpha+\delta)(1-R)^{-1}R\right]R^{n-1}u_0$ ………..(10)

Set $\alpha_n = (1-R)^{-1}R^n u_0$and

$$\beta_n = (1-\gamma)^{-1}\left[(\alpha+\beta+\gamma) + (\alpha+\delta)(1-R)^{-1}R\right]R^{n-1}u_0 .$$

As $r(R) < 1$ and $R^n \to 0$ $(n \to \infty)$, we know αn, $\beta_n \to 0$ $(n \to \infty)$.Thus, by (9), (10), and Lemma 3, we get that $fu = u$.In the following we shall show the fixed point is unique.

Firstly, we have to prove $\left(\frac{1}{M(u,u,t)}-1\right) = 0$. Making full use of (1),we get

$$\left(\frac{1}{M(u,u,t)}-1\right) = \left(\frac{1}{M(fu,fu,t)}-1\right)$$
$$\leq \alpha\left(\frac{1}{M(u,u,t)}-1\right) + \beta\left(\frac{1}{M(u,fu,t)}-1\right) + \gamma\left(\frac{1}{M(u,fu,t)}-1\right) + \delta\left(\frac{1}{M(fu,u,t)}-1\right)$$

$= (\alpha + \beta + \gamma + \delta)\left(\frac{1}{M(u,u,t)} - 1\right)$………………..(11)

In view of $\alpha + \beta + \gamma + \delta \leq \alpha + \beta + \gamma + 2\delta$, $r(\alpha + \beta + \gamma + 2\delta) < 1 - r(\gamma) < 1$, and Lemma 4, we have $\left(\frac{1}{M(u,u,t)} - 1\right) = 0$. Secondly, if there is another fixed point v. Then by (1), we get

$$\left(\frac{1}{M(v,u,t)} - 1\right) = \left(\frac{1}{M(fv,fu,t)} - 1\right)$$
$$\leq \alpha\left(\frac{1}{M(v,u,t)} - 1\right) + \beta\left(\frac{1}{M(v,fv,t)} - 1\right) + \gamma\left(\frac{1}{M(v,fu,t)} - 1\right) + \delta\left(\frac{1}{M(fv,u,t)} - 1\right)$$
$$\leq \alpha\left(\frac{1}{M(v,u,t)} - 1\right) + \beta\left(\frac{1}{M(v,v,t)} - 1\right) + \gamma\left(\frac{1}{M(v,u,t)} - 1\right) + \delta\left(\frac{1}{M(v,u,t)} - 1\right)$$

which establishes that

$\left(\frac{1}{M(v,u,t)} - 1\right) \leq (\alpha + \gamma + \delta)\left(\frac{1}{M(v,u,t)} - 1\right)$……… (12)

Since $\alpha + \gamma + \delta \leq \alpha + \beta + \gamma + 2\delta$ *and* $r(\alpha + \beta + \gamma + 2\delta) < 1 - r(\gamma) < 1$, it follows immediately from Lemma 1.4 that $\left(\frac{1}{M(v,u,t)} - 1\right) = 0$.

Actually, by (2), we also have that $\left(\frac{1}{M(u,v,t)} - 1\right) \leq (\alpha + \gamma + \delta)\left(\frac{1}{M(u,v,t)} - 1\right)$

Similar to the above proof, it is not difficult to obtain that $\left(\frac{1}{M(u,v,t)} - 1\right) = 0$.

Thus, $u = v$, this concludes the theorem.

Corollary 1. Let (X, M, *) be a complete fuzzy metric space over Banach algebra A. Let M be a c-fuzzy metric on X. Suppose the mapping M satisfies generalized Lipchitz conditions:

$$\left(\frac{1}{\mathrm{M\ (fx, fy, t)}} - 1\right) \leq \alpha\left(\frac{1}{\mathrm{M(x, y, t)}} - 1\right) + \beta\left(\frac{1}{\mathrm{M(x, fx, t)}} - 1\right) + \gamma\left(\frac{1}{\mathrm{M(x, fy, t)}} - 1\right) + \delta\left(\frac{1}{\mathrm{M(fx, y, t)}} - 1\right) \quad \text{………………(1)}$$

$$\left(\frac{1}{\mathrm{M\ (fy, fx, t)}} - 1\right) \leq \alpha\left(\frac{1}{\mathrm{M(y, x, t)}} - 1\right) + \beta\left(\frac{1}{\mathrm{M(fx, x, t)}} - 1\right) + \gamma\left(\frac{1}{\mathrm{M(fy, x, , t)}} - 1\right) + \delta\left(\frac{1}{\mathrm{M(y, fx, t)}} - 1\right) \quad \text{………………(2)}$$

for all $x, y \in X$, where $\alpha, \beta, \gamma, \delta \in \mathrm{R}^+$ and satisfies $\alpha + \beta + \gamma + 2\delta < 1$. Then, f has a unique fixed point.

Proof. By taking $r(\alpha) = \alpha$, $r(\beta) = \beta$, $r(\gamma) = \gamma$ and $r(\delta) = \delta$ for each $\alpha, \beta, \gamma, \delta \in \mathrm{R}^+$ in above theorem, we obtain the desired result.

REFERENCES

[1] M. Abbas and B. E. Rhoades, "Common fixed point theorems for hybrid pairs of occasionally weakly compatible mappings satisfying generalized condition of integral type", Fixed Point Theory Appl., **(2007), Art. ID 54101, 9pp.**

[2] M. A. Al-Thagafi and N.Shahzad," A note on occasionally weakly compatible maps", Int. J. Math. Anal.**3(2), (2009), 55-58**.

[3] M. A. Al-Thagafi and N. Shahzad, "Generalized I-non expansive self-maps and invariantapproximation", ActaMathematicaSinica, English Series **24, (2008), 867-876**.

[4] H. Aydi, E. Karapinar, and S. Moradi, "Coincidence points for expansive mappingsunder c-distance in cone metric spaces," International Journal of Mathematics andMathematical Sciences, vol. 2012,Article **ID 308921, 2012.**

[5] Y. J. Cho, R. Saadati, and S. H. Wang, "Common fixed point theorems on generalizeddistance in ordered cone metric spaces," Computers & Mathematics with Applications,vol. **61**, no.**4**, pp. **1254–1260, 2011.**

[6] H. Huang, S. Radenovi´c, and T.Doˇsenovi´c, "Some common fixed point theorems onc-distance in cone metric spaces over banach algebras," Applied and ComputationalMathematics, vol.**14,** no. **2**, pp. **180–193, 2015**

[7] H. Huang, S. Hu, B. Z. Popovi´c, and S. Radenovi´c, "Common fixed point theorems forfour mappings on cone b-metric spaces over Banach algebras," Journal of NonlinearScience and Applications, vol. **9**, no. **6**, pp. **3655–3671, 2016.**

[8] Dr. R Krishnakumar and NagaralPanditSanatammappa, study on two Non-ArchimedianMenger Probability metric space, International Journal of Statistics and Applied Mathematics **1(3), 2016, 01-04**

[9] R. Krishnakumar, R. Livingston, "Study On Fixed Point Theorems in Complete Cone Metric Spaces"InternationalJournal of Innovative Research in Science, Engineering and Technology,Volume **6**,Issue **7**, July **2017, 13526 – 13531.**

[10] R. Krishnakumar , K. Dinesh , D. Dhamodharan, Some fixed point theoremsφ- ψ weak contractiononFuzzyMetricspaces, International Journal of Scientific Research in Mathematicaland Statistical Sciences, Volume-**5**, Issue-**3**,pp.**146- 152,** June **(2018)**

[11] W. Rudin, "Functional Analysis", McGraw-Hill, New York, NY, **USA**,**2**nd edition, **1991.**

[12]S. Wang and B. Guo, "Distance in cone metric spaces and common fixed pointtheorems," Applied Mathematics Letters, vol. **24**, no. **10**, pp. **1735–1739, 2011.**

[13] S. Xu and S. Radenovi´c, "Fixed point theorems of generalized Lipschitz mappings oncone metric spaces over Banach algebras without assumption of normality," Fixed PointTheory and Applications, vol. **2**, no. **1**, pp. **102 -111, 2014.**

