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Research Paper

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Some Problems of Non Homogeneous Differential Equation of First Order and First Degree, Solved By the Method of Homogeneous Differential Equation of First Order and First Degree

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***Abstract*—** This is based on the some Non homogeneous differential equation of first order and first degree which is solved by the method of homogeneous differential equation of first order and first degree, because some types of non homogeneous differential equation problems are not solved by all the method of first order and first degree except homogeneous differential equation method of first order and first degree. This method gives a perfect general solution of some non homogeneous differential equations of first order and first degree.

Keywords—Differential equation, First order, First degree, Variable seprable, Homogeneous differential equation, Non homogeneous differential equation, Linear differential equation, bernoulli difference equation, Exact differential equation.

I. INTRODUCTION

Some problems which is Non-homogeneous differential equation of first order and first degree solved by the method of homogeneous differential equation of first order and first degree. In Homogeneous differential equations the power of variables 'x' and 'y' are equal in the left hand side and right hand side in the differential equation. If the 'x' and 'y' variables are in multiplication then the power of both get add but if the variables are in divisible i.e. x/y then the total power of 'x' and 'y' variables get subtract. If the power of 'x' and 'y' variables are not equal in the differential equations then these types of differential equation is non homogeneous differential equation of first order and first degree. Many problems of non-homogeneous differential equation of first order and first degree are very complicated to solve. All the methods to solve first order and first degree differential equations are variable separable method, homogeneous differential equation method, linear differential equation method, nonlinear differential equation method, exact differential equation method. Then, we are discussing some type of problems, which is non homogenous differential equations but solved only by homogenous differential equation method because all the methods of first order and first degree are not applicable except homogeneous differential equation method of first order and first degree for perfect solution.

Sankar Prasad Mondal, Tapan Kumar Roy, introduced the first order non homogenous differential equation [1]. DjilaliBehloul, Sui Sun Cheng, discussed the first-order nonlinear differential equation [2]. Sankar Prasad Mondal, Tapan Kumar Roy, introduced the First order linear non homogeneous ordinary differential equation [3].MahtabUddin, M. A. Ullah, discussed the Ordinary Differential Equation of First Order and First Degree [4]. E.L.Ince, Discussed the Ordinary Differential Equations [5]. U.S.Rana, introduced the differential equation of first order and first degree and also discuss the identification or solution of types of differential equation of first order and first degree[6].

V.P.Mishra, Jyoti Sinha, pratibha Mishra,discussed the introduction and identification of differential equations of first order and first degree. After that discuss about the types of differential equations of first order and first degreesolutions[7]. P.Sivaramakrishna Das, C.Vijayakumari, discussed the differential equations of first order and first degree and how to solve the types of differential equations of first order and first degree [8].

In this paper we are discussing the Some problems of Non homogeneous differential equation of first order and first degree which are solved by the method of homogeneous differential equation of first order and first degree.

Rest of the paper is organized as follows: Section II, Based on the related work which shows how to Implement the first order and first degree differential equation methods in differential equations. Section III, Based on the methodology of the homogeneous differential equation method of first order and first degree. Section IV, Discussing the problems of non-homogeneous differential equation of first order and first degree solved by the method of homogeneous differential equation of first order and first degree. Also solving for the perfect solution. SectionV,Based on the conclusion of the problem and Discussing the future scope of the non-homogeneous differential equation of first order and firstdegree problems. Section VI,section is based on the references, which is helpful for the identification and suitable perfect solution of differential equation of first order and first degree.

II. RELATED WORK

[1] Sankar Prasad Mondal ,Tapan Kumar Roy, " Application of first order non homogenous fuzzy differential equation", Number 1, 2013.

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III. METHODOLOGY

The method to solve homogeneous differential equation of first order and first degree method is the best method to solve some non-homogeneous problems of first order and first degree, which are not solved by all other methods fo first order and first degree. The equations given below (11) and (21) are non-homogeneous differential equations, but these equations only can solve by the homogeneous equation method of first order and first degree because these equations (11) and (21) give a perfect solution by using homogeneous differential equation method of first order and first degree.

IV. RESULT AND DISCUSSION

Let us consider the two special types of problems which are Non homogeneous differential equations of first order and first degree. It means that the power of variables'x' and 'y' in the left hand side and right hand sides are not equal in differential equations.

$$(x\frac{dy}{dx} - y)^2 = x^2(x^2 + y^2) \quad (11)$$

$$(y\frac{dx}{dy} - x)^2 = y^2(x^2 + y^2) \quad (21)$$

These above equations (11) and (21) are non-homogeneous differential equations of first order and first degree. If we apply all the methods of differential equation of first order and first degree in the above differential equations. Then, these equation (11) and (21) are not satisfying the conditions and forms of the differential equation of first order and first degree except homogeneous differential equation method of first order and first degree. Now, verifying all the methods of first order and first degree which are given below.

- Variable separable method- If we apply the Variable separable method in the above equations (11) and (21) are not satisfying the condition and form, because the variables of 'x' and 'y' are not completely separated in the dx = dy sides respectively. Therefore, this method is not applicable to find the solution of (11) and (21) equations.
- Linear differential equation method- These two differential equations (11) and (21) are not making the linear differential equation form of first order and first degree.

  i.e . $\frac{dy}{dx} + Py = Q$ or $\frac{dx}{dy} + Px = Q$

  Therefore, equations (11) and (21) can't solved by the linear differential equation method.
- Bernoulli differential equation (Reducible linear differential equation) - Equations (11) and (21) are not making the form of Bernoulli (Reducible linear) differential equation of first order and first degree.

  i.e. $\frac{dy}{dx} + Py = Qy^n$ or $\frac{dx}{dy} + Px = Qx^n$

  Therefore, equations (11) and (21) can't solved by Bernoulli differential equation method.
- Homogeneous differential equation- The first step of homogeneous differential equation is to check the power of the variable 'x' and 'y' are equal or not. After that second step is to free the derivative from the differential equation.

  (i.e. $\frac{dy}{dx}$ or $\frac{dx}{dy}$), then the third step is put the value of $y = tx$ and $\frac{dy}{dx} = t + x\frac{dt}{dx}$ in the given problem, when the derivative of the differential equation in the form of $\frac{dy}{dx}$. If the derivative of the differential equation in the

form of $\frac{dx}{dy}$, then put the value in the given problem of differential equation is $x = ty$ and $\frac{dx}{dy} = t + y\frac{dt}{dy}$

After that the homogeneous differential equation converted into the variable separable form and the Last step is separate the variables of 'x' and 'y' in the side of dx = dy respectively, then integrate and after that replace all the value of 't' in the solution.

These above equations (11) and (21) can solved by only one method of differential equation of first order and first degree. i.e. homogeneous differential equation method because all other methods which was used in the equations (11) and (21) but these equations is not fulfilling the conditions and forms to find the perfect general solution. The method of homogeneous differential equation of first order and first degree gives the perfect general solution of these above non homogeneous differential equation first order and first degree. We are discussing the problem of equations (11) and (21) in the below.

Problem 1- $(x\frac{dy}{dx} - y)^2 = x^2(x^2 + y^2)$ (11)

$$x\frac{dy}{dx} - y = x\sqrt{x^2 + y^2}$$

Free from the derivative of the above equation

$$\frac{dy}{dx} = \frac{x\sqrt{x^2+y^2}+y}{x} \quad (12)$$

Put *y*= *tx* then differentiate w.r.t *x*

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

Now put these values of *y* and $\frac{dy}{dx}$ in equation (12) we get,

$$t + x\frac{dt}{dx} = \frac{x\sqrt{x^2 + t^2x^2} + tx}{x}$$

$$t + x\frac{dt}{dx} = x\sqrt{1 + t^2} + t$$

$$\frac{dt}{dx} = \sqrt{1 + t^2}$$

(By using the variable separable condition i.e seprate and integrate)

$$\int \frac{1}{\sqrt{1+t^2}}dt = \int dx$$

$$sinh^{-1} t = x + c$$

$$t = sinh(x + c)$$

$$y = x\, sinh(x + c) \ \ (\because t=\tfrac{y}{x}) \quad (13)$$

Problem 2- $(y\frac{dx}{dy} - x)^2 = y^2(x^2 + y^2)$ (21)

$$y\frac{dx}{dy} - x = y\sqrt{x^2 + y^2}$$

Free from the derivative of the above equation

$$\frac{dx}{dy} = \frac{y\sqrt{x^2+y^2}+x}{y} \quad (22)$$

Put *x*= *ty* then differentiate w.r.t *y*

$$\frac{dx}{dy} = t + x\frac{dt}{dy}$$

Now put these values of *x* and $\frac{dx}{dy}$ in equation (22) we get,

$$t + y\frac{dt}{dx} = \frac{y\sqrt{y^2 + t^2y^2} + ty}{y}$$

$$t + y\frac{dt}{dy} = y\sqrt{1 + t^2} + t$$

$$\frac{dt}{dy} = \sqrt{1 + t^2}$$

(By using the variable separable condition i.e seprate and integrate)

$$\int \frac{1}{\sqrt{1+t^2}}dt = \int dy$$

$$sinh^{-1} t = y + c$$

$$t = sinh(y + c)$$

$$x = y\, sinh(y + c) \ \ (\because t=\tfrac{x}{y}) \quad (23)$$

In the above we already discussed all the methods of first order and first degree, Then only one method is applicable for perfect solution. These Equations (11) and (21) can be solved by the method of homogeneous differential equation of first order and first degree. Because all the method of first order and first degree are not applicable. Equation (13) and (23) be the perfect or general solution of equation (11) and (21).

V. CONCLUSION AND FUTURE SCOPE

In the above the two differential equations (11) and (21) proves that some Non homogeneous differential equations of first order and first degree problem can be solved by the method of homogeneous differential equation of first order and first degree, because all the methods of first order and first degree are not applicable except homogeneous differential equation method of first order and first degree.

The homogeneous method is the best method for the Non homogeneous differential equation problems of first order and first degree, because these types of problems are not fulfilling the condition and forms of the diffrential equation of first order and first degree for the perfect general solution. If any non homogneous differential equations first order and first degree are not solved by the Variable seprable method, linear differential equation method, Bernoulli differential equation method, Exact differential equation method. Then that types of problem only can solved by the homogeneous differential equation method because all method of first order and first degree differential equations are not applicable.

So many upcoming that type of problem which is non homogenous differential equation of first order and first degree, which is not solved by the other method of first order and first degree. Then homogeneous differential equation of first order and first degree method is best

method to get perfect general solution of non homogeneous differential equation of first order and first degree.

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