

## On Plurimum Discidium

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**Abstract**— This Paper deals with the divisions of really big numbers, some results will explain by using its proof, for example, method of mathematical induction, etc. for proving the theorems authors use the concept of exponents, limit, series, sequence etc. So, for understand the proof of the theorems this knowledge of Limits, Series, sequence etc required. The results that authors will introduce are really a difficult task to think because there required an assumption that such big numbers are exists may in future mathematicians take some actions to prove such numbers are existing one. The proof demands many steps and every one step of its open new doors to thing further as a result we discussed some results on behalf of which there some other generalizations will take place.

**Keywords**— Division, Bar, Very Big Numbers, Limit, continuous, sequence, increasing and decreasing etc.

### I. INTRODUCTION

We have a long history about numbers and we (humans) spent their time with numbers also from a long time, some of the curious mans in our history gave us many things related to numbers like Addition, Subtraction, multiplication, division, Fibonacci numbers, Lucas Numbers, Pell numbers etc. we are living in such a situation in which without knowing numbers and their mathematics it is really hard to understand phenomena's of nature and also we can't make anything with a perfection for that we must have to use measurements to make something perfect. Now a days we have many other subjects in mathematics like Number Theory, Geometry, Trigonometry, Calculus, Algebra etc some the peoples quoted that "Physics is the King of Science while Mathematics is the Queen" further on we found Gauss Quoted that "Mathematics is the Queen of the science and Number Theory (Arithmetic) is the Queen of Mathematics" [1] we can also observe that in every subject there is a use of every other subject as we can use geometry, trigonometry easily and we can't easily say that do some research in geometry without using trigonometry because there is a very big path that join both subjects like it there is really big place of number theory in mathematics that plays a vital role in every subject respectively. Here we will assuming very big numbers with a continuous repetition in every single number with it we have formed a number and divide it with also a big number that we will discuss in further sections of the paper respectively.

The title of this paper "*On Plurimum Discidium*" consists of two Latin words *Plurimum* and *Discidium* which means Big Numbers and Division respectively, we could change our title but because we want to give respect to the work of Latin Mathematician that is also available in English now but at that time mathematicians like C.F Gauss published his work in their own native language as *Disquisitiones Arithmeticae* [2]. in going details we found our work in number theory and we all knows Gauss called as the King of Number Theory as a result we are giving respect him by using Latin language in our main title.

We are assuming very big numbers like 999... as it is continuous and never ending for such numbers, we use bar i.e., represented by (—) above the number [3][4][5] and looking like.  $999.\overline{9} = 9$  this is the basic information that we will use to rewrite such big numbers easily respectively.

All that we will taking further. gets start from the question. If very big numbers exist then show that

$$\frac{\overline{987654321}}{\overline{123456789}} = 9$$

We obtain it equal to 9 with a very simple method that we will explain in further sections.

**II. OVERVIEW**

This paper consists of different sections are as follows:

**Abstract and keywords**

1. **Introduction:** in this section author briefly discuss meaning of the title and basic knowledge required
2. **Overview:** in this section we can observe content of the paper.
3. **Related Work:** this section deals with the published work related to the main title.
4. **Main Result:** this section is the main section which carried new theorems, work etc.
5. **Generalization:** in this section, by adding all observations together there expressed a generalization that have a capacity to express our work.
6. **Discussion:** in this section we have seen some remarkable graphs and observations from generalizations.
7. **Open problem:** in this section there are some open problems for renders to found and further discussion
8. **Conclusion:** this section deals with uses of introduced method, further scope of the method, and summary of the work.

**References**

**Authors Profile**

**III. RELATED WORK**

We thought this is a new idea to think in this direction as a result, we can say all the work related to Bar is related to our work and all the divisions of big numbers are related to our work.

**IV. MAIN RESULT**

A year ago, Ankush tells Toyesh that

$$\frac{987654321}{123456789} = 9$$

He **proved** this result as

$$\text{As, } \frac{987654321}{123456789} \approx 8.0,$$

$$\frac{998877665544332211}{112233445566778899} \approx 8.9,$$

$$\frac{999888777666555444333222111}{111222333444555666777888999} \approx 8.99,$$

Likewise, for  $n > 0$ .

$$\frac{99..988..877..766..655..544..433..322..211..1}{11..122..233..344..455..566..677..788..899..9} \approx 8.99..9$$

And

$$\lim_{n \rightarrow \infty} \frac{99..988..877..766..655..544..433..322..211..1}{11..122..233..344..455..566..677..788..899..9} = \lim_{n \rightarrow \infty} 8.99..9$$

As,  $\lim_{n \rightarrow \infty} 99..9 = 9$  then,

$$\frac{987654321}{123456789} = 8.99...$$

Now,

**Note:** - consider  $x = 8.99...$ , multiply 10 on both side as a result  $10x = 89.99...$  by subtract  $10x - x = 89.99... - 8.99...$  further  $9x = 81 \Rightarrow x = 9$ .

Hence

$$\frac{\overline{987654321}}{\overline{123456789}} = 9$$

We have submitted it in many journals but no one accept it may they want some thing else like other proof in which there is no such use of induction that we used here respectively. Although that time we found some other results like

$$\frac{\overline{21}}{\overline{12}} \approx 2 . \text{we give its proof like above } \mathbf{Proof: -}$$

As,  $\frac{21}{12} \approx 1.75,$

$$\frac{2211}{1122} \approx 1.9705,$$

$$\frac{222111}{111222} \approx 1.997005,$$

$$\frac{22221111}{11112222} \approx 1.99970005$$

With observing a pattern, we can say that for  $n > 0$ .

$$\frac{\overline{22\dots211\dots1}}{\overline{11\dots122\dots2}} \approx 1.99\dots9700\dots05$$

$$11\dots122\dots2$$

And further more

$$\lim_{n \rightarrow \infty} \frac{\overline{22\dots211\dots1}}{\overline{11\dots122\dots2}} \approx \lim_{n \rightarrow \infty} 1.99\dots9700\dots05$$

$$\frac{\overline{21}}{\overline{12}} \approx \lim_{n \rightarrow \infty} 1.99\dots9700\dots05$$

$$\frac{\overline{21}}{\overline{12}} \approx 2$$

$$\overline{12}$$

We found many other examples like

$$\frac{\overline{321}}{\overline{123}} = 3, \frac{\overline{8421}}{\overline{1248}} = 8, \frac{\overline{4321}}{\overline{1234}} = 4, \frac{\overline{12}}{\overline{21}} = \frac{1}{2} \text{ etc.}$$

**Observations**

From the above examples, we have some points that must be note down are as follows

1. Both the numbers in the numerator and in the denominator are very Big,
2. There we have descending order in numerator while ascending in the denominator of the same number,
3. Total number of digits in the numerator must be equal to the digits of denominator without Bar.

Some time ago, Sahil gives his proof that is

**Results and their Proof**

As we already take this example above, now we are proving it.

**Result-1**

$$\frac{\overline{21}}{\overline{12}} \approx 2$$

**Proof:**

As,

$$21 = 2 \cdot 10 + 1$$

$$2211 = 2 \cdot 10^3 + 2 \cdot 10^2 + 10 + 1 = (10 + 1)(2 \cdot 10^2 + 1)$$

$$222111 = 2 \cdot 10^5 + 2 \cdot 10^4 + 2 \cdot 10^3 + 10^2 + 10 + 1 = (10^2 + 10 + 1)(2 \cdot 10^3 + 1)$$

So, by observing pattern in it we can say that, for  $n > 0$

$$\overset{n}{2}2\dots\overset{n}{2}11\dots1 = (10^{n-1} + 10^{n-2} + \dots + 10 + 1)(2 \cdot 10^n + 1) \tag{1}$$

Likewise

$$12 = 1 \cdot 10 + 2$$

$$1122 = 1 \cdot 10^3 + 1 \cdot 10^2 + 2 \cdot 10 + 2 = (10 + 1)(1 \cdot 10^2 + 2)$$

$$111222 = 1 \cdot 10^5 + 1 \cdot 10^4 + 1 \cdot 10^3 + 2 \cdot 10^2 + 2 \cdot 10 + 2 = (10^2 + 10 + 1)(1 \cdot 10^3 + 2)$$

Then, for  $n > 0$

$$\overset{n}{1}1\dots\overset{n}{1}22\dots2 = (10^{n-1} + 10^{n-2} + \dots + 10 + 1)(1 \cdot 10^n + 2) \tag{2}$$

So from (1) and (2) we have,

$$\frac{\overset{n}{2}2\dots\overset{n}{2}11\dots1}{\overset{n}{1}1\dots\overset{n}{1}22\dots2} = \frac{(10^{n-1} + 10^{n-2} + \dots + 10 + 1)(2 \cdot 10^n + 1)}{(10^{n-1} + 10^{n-2} + \dots + 10 + 1)(1 \cdot 10^n + 2)}$$

$$\frac{\overset{n}{2}2\dots\overset{n}{2}11\dots1}{\overset{n}{1}1\dots\overset{n}{1}22\dots2} = \frac{2 \cdot 10^n + 1}{1 \cdot 10^n + 2}$$

Further,

$$\lim_{n \rightarrow \infty} \frac{\overset{n}{2}2\dots\overset{n}{2}11\dots1}{\overset{n}{1}1\dots\overset{n}{1}22\dots2} = \lim_{n \rightarrow \infty} \frac{2 \cdot 10^n + 1}{1 \cdot 10^n + 2}$$

Now,

$$\frac{\overset{-}{2}1}{\overset{-}{1}2} = \lim_{n \rightarrow \infty} \frac{2 + \frac{1}{10^n}}{1 + \frac{2}{10^n}} = 2$$

Or by using L-Hopital Rule [6]

$$\frac{\overset{-}{2}1}{\overset{-}{1}2} = \lim_{n \rightarrow \infty} \frac{2 \cdot 10^n + 1}{1 \cdot 10^n + 2} \stackrel{L-H, rule}{=} \lim_{n \rightarrow \infty} \frac{2 \cdot 10^n \ln 10 + 0}{1 \cdot 10^n \ln 10 + 0} = 2$$

Although this was the basics proof, but it was not that Sahil gives, Sahil used the same method for proving-

$$\frac{\overset{-}{9}87654321}{\overset{-}{1}23456789} = 9 \text{ that we will explain. For giving better understanding to all, we have explained the proof of } \frac{\overset{-}{2}1}{\overset{-}{1}2} = 2$$

because there are no complications in calculations of it but there, we found some messy calculations respectively.

So,

**Result-2**

$$\frac{\overline{\overline{987654321}}}{\overline{\overline{123456789}}} = 9$$

**Proof**

As,

$$\begin{aligned} 987654321 &= 9 \cdot 10^8 + 8 \cdot 10^7 + 7 \cdot 10^6 + 6 \cdot 10^5 + 5 \cdot 10^4 + 4 \cdot 10^3 + 3 \cdot 10^2 + 2 \cdot 10 + 1 \\ 998877665544332211 &= 9 \cdot 10^{17} + 9 \cdot 10^{16} + 8 \cdot 10^{15} + 8 \cdot 10^{14} + 7 \cdot 10^{13} + 7 \cdot 10^{12} + 6 \cdot 10^{11} + 6 \cdot 10^{10} + \\ &\quad 5 \cdot 10^9 + 5 \cdot 10^8 + 4 \cdot 10^7 + 4 \cdot 10^6 + 3 \cdot 10^5 + 3 \cdot 10^4 + 2 \cdot 10^3 + 2 \cdot 10^2 + 10 + 1 \\ &= 9 \cdot 10^{16} \cdot (10 + 2) + 8 \cdot 10^{14} \cdot (10 + 1) + 7 \cdot 10^{12} \cdot (10 + 1) + 6 \cdot 10^{10} \cdot (10 + 1) + 5 \cdot 10^8 \cdot (10 + 1) + \\ &\quad 4 \cdot 10^6 \cdot (10 + 1) + 3 \cdot 10^4 \cdot (10 + 1) + 2 \cdot 10^2 \cdot (10 + 1) + (10 + 1) \\ &= (10 + 1)(9 \cdot 10^{16} + 8 \cdot 10^{14} + 7 \cdot 10^{12} + 6 \cdot 10^{10} + 5 \cdot 10^8 + 4 \cdot 10^6 + 3 \cdot 10^4 + 2 \cdot 10^2 + 1) \end{aligned}$$

Likewise,

$$\begin{aligned} 999888777666555444333222111 &= 9 \cdot 10^{26} + 9 \cdot 10^{25} + 9 \cdot 10^{24} + 8 \cdot 10^{23} + 8 \cdot 10^{22} + 8 \cdot 10^{21} + \\ &\quad 7 \cdot 10^{20} + 7 \cdot 10^{19} + 7 \cdot 10^{18} + 6 \cdot 10^{17} + 6 \cdot 10^{16} + 6 \cdot 10^{15} + 5 \cdot 10^{14} + 5 \cdot 10^{13} + 5 \cdot 10^{12} + \\ &\quad 4 \cdot 10^{11} + 4 \cdot 10^{10} + 4 \cdot 10^9 + 3 \cdot 10^8 + 3 \cdot 10^7 + 3 \cdot 10^6 + 2 \cdot 10^5 + 2 \cdot 10^4 + 2 \cdot 10^2 + \\ &\quad 10^2 + 10 + 1 \\ &= (10^2 + 10 + 1) \left( \begin{matrix} 9 \cdot 10^{24} + 8 \cdot 10^{21} + 7 \cdot 10^{18} + 6 \cdot 10^{15} + 5 \cdot 10^{12} + 4 \cdot 10^9 + \\ 3 \cdot 10^6 + 2 \cdot 10^2 + 1 \end{matrix} \right) \end{aligned}$$

So, by observing patter we can say that for  $n > 0$

$$\begin{aligned} \overbrace{99}^n \dots \overbrace{988}^n \dots \overbrace{877}^n \dots \overbrace{766}^n \dots \overbrace{655}^n \dots \overbrace{544}^n \dots \overbrace{433}^n \dots \overbrace{322}^n \dots \overbrace{211}^n \dots 1 \\ = (10^{n-1} + 10^{n-2} + \dots + 10 + 1) (9 \cdot 10^{8n} + 8 \cdot 10^{7n} + 7 \cdot 10^{6n} + 6 \cdot 10^{5n} + 5 \cdot 10^{4n} + 4 \cdot 10^{3n} + 3 \cdot 10^{2n} + 2 \cdot 10^n + 1) \end{aligned}$$

We will also prove the same result with the help of Mathematical Induction.

$$\begin{aligned} 11 \dots \overbrace{122}^n \dots \overbrace{233}^n \dots \overbrace{344}^n \dots \overbrace{455}^n \dots \overbrace{566}^n \dots \overbrace{677}^n \dots \overbrace{788}^n \dots \overbrace{899}^n \dots 9 \\ = (10^{n-1} + 10^{n-2} + \dots + 10 + 1) (1 \cdot 10^{8n} + 2 \cdot 10^{7n} + 3 \cdot 10^{6n} + 4 \cdot 10^{5n} + 5 \cdot 10^{4n} + 6 \cdot 10^{3n} + 7 \cdot 10^{2n} + 8 \cdot 10^n + 9) \end{aligned}$$

Then, from both the above result, with consider  $10^{n-1} + 10^{n-2} + \dots + 10 + 1 = S$

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{\overbrace{99}^n \dots \overbrace{988}^n \dots \overbrace{877}^n \dots \overbrace{766}^n \dots \overbrace{655}^n \dots \overbrace{544}^n \dots \overbrace{433}^n \dots \overbrace{322}^n \dots \overbrace{211}^n \dots 1}{\overbrace{11}^n \dots \overbrace{122}^n \dots \overbrace{233}^n \dots \overbrace{344}^n \dots \overbrace{455}^n \dots \overbrace{566}^n \dots \overbrace{677}^n \dots \overbrace{788}^n \dots \overbrace{899}^n \dots 9} \\ = \lim_{n \rightarrow \infty} \frac{S \cdot (9 \cdot 10^{8n} + 8 \cdot 10^{7n} + 7 \cdot 10^{6n} + 6 \cdot 10^{5n} + 5 \cdot 10^{4n} + 4 \cdot 10^{3n} + 3 \cdot 10^{2n} + 2 \cdot 10^n + 1)}{S \cdot (1 \cdot 10^{8n} + 2 \cdot 10^{7n} + 3 \cdot 10^{6n} + 4 \cdot 10^{5n} + 5 \cdot 10^{4n} + 6 \cdot 10^{3n} + 7 \cdot 10^{2n} + 8 \cdot 10^n + 9)} \\ = \lim_{n \rightarrow \infty} \frac{(9 \cdot 10^{8n} + 8 \cdot 10^{7n} + 7 \cdot 10^{6n} + 6 \cdot 10^{5n} + 5 \cdot 10^{4n} + 4 \cdot 10^{3n} + 3 \cdot 10^{2n} + 2 \cdot 10^n + 1)}{(1 \cdot 10^{8n} + 2 \cdot 10^{7n} + 3 \cdot 10^{6n} + 4 \cdot 10^{5n} + 5 \cdot 10^{4n} + 6 \cdot 10^{3n} + 7 \cdot 10^{2n} + 8 \cdot 10^n + 9)} \end{aligned}$$

**Method-I**

Let,

$$l = \lim_{n \rightarrow \infty} \frac{(9 \cdot 10^{8n} + 8 \cdot 10^{7n} + 7 \cdot 10^{6n} + 6 \cdot 10^{5n} + 5 \cdot 10^{4n} + 4 \cdot 10^{3n} + 3 \cdot 10^{2n} + 2 \cdot 10^n + 1)}{(1 \cdot 10^{8n} + 2 \cdot 10^{7n} + 3 \cdot 10^{6n} + 4 \cdot 10^{5n} + 5 \cdot 10^{4n} + 6 \cdot 10^{3n} + 7 \cdot 10^{2n} + 8 \cdot 10^n + 9)}$$

Now, Applying L-Hopital Rule



V. GENERALIZATIONS

**Generalization-I**

After it Toyesh demands to both Ankush and Sahil to find out a **generalization** or proof a result that can express till explained results and gives an idea after observing the results that for r belongs to natural number.,

$$\frac{\overline{r(r-1)\dots 321}}{\overline{123\dots r(r-1)}} = r$$

On the same day, Sahil give the proof of the above result by using the his Sahil used.

He note down the following two equations on behalf of the pattern can be obtained

$$\begin{aligned} & \overbrace{rr\dots r}^n \overbrace{(r-1)(r-1)\dots(r-1)}^n \overbrace{33\dots 3}^n \overbrace{22\dots 2}^n \overbrace{211\dots 1}^n \\ &= (10^{n-1} + 10^{n-2} + \dots + 10 + 1) (r \cdot 10^{(r-1)n} + (r-1) \cdot 10^{(r-2)n} + \dots + 3 \cdot 10^{2n} + 2 \cdot 10^n + 1) \end{aligned}$$

And another one is

$$\begin{aligned} & \overbrace{11\dots 1}^n \overbrace{22\dots 2}^n \overbrace{33\dots 3}^n \overbrace{(r-1)(r-1)\dots(r-1)}^n \overbrace{rr\dots r}^n \\ &= (10^{n-1} + 10^{n-2} + \dots + 10 + 1) (1 \cdot 10^{(r-1)n} + 2 \cdot 10^{(r-2)n} + \dots + (r-2) \cdot 10^{2n} + (r-1) \cdot 10^n + r) \end{aligned}$$

Then

$$\begin{aligned} \frac{\overline{r(r-1)\dots 321}}{\overline{123\dots r(r-1)}} &= \lim_{n \rightarrow \infty} \frac{\overbrace{rr\dots r}^n \overbrace{(r-1)(r-1)\dots(r-1)}^n \overbrace{33\dots 3}^n \overbrace{22\dots 2}^n \overbrace{211\dots 1}^n}{\overbrace{11\dots 1}^n \overbrace{22\dots 2}^n \overbrace{33\dots 3}^n \overbrace{(r-1)(r-1)\dots(r-1)}^n \overbrace{rr\dots r}^n} \\ &= \lim_{n \rightarrow \infty} \frac{(10^{n-1} + 10^{n-2} + \dots + 10 + 1) (r \cdot 10^{(r-1)n} + (r-1) \cdot 10^{(r-2)n} + \dots + 3 \cdot 10^{2n} + 2 \cdot 10^n + 1)}{(10^{n-1} + 10^{n-2} + \dots + 10 + 1) (1 \cdot 10^{(r-1)n} + 2 \cdot 10^{(r-2)n} + \dots + (r-2) \cdot 10^{2n} + (r-1) \cdot 10^n + r)} \\ &= \lim_{n \rightarrow \infty} \frac{(r \cdot 10^{(r-1)n} + (r-1) \cdot 10^{(r-2)n} + \dots + 3 \cdot 10^{2n} + 2 \cdot 10^n + 1)}{(1 \cdot 10^{(r-1)n} + 2 \cdot 10^{(r-2)n} + \dots + (r-2) \cdot 10^{2n} + (r-1) \cdot 10^n + r)} \\ &= \lim_{n \rightarrow \infty} \frac{10^{(r-1)n} \left( r + (r-1) \cdot \frac{1}{10^n} + \dots + 3 \cdot \frac{1}{10^{n(r-3)}} + 2 \cdot \frac{1}{10^{n(r-2)}} + \frac{1}{10^{(r-1)n}} \right)}{10^{(r-1)n} \left( 1 + 2 \cdot \frac{1}{10^n} + \dots + (r-2) \cdot \frac{1}{10^{n(r-3)}} + (r-1) \cdot \frac{1}{10^{n(r-2)}} + \frac{r}{10^{(r-1)n}} \right)} \\ &= \lim_{n \rightarrow \infty} \frac{\left( r + (r-1) \cdot \frac{1}{10^n} + \dots + 3 \cdot \frac{1}{10^{n(r-3)}} + 2 \cdot \frac{1}{10^{n(r-2)}} + \frac{1}{10^{(r-1)n}} \right)}{\left( 1 + 2 \cdot \frac{1}{10^n} + \dots + (r-2) \cdot \frac{1}{10^{n(r-3)}} + (r-1) \cdot \frac{1}{10^{n(r-2)}} + \frac{r}{10^{(r-1)n}} \right)} \\ &= r \end{aligned}$$

Hence Proof.

**Generalization-II**

After it Toyesh Proposed his **generalization**: -

If  $n, r > 0$  and  $n > r$  with that both the numbers belong to natural numbers then,

$$\frac{\overline{n(n-1)\dots(n-(r-1))(n-r)}}{(n-r)(n-(r-1))\dots(n-1)n} = \frac{n}{(n-r)}$$

As we can understand till from above methods that  $(10^{n-1} + 10^{n-2} + \dots + 10 + 1)$  is only due to numbers of digits after the first number So,

$$\frac{\overline{n(n-1)} \dots \overline{(n-(r-1))} \overline{(n-r)}}{\overline{(n-r)} \overline{(n-(r-1))} \dots \overline{(n-1)} \overline{n}}$$

$$= \lim_{x \rightarrow \infty} \frac{\overbrace{nn \dots n}^x \overbrace{(n-1)(n-1) \dots (n-1)}^x \dots \overbrace{(n-(r-1))(n-(r-1)) \dots (n-(r-1))}^x \overbrace{(n-r)(n-r) \dots (n-r)}^x}{\overbrace{(n-r)(n-r) \dots (n-r)}^x \overbrace{(n-(r-1))(n-(r-1)) \dots (n-(r-1))}^x \dots \overbrace{(n-1)(n-1) \dots (n-1)}^x \overbrace{nn \dots n}^x}$$

If we expand both numerator and denominator then we found  $(10^{x-1} + 10^{x-2} + \dots + 10 + 1)$  in both part as a result we cut off that became limit as.

$$= \lim_{x \rightarrow \infty} \frac{(n \cdot 10^{xT_n} + (n-1) \cdot 10^{xT_{n-1}} + \dots + (n-(r-1)) \cdot 10^{xT_{n-(r-1)}} + (n-r) \cdot 10^{xT_{n-r}})}{((n-r)10^{xT_n} + (n-(r-1)) \cdot 10^{xT_{n-1}} + \dots + (n-1) \cdot 10^{xT_{n-(r-1)}} + n \cdot 10^{xT_{n-r}})}$$

Where, Number  $T_n$  defined as the total number of digits before  $n$  in its simple form like  $n(n-1) \dots (n-(r-1))(n-r)$  let's take some examples for better understanding in 98765431 this  $T_9$  is 8 because there are 8 digits before 9,  $T_8$  is 7 like wise  $T_7=6, T_6=5, T_5=4, T_4=3, T_2=1$ , and  $T_1=0$  as a result while doing limits with that we have numerator  $(9 \cdot 10^{8n} + 8 \cdot 10^{7n} + 7 \cdot 10^{6n} + 6 \cdot 10^{5n} + 5 \cdot 10^{4n} + 4 \cdot 10^{3n} + 3 \cdot 10^{2n} + 2 \cdot 10^n + 1)$  that we can also

rewrite as  $(9 \cdot 10^{T_9n} + 8 \cdot 10^{T_8n} + 7 \cdot 10^{T_7n} + 6 \cdot 10^{T_6n} + 5 \cdot 10^{T_5n} + 4 \cdot 10^{T_4n} + 3 \cdot 10^{T_3n} + 2 \cdot 10^{T_2n} + 1 \cdot 10^{T_1n})$  likewise in 21,  $T_2=1$  and  $T_1=0$  because there is nothing after 1. Here we can observe that  $T_n=(n-1)$  it is due to the 1 at the end of the number or last digit of the number as far now considered generalization we have  $(n-r)$  as the last digit of the number as a result we can't conform  $T_n$  by just seen 1 at the end of the number. Example in 98,  $T_9$  is 1 not 9 as a result it is not necessary to consider 1 as a last digit respectively but if in current generalization, we supposed  $r=(n-1)$  then it becomes above Ankush's generalisation form. Here in the limit  $T_{n-r}=0$  Now,

$$= \lim_{x \rightarrow \infty} \frac{(n \cdot 10^{xT_n} + (n-1) \cdot 10^{xT_{n-1}} + \dots + (n-(r-1)) \cdot 10^{xT_{n-(r-1)}} + (n-r))}{((n-r)10^{xT_n} + (n-(r-1)) \cdot 10^{xT_{n-1}} + \dots + (n-1) \cdot 10^{xT_{n-(r-1)}} + n)}$$

$$= \lim_{x \rightarrow \infty} \frac{10^{xT_n} \left( n + (n-1) \cdot \frac{1}{10^{x(T_n-T_{n-1})}} + \dots + (n-(r-1)) \cdot \frac{1}{10^{x(T_n-T_{n-r+1})}} + (n-r) \cdot \frac{1}{10^{xT_n}} \right)}{10^{xT_n} \left( (n-r) + (n-(r-1)) \cdot \frac{1}{10^{x(T_n-T_{n-1})}} + \dots + (n-1) \cdot \frac{1}{10^{x(T_n-T_{n-r+1})}} + n \cdot \frac{1}{10^{xT_n}} \right)}$$

As,

$$T_n - T_{n-1}, \dots, T_n - T_{n-r+1} > 0$$

Then,

$$\frac{\overline{n(n-1)} \dots \overline{(n-(r-1))} \overline{(n-r)}}{\overline{(n-r)} \overline{(n-(r-1))} \dots \overline{(n-1)} \overline{n}} = \frac{n}{(n-r)}$$

**Generalization-III**

There is a point to be noted that there is a perfect sequence in both numerator and denominator further on a day after Sahil provide another generalization in which he considered random numbers. Let suppose a,b,c,...,x,y,z belongs to natural numbers.



$$\overbrace{abc\dots xyz}^{n\text{-digits}} = a \cdot 10^{n-1} + b \cdot 10^{n-2} + c \cdot 10^{n-3} \dots + x \cdot 10^2 + y \cdot 10 + z$$

$$\overbrace{aa\dots a bb\dots b cc\dots c \dots xx\dots x yy\dots y zz\dots z}^{nr} = (10^{n-1} + 10^{n-2} + \dots + 10 + 1) (a \cdot 10^{r(n-1)} + b \cdot 10^{r(n-2)} + c \cdot 10^{r(n-3)} \dots + x \cdot 10^{2r} + y \cdot 10^r + z)$$

Likewise

$$\overbrace{zyx\dots cba}^{n\text{-digits}} = z \cdot 10^{n-1} + y \cdot 10^{n-2} + x \cdot 10^{n-3} \dots + c \cdot 10^2 + b \cdot 10 + a$$

$$\overbrace{zz\dots z yy\dots y xx\dots x \dots cc\dots c bb\dots b aa\dots a}^{nr} = (10^{n-1} + 10^{n-2} + \dots + 10 + 1) (z \cdot 10^{r(n-1)} + y \cdot 10^{r(n-2)} + x \cdot 10^{r(n-3)} \dots + c \cdot 10^{2r} + b \cdot 10^r + a)$$

Now,

$$\begin{aligned} & \lim_{r \rightarrow \infty} \frac{\overbrace{aa\dots a bb\dots b cc\dots c \dots xx\dots x yy\dots y zz\dots z}^{nr}}{\overbrace{zz\dots z yy\dots y xx\dots x \dots cc\dots c bb\dots b aa\dots a}^{nr}} \\ &= \lim_{r \rightarrow \infty} \frac{(10^{r-1} + 10^{r-2} + \dots + 10 + 1) (a \cdot 10^{r(n-1)} + b \cdot 10^{r(n-2)} + c \cdot 10^{r(n-3)} + \dots + y \cdot 10^{2r} + x \cdot 10^r + z)}{(10^{r-1} + 10^{r-2} + \dots + 10 + 1) (z \cdot 10^{r(n-1)} + y \cdot 10^{r(n-2)} + x \cdot 10^{r(n-3)} \dots + c \cdot 10^{2r} + b \cdot 10^r + a)} \\ &= \lim_{r \rightarrow \infty} \frac{(a \cdot 10^{r(n-1)} + b \cdot 10^{r(n-2)} + c \cdot 10^{r(n-3)} + \dots + y \cdot 10^{2r} + x \cdot 10^r + z)}{(z \cdot 10^{r(n-1)} + y \cdot 10^{r(n-2)} + x \cdot 10^{r(n-3)} + \dots + c \cdot 10^{2r} + b \cdot 10^r + a)} \\ &= \lim_{r \rightarrow \infty} \frac{10^{r(n-1)} \cdot \left( a + b \cdot \frac{1}{10^r} + c \cdot \frac{1}{10^{2r}} + \dots + y \cdot \frac{1}{10^{r(n-3)}} + x \cdot \frac{1}{10^{r(n-2)}} + z \cdot \frac{1}{10^{r(n-1)}} \right)}{10^{r(n-1)} \cdot \left( z + y \cdot \frac{1}{10^r} + x \cdot \frac{1}{10^{2r}} + \dots + c \cdot \frac{1}{10^{r(n-3)}} + b \cdot \frac{1}{10^{r(n-2)}} + a \cdot \frac{1}{10^{r(n-1)}} \right)} \\ &= \frac{a}{z} \end{aligned}$$

So,

$$\frac{\overbrace{abc\dots xyz}}{\overbrace{zyx\dots cba}} = \frac{a}{z}$$

Where a,b,c,...x,y,z>0

**Generalization-IV**

Further, Toyesh Proposed that for all a,b,c,...,x,y,z, p,q,r,... j,k,l belongs to natural numbers.

$$\frac{\overbrace{abc\dots xyz}}{\overbrace{pqr\dots jkl}} = \frac{a}{p}$$

Let's take an example

What's the value of  $\frac{\overline{\overline{24}}}{\overline{\overline{21}}}$  ?

**Solution:** - by using the above generalization we can say that  $\frac{\overline{\overline{24}}}{\overline{\overline{21}}} = \frac{2}{2} = 1$ , Now we will prove it

As, for n>0.

$$\overbrace{22\dots 244\dots 4}^n = (10^{n-1} + 10^{n-2} + \dots + 10 + 1) (2 \cdot 10^n + 4)$$

And

$$22\dots211\dots1 = (10^{n-1} + 10^{n-2} + \dots + 10 + 1)(2 \cdot 10^n + 1)$$

Then,

$$\lim_{n \rightarrow \infty} \frac{22\dots2\overbrace{44\dots4}^n}{22\dots211\dots1} = \lim_{n \rightarrow \infty} \frac{(10^{n-1} + 10^{n-2} + \dots + 10 + 1)(2 \cdot 10^n + 4)}{(10^{n-1} + 10^{n-2} + \dots + 10 + 1)(2 \cdot 10^n + 1)}$$

$$\lim_{n \rightarrow \infty} \frac{22\dots2\overbrace{44\dots4}^n}{22\dots211\dots1} = \lim_{n \rightarrow \infty} \frac{(2 \cdot 10^n + 4)}{(2 \cdot 10^n + 1)}$$

$$\lim_{n \rightarrow \infty} \frac{22\dots2\overbrace{44\dots4}^n}{22\dots211\dots1} = \lim_{n \rightarrow \infty} \left( \frac{2 + \frac{4}{10^n}}{2 + \frac{1}{10^n}} \right)$$

$$\frac{\overline{24}}{\overline{21}} = 1$$

Likewise, there are many possible examples and we come across such a situation in which we may answer to a wide range of questions respectively.

**VI. DISCUSSION**

Now, let suppose an expression  $\frac{\overline{n(n-1)}}{(n-1)n}$  on behalf of second most generalization it is equal to  $\frac{\overline{n(n-1)}}{(n-1)n} = \frac{n}{(n-1)}$  if n

belongs to natural number. Let's doing some graphs so, consider  $f(x) = \frac{x}{(x-1)}$  then,

$$\frac{\overline{21}}{\overline{12}} = f(2) = 2, \frac{\overline{32}}{\overline{23}} = f(3) = \frac{3}{2}, \frac{\overline{43}}{\overline{34}} = f(4) = \frac{4}{3}, \text{ etc}$$

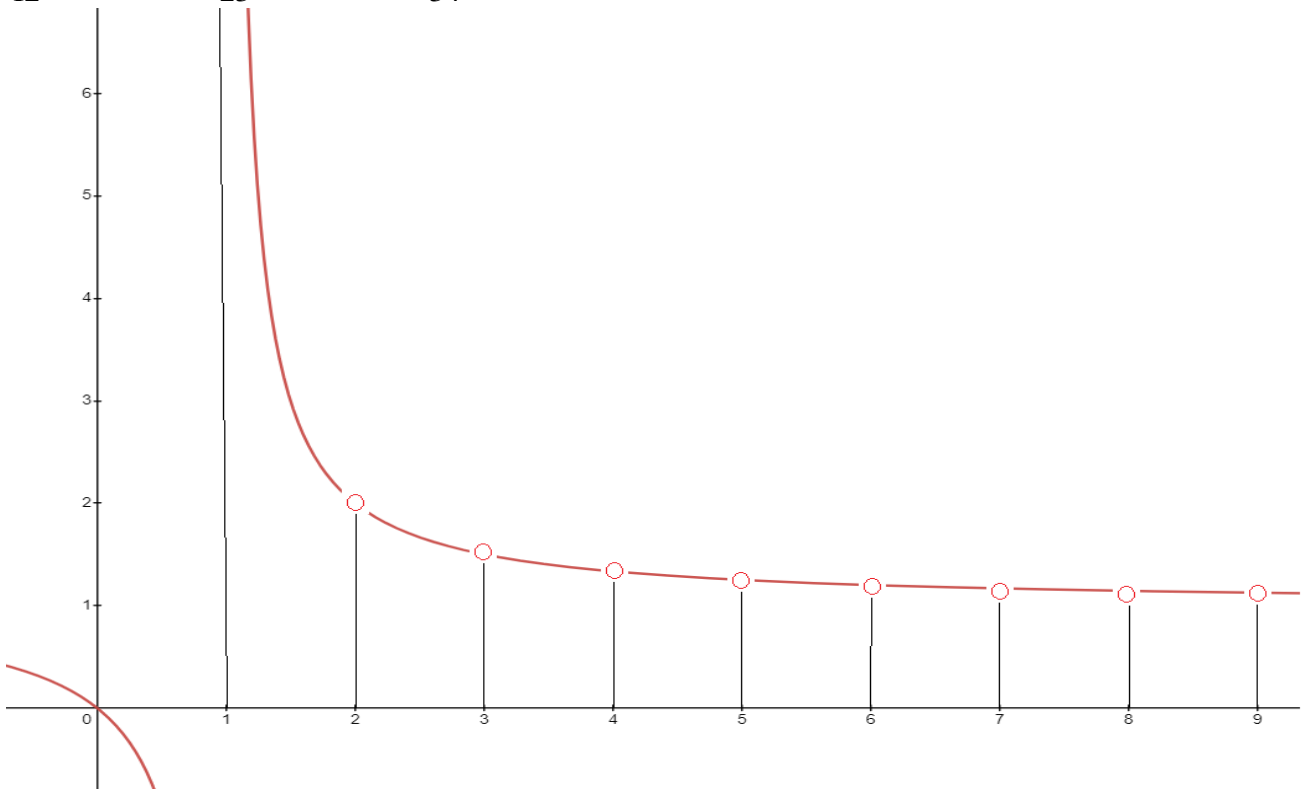


Fig.1.

In this given figure,  $f(x)=x/(x-1)$  have been shown with red coloured curve, as our considered expression is only for natural number as a result here provided circles on the curve are the only values of the expression.

The above graph telling the same points are as follows:

1. As  $x$  increases, our function decreases. As for  $x=1$ ,  $f(x)$  becomes infinite while taking limit of  $f(x)$ ,  $x$  tends to infinite then we got: -

$$l = \lim_{x \rightarrow \infty} \frac{x}{(x-1)} \Rightarrow \lim_{x \rightarrow \infty} \frac{1}{\left(1 - \frac{1}{x}\right)} = 1, \text{ here we obtained 1 as a result from 1 to infinite consider function}$$

is continuously decreasing.

2.  $2 \geq f(x) \geq 1, x \in [2, \infty)$

**3. Open Problem.**

We have,  $f(x)=x/(x-1)$  that defined further for  $x>9$  but till we have no idea about did

$$\frac{\overline{n(n-1)}}{(n-1)n} = \frac{n}{(n-1)}$$

is equally valid or not as far we are in confusion about to it. Can someone show that it

is equally valid for  $n=10,11, 12, \dots$  if not then why not?

As far we have,

$$\frac{\overline{n(n-1)}}{(n-1)n} = \frac{n}{(n-1)}, \text{ likewise, we also have:- } \frac{\overline{n(n-1)(n-2)}}{(n-2)(n-1)n} = \frac{n}{(n-2)}, \frac{\overline{n(n-1)(n-2)(n-3)}}{(n-3)(n-2)(n-1)n} = \frac{n}{(n-3)},$$

$$\frac{\overline{n(n-1)(n-2)(n-3)(n-4)}}{(n-4)(n-3)(n-2)(n-1)n} = \frac{n}{(n-4)}$$

There we can see a graph of the function.

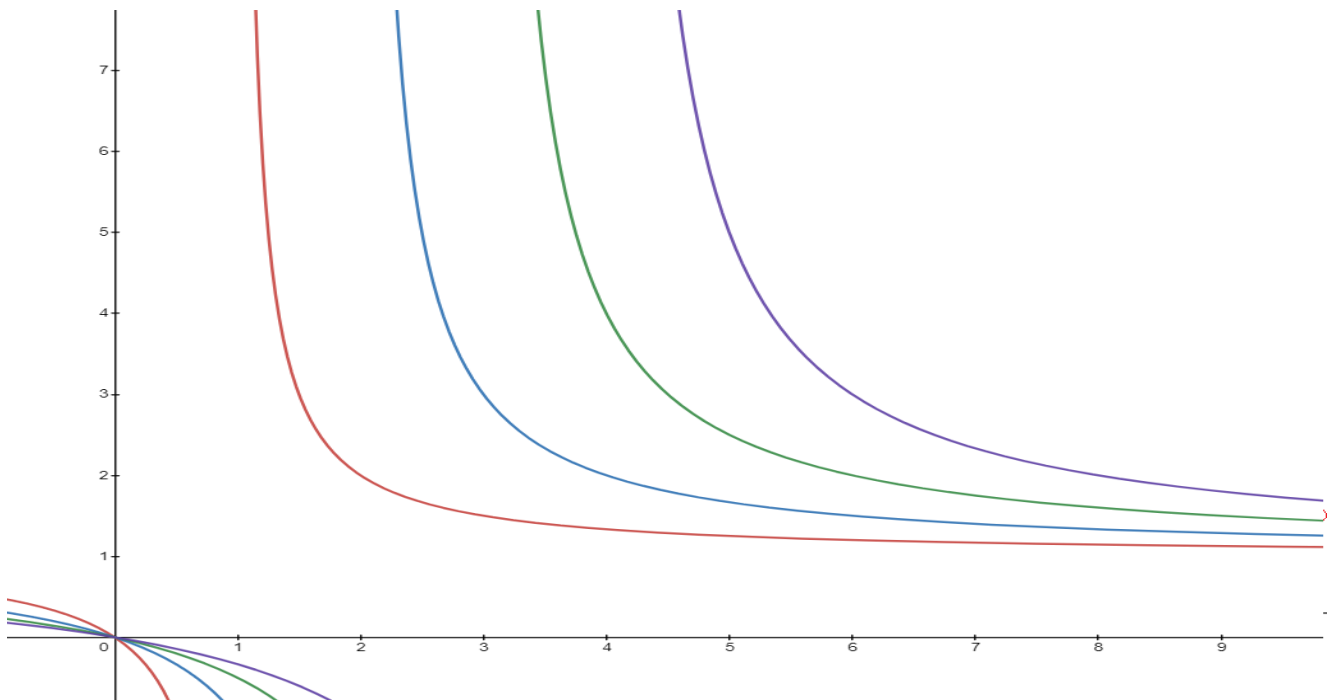


Fig.2.

In this given figure, there  $y=x/(x-1)$  shown in red coloured curve,  $y=x/(x-2)$  shown in blue coloured curve,  $y=x/(x-3)$  in green coloured curve, and  $y=x/(x-4)$  shown in violet coloured curve. All the curves are in their own order.

**VII. OPEN PROBLEM**

In the paper, we deal with many different hard situations but we did our best to answer them, now we are presenting open problems for solving by someone.

1. In the whole paper, we deal with divisions of very big numbers by the assumption that these numbers exist. So, now we are asking all the readers to give some clues to say yes! These numbers are existing ones.
2. Prove these results by another method other than that we described in the paper.
3. This one is already asked above in VI section of the paper.

### VIII. CONCLUSION

In this paper, the authors deal with the division of very large numbers. They provide many results and generalizations that on behalf of them by judge the situation we use here provided results and generalizations both for it, it may save our time we are thinking that may in coming future our provided results take place in mathematics as a number like other numbers. After doing this solution we found ourself happy and we thought this feeling is very essential for all of us because we obtain the solution after a long duration of time.

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