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Construction of Diophantine Triples Involving Hexagonal Pyramidal Numbers

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Abstract— We scrutinize for three particular polynomials with integer coefficients to such an extent that the result of any two numbers expanded by a non-zero number (or polynomials with number coefficients) is a perfect square.

Keywords— Dio triples, pyramidal numbers, Hexagonal pyramidal number, polynomials, integers, perfect square.

I. INTRODUCTION

In Math, a Diophantine equation is a polynomial equation, the word Diophantine suggests the Greek mathematician of the third century. Diophantus of Alexandria, who made an examination of such conditions and was one of the foremost mathematicians to bring symbolism into variable based math. The mathematical examination of Diophantine problems that Diophantus began is as of now called Diophantine analysis. While particular conditions present such a confound and have been considered from the start of time, the meaning of general speculations of Diophantine equations (past the theory of quadratic constructions) was an achievement of the 20th century.

In [1-5], speculation of numbers was discussed. In [6-12], Diophantine triples and quadruples with the property for any number and besides for any straight polynomials were discussed. This paper targets creating Dio-Triples where the consequence of any two members from the triple expanded by a non-zero number (or polynomials with integer coefficients) satisfies the property. In this manner, we present three fragments where in all of which we find the Dio triples from Hexagonal Pyramidal number of different cases with their relating properties.

II. RELATED WORK

NOTATION:

p_n^6 : Hexagonal pyramidal number of rank n.

BASIC DEFINITION:

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be **Dio triple** with property $D(n)$ if $a_i * a_j + n$ is a perfect square for all $1 \le i < j \le 3$, where n may be non-zero number or a polynomial with integer coefficients.

III. METHODOLOGY

Case 1:

Construction of the Diophantine triples involving hexagonal pyramidal number of rank n and n+1:

Let $a = 6p_n^6$ and $b = 6p_{n+1}^6$ be hexagonal pyramidal numbers of rank n and $n+1$ respectively.

Now, $a = 6p_n^6$ and $b = 6p_{n+1}^6$

$$ab + (-4n^4 + 2n^3 + 26n^2 + 12n + 1) = 16n^6 + 72n^5 + 109n^4 + 60n^3 + n^2 - 6n$$
$$= (4n^3 + 9n^2 + 3n + 1)^2$$
$$ab + (-4n^4 + 2n^3 + 26n^2 + 12n + 1) = (4n^3 + 9n^2 + 3n + 1)^2 = \alpha^2 \quad \to (1)$$

Equation (1) is a perfect square,

$$ab + (-4n^4 + 2n^3 + 26n^2 + 12n + 1) = \alpha^2$$

where $\alpha = 4n^3 + 9n^2 + 3n + 1$

Let c be non-zero integer such that,

$$bc + (-4n^4 + 2n^3 + 26n^2 + 12n + 1) = \beta^2 \quad \to (2)$$
$$ca + (-4n^4 + 2n^3 + 26n^2 + 12n + 1) = \gamma^2 \quad \to (3)$$

Choose $\beta = x + by$ and $\gamma = x + ay$,

$$(2) - (3) \Rightarrow c(b - a) = \gamma^2 - \beta^2$$
$$\Rightarrow c(b - a) = (b + a + 2\alpha)(b - a)$$
$$\Rightarrow c = a + b + 2\alpha$$
$$c = 16n^3 + 36n^2 + 22n + 8$$
$$\Rightarrow c = (2(a + b - 10n - 4))$$

Therefore, the triple $\{a, b, (2(a + b - 10n - 4))\} = \{6p_n^6, 6p_{n+1}^6, (2(6p_n^6 + 6p_{n+1}^6 - 10n - 4))\}$ is a Dio triple with the property $D(-4n^4 + 2n^3 + 26n^2 + 12n + 1)$.

Some numerical examples satisfying the above triple are given below in the following table.

TABLE 1

n	Dio Triples	$D(-4n^4+2n^3+26n^2+12n+1)$
0	$(0,6,8)$	1
1	$(6,42,82)$	37
2	$(42,132,324)$	81

Section-B:

Construction of the Diophantine triples involving hexagonal pyramidal number of rank n and n+2:

Let $a=6p_n^6$ and $b=6p_{n+2}^6$ be hexagonal pyramidal numbers of rank n and $n+2$ respectively.

Now, $a=6p_n^6$ and $b=6p_{n+2}^6$

$$ab+(20n^3+84n^2+64n+1)=16n^6+120n^5+313n^4+318n^3+67n^2+42n$$

$$=(4n^3+15n^2+11n+1)^2$$

$$ab+(20n^3+84n^2+64n+1)=(4n^3+15n^2+11n+1)^2=\alpha^2 \quad \rightarrow(4)$$

Equation (4) is a perfect square,

$ab+(20n^3+84n^2+64n+1)=\alpha^2$

where $\alpha=4n^3+15n^2+11n+1$

Let c be non-zero integer such that,

$$bc+(20n^3+84n^2+64n+1)=\beta^2 \quad \rightarrow(5)$$

$$ca+(20n^3+84n^2+64n+1)=\gamma^2 \quad \rightarrow(6)$$

Choose $\beta=x+by$ and $\gamma=x+ay$

$$(5)-(6)\Rightarrow c(b-a)=\gamma^2-\beta^2$$

$$\Rightarrow c(b-a)=(b+a+2\alpha)(b-a)$$

$$\Rightarrow c=a+b+2\alpha$$

$$c=16n^3+60n^2+80n+44$$

$$\Rightarrow c=(2(a+b-36n-40))$$

Hence, the triple $\{a,b,(2(a+b-36n-40))\}=\{6p_n^6,6p_{n+2}^6,(2(6p_n^6+6p_{n+2}^6-36n-40))\}$ is a Dio triple with the property $D(20n^3+84n^2+64n+1)$.

Some numerical examples satisfying the above triple are given below in the following table.

TABLE 2

n	Dio Triples	$D(20n^3+84n^2+64n+1)$
0	$(0,42,44)$	1
1	$(6,132,200)$	169
2	$(42,300,572)$	625

Section-C:

Construction of the Diophantine triples involving hexagonal pyramidal number of rank n and n+3:

Let $a=6p_n^6$ and $b=6p_{n+3}^6$ be hexagonal pyramidal numbers of rank n and $n+3$ respectively.

Now, $a=6p_n^6$ and $b=6p_{n+3}^6$

$$ab+(-4n^4+26n^3+212n^2+174n+1)=16n^6+168n^5+613n^4+864n^3+271n^2-132n$$

$$=(4n^3+21n^2+21n+1)^2$$

$$ab+(-4n^4+26n^3+212n^2+174n+1)=(4n^3+21n^2+21n+1)^2=\alpha^2$$

$\rightarrow(7)$

Equation (7) is a perfect square,

$ab+(-4n^4+26n^3+212n^2+174n+1)=\alpha^2$ where

$\alpha=4n^3+21n^2+21n+1$

Let c be non-zero integer such that,

$$bc+(-4n^4+26n^3+212n^2+174n+1)=\beta^2 \quad \rightarrow(8)$$

$$ca+(-4n^4+26n^3+212n^2+174+1)=\gamma^2 \quad \rightarrow(9)$$

Choose $\beta=x+by$ and $\gamma=x+ay$,

$$(8)-(9)\Rightarrow c(b-a)=\gamma^2-\beta^2$$

$$\Rightarrow c(b-a)=(b+a+2\alpha)(b-a)$$

$$\Rightarrow c=a+b+2\alpha$$

$$c=16n^3+84n^2+166n+134$$

$$\Rightarrow c=(2(a+b-84n-130))$$

Therefore, the triple $\{a,b,(2(a+b-84n-130))\}=\{6p_n^6,6p_{n+3}^6,(2(6p_n^6+6p_{n+3}^6-84n-130))\}$ is a Dio triple with the property $D(-4n^3+26n^3+212n^2+174n+1)$.

Some numerical values satisfying the above triple are given below in the following table.

TABLE 3

n	Dio Triples	$D(-4n^4+26n^3+212n^2+174n+1)$
0	$(0,132,134)$	1
1	$(6,300,400)$	409
2	$(42,570,930)$	1341

IV. CONCLUSION AND FUTURE SCOPE

We have presented the Dio triples involving hexagonal pyramidal numbers. To conclude one may look for triples or quadruples for different numbers with their relating properties.

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