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Ikuemuya's Formula III: An Alternative Formula to the Law of Cosines for The Computation of Magnitude of Resultant Vector

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Abstract— This paper presents a simple formula for the use in Mathematics, Physics and Engineering in solving geometry related problems. It is a simple alternative formula to the law of cosines for use in the computation of magnitude of resultant vector in geometry. The formula is named the Ikuemuya's formula III. It is simple and less cumbersome than the usual law of cosines and it does not involve the finding of squares and square roots of numbers when used in the computation of magnitude of resultant vectors as common with the law of cosines, it only makes use of basic arithmetic operation like addition, subtraction and multiplication, and the magnitude of the resultant vector can be computed quite easily.

Keywords—Alternative formula, Average constant factor, Ikuemuya's formula III, Law of cosines, Magnitude of resultant vector, Non-perfect squares.

I. INTRODUCTION

The law of cosines is a powerful tool or topic when it comes to finding the magnitude of the resultant of two vectors acting at an obtuse angle [1] but you have to work with squares of numbers and at the end find the square root of a certain number which is sometimes very hard to figure out without the use of a calculator most especially if the number is not a perfect square, for example $\sqrt{36}$ is easy to evaluate because it is a perfect square of 6, but $\sqrt{67}$ is hard to work out without the use of a calculator since 67 is not a perfect square. The square root of such non-perfect squares often come up when using the law of cosines to compute the magnitude of resultant of two vectors acting at an obtuse angle and this problem is eliminated with the use of the Ikuemuya's formula III, since one does not have to worry about working with squares and square roots of numbers when computing the magnitude of resultant of two vectors at obtuse angle. It is important to note that the Pythagoras theorem is a special case of the law of cosines [2, 3] and this idea was used in this paper in the formulation of the Ikuemuya's formula III. In words, the law of cosines states that; the square of any side triangle is equal to the sum of the squares of the other two sides minus twice the product of those two sides' times the cosine of the included angle [4].

The motivation to come up with this simple alternative formula to the cosine rule arises out of the perceived increasing dread and complexity in the use of the cosine rule in recent times among students of mathematics and physics particularly in secondary or high school level which I was also a victim of while in high school. Also, another motivation was that, the world needs alternatives, to leave users and students with formula choices when computing the magnitude of resultant vector. It is hoped that the proposed Ikuemuya's formula III would help to assuage the dread among secondary or high school student.

In the next section, the theoretical formulation of the proposed Ikuemuya's formula III are presented and this is followed by results, discussion and concluding remarks in the last section.

II. RELATED WORK

Jarno Mielikainen showed that vector quantization (VQ) is an essential tool in signal processing and that the classical generalized Lloyd algorithm (GLA) is still widely used due to its simplicity and good performance and using the law of cosines, this letter presents a simple improved method for nearest neighbor search in GLA [5]. RB Kershner, introduced some sets of equations and showed that like the law of sine's for triangles, each of these equations involves all but one of the sides and all but of the angles of the polygon. In the same way, a direct generalization of the law of cosines for triangles, it can be rewritten in five different ways [6].

III. METHODOLOGY

THEORETICAL FORMULATION OF THE PROPOSED IKUEMUYA'S FORMULA III

Alternative to the Law of Cosines for Computing the Magnitude of Resultant Vector

When the angle between two vectors is obtuse, that is, greater than 90 degrees but less than 180 degrees as shown in the parallelogram below (Figure 1), we use the law of cosines to compute the magnitude of the resultant vector R [7].

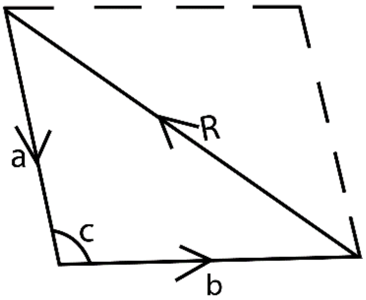


Figure 1.0: showing two vectors (a & b) acting at an obtuse angle C with resultant R

The law of cosines in this case can be stated as:

$$R^2 = a^2 + b^2 - 2 \times a \times b \times CosC \quad (1)$$

where C is the obtuse angle between the two vectors, when the angle C is exactly 180 degrees we have;

$$R^2 = a^2 + b^2 - 2 \times a \times b \times Cos(180)$$

$$R^2 = a^2 + b^2 - 2 \times a \times b \times (-1)$$

$$R^2 = a^2 + b^2 + 2ab$$

But,

$$a^2 + b^2 + 2ab = (a+b)^2$$

Therefore,

$$R^2 = (a+b)^2$$

$$R = a + b \quad (2)$$

The Ikuemuya's formula III was formulated as:

$$R = (|a| + |b|) \times 0.7 - (|a| + |b|) \times 0.3 \times CosC \quad (3)$$

The first term $(|a| + |b|) \times 0.7$ in the proposed Ikuemuya's formula III was obtained from the established fact that, for a right angle triangle or for two vectors acting perpendicularly with each other, the approximate length of the hypotenuse which also represent the magnitude of the resultant vector is equal to the algebraic sum of the magnitude of the two vectors or the sum of the magnitude of the other two smaller sides (opposite side and adjacent side) of the right angled triangle multiplied by an average constant factor of 0.7 [8], this first term, is also similar to the Pythagoras term $a^2 + b^2$ in the law of cosines . The last term $-(|a| + |b|) \times 0.3 \times CosC$ in the proposed Ikuemuya's formula III was formulated and added to the first term so that equation (3) which is the proposed Ikuemuya's formula III will reduce to equation (2) which was obtained from the law of cosines (equation 1) when the angle C was exactly equated to 180 degrees.

Now, let us test and see if when the angle between the two vectors are exactly equal to 180 degrees, if the Ikuemuya's formula III will reduce to the same $a+b$ as obtained using the law of cosines.

$$R = (|a| + |b|) \times 0.7 - (|a| + |b|) \times 0.3 \times Cos(180)$$

$$R = (|a| + |b|) \times 0.7 - (|a| + |b|) \times 0.3 \times (-1)$$

$$R = (|a| + |b|) \times 0.7 + (|a| + |b|) \times 0.3$$

$$R = |a| + |b| \quad (4)$$

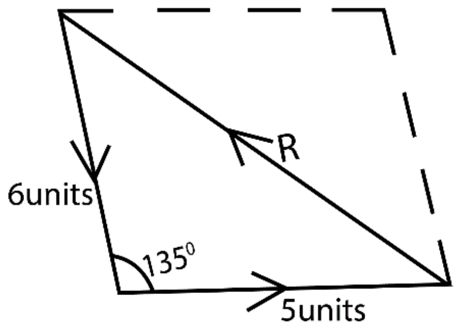
It is verified that equation (4) which was obtained from the proposed Ikuemuya's formula III is equivalent to equation (2) which was obtained from the law of cosines. Thus, both formulas agree.

It should be noted that, the Ikuemuya's formula III (equation 3) was formulated mainly for the computation of the magnitude of the resultant vector or for finding the length of the diagonal of the parallelogram or the longest side of an obtuse triangle. The Ikuemuya's formula III cannot be used to compute the magnitude or length of other two sides or component vectors.

IV. RESULTS AND DISCUSSION

Verification of the proposed Ikuemuya's formula III: alternative to the cosine rule for the computation of the magnitude of resultant vector.

Case I: The magnitude of the resultant R of the two vectors of magnitude 6 units and 5 units both acting at an angle of 135 degrees can be computed using the law of cosine or the cosine rule as follows:



$$R^2 = a^2 + b^2 - 2 \times a \times b \times CosC$$

$$R^2 = 6^2 + 5^2 - 2 \times 6 \times 5 \times Cos(135)$$

$$R^2 = 36 + 25 - 60 \times (-0.70711)$$

$$R^2 = 61 + 42.426$$

$$R^2 = 103.426$$

$$R = \sqrt{103.426}$$

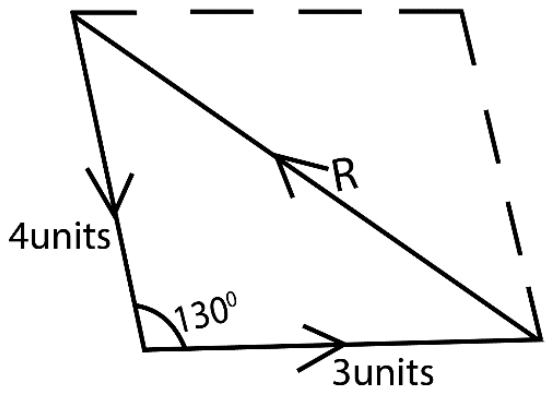
$$R = 10.17 units$$

On the other hand, using the proposed Ikuemuya's formula III (eqn. 3), we have;

$$R = (|a| + |b|) \times 0.7 - (|a| + |b|) \times 0.3 \times CosC$$
$$R = (6 + 5) \times 0.7 - (6 + 5) \times 0.3 \times Cos(135)$$
$$R = 7.7 - 3.3 \times (-0.70711)$$
$$R = 7.7 + 2.333$$
$$R = 10.03 units$$

This result is in agreement with that obtained with the law of cosines.

Case II: If two vectors of magnitude 4 units and 3 units act at an angle of 130 degrees as shown below;



The magnitude of the resultant vector R can be calculated using the law of cosines as follows:

$$R^2 = a^2 + b^2 - 2 \times a \times b \times CosC$$
$$R^2 = 4^2 + 3^2 - 2 \times 4 \times 3 \times Cos(130)$$
$$R^2 = 16 + 9 - 24 \times (-0.642788)$$
$$R^2 = 25 + 15.427$$
$$R^2 = 40.427$$
$$R = \sqrt{40.427}$$
$$R = 6.358 units$$

On the other hand, using the proposed Ikuemuya's formula III, we have that;

$$R = (|a| + |b|) \times 0.7 - (|a| + |b|) \times 0.3 \times CosC$$
$$R = (4 + 3) \times 0.7 - (4 + 3) \times 0.3 \times Cos(130)$$
$$R = 4.9 - 2.1 \times (-0.64279)$$
$$R = 4.9 + 1.350$$
$$R = 6.250 units$$

It is seen that the result obtained using the Ikuemuya's formula III which is 6.250 units compare reasonably well with 6.358 units obtained using the law of cosines.
The percentage error in using the Ikuemuya's formula III in the above two cases are shown in table 1.0 below;

Table 1.0: Table showing the percentage error in using the Ikuemuya's formula III for computing the magnitude of resultant vector.

Results from the law of cosine (Exact value)	Results from the proposed Ikuemuya's formula III (Computed value)	Percentage error in Ikuemuya's formula III
10.170 units	10.030 units	1.38%
6.358 units	6.250 units	1.70%

The table above shows that the Ikuemuya's formula III agrees quite well with the law of cosines when used in the computation of the magnitude of resultant vector.

Discussion
One can see that, the proposed Ikuemuya's Formula III gives answers that are in reasonably good agreement with the law of cosines when used in the computation of the magnitude of resultant vector with a very minimal percentage error and with a degree of accuracy of over 98%, for example, in case one above, we got 10.17 units using the law of cosine while 10.03 units while using the Ikuemuya's formula III, looking critically, it can be seen and concluded that these two results are similar and are in agreement. Therefore, students and users in science and engineering do not have to worry about working with squares and square roots of numbers anymore in arriving at a final result when using the proposed Ikuemuya's formula III as they were made to go through hitherto using the law of cosines.

V. CONCLUSION AND FUTURE SCOPE

In this paper, we have used the fact that, the sum of the magnitude of the two smaller sides (opposite and adjacent sides) of a right angled triangle or the sum of the magnitude of two vectors acting perpendicularly is being reduced by an average constant factor of 0.7 to give the approximate length of the hypotenuse or the magnitude of the resultant vector in the formulation of the proposed Ikuemuya's formula III (eqn. 3). This formula gives similar result as the law of cosines when used in the computation of the magnitude of resultant vector of two vectors acting at an obtuse angle.

The use of the proposed Ikuemuya's formula III does not require the finding of squares and square roots of numbers which is peculiar with the law of cosines when used in the computation of the magnitude of resultant of two vectors acting at obtuse angle, thus making it simple to use. It is thus hopeful that the proposed new formula would serve as a better alternative for students and users particularly at secondary or high school levels in solving mathematical and physical related problems in geometry due to its simplicity.

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Olusegun Eniola Ikuemuya is currently a postgraduate student at the Department of Physics, Federal University of Technology Akure, Nigeria. He obtained a Bachelor of Technology (B.Tech.) in Physics with a first class honor from same institution in 2017. He currently has two publications with his name on it in reputable international journals, he propounded the Ikuemuya's formula I &II which are alternatives to the Pythagoras theorem for computing the magnitude of resultant vector in 2 and 3-dimensions which appeared in ISROSET, IJSRMSS journal, he is analytical and a highly inquisitive student/teacher. His research area focuses on the theoretical and mathematical aspect of Physics.