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Comparative study on Type I, Type II and gestational Diabetes Patients Using a Weighted Power Prakaamy Distribution

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Abstract- In the current study, a novel model of power prakaamy distribution known as the weighted power prakaamy distribution has been examined. Moments, Order statistics, the Likelihood Ratio test, entropies, the Bonferroni and Lorenz curves, and other statistical properties of the proposed distribution have been computed and analysed. The parameters of the new distribution are examined using the (MLE) maximum likelihood estimator, and the (FIM) Fisher's information matrix has been addressed. The Weighted Power prakaamy distribution used in Type I, Type II and Gestational Diabetes datasets. In this paper, we have to study on these three types of diabetes and we will find out, which one is mostly affected and then which age group of people mostly affected in of diabetes. Finally, Comparative study on this distribution is executed to show its flexibility and superiority over other distributions.

Keywords- Diabetes mellitus, Weighted Power Prakaamy distribution, Survival analysis, Moment, Maximum likelihood estimation.

I. INTRODUCTION

A survey done the previous year (2021) revealed that type I diabetes affects 11% of individuals globally, type II diabetes affects more than 90%, and gestational diabetes affects 16.5% of pregnant women. The 6.8% million deaths reported in the global death survey for 2021. Between the ages of 20 and 80, 538 million people will have diabetes. By 2030, 644 million individuals will have diabetes mellitus worldwide, and by 2045, 783 million would. This comparative analysis of patients with type I, type II, and gestational diabetes is based on the weighted power prakaamy distribution. In 2021, Shanker and Shukla created a brand-new variation of the prakaamy distribution called as the quasi shanker distribution. They talked about its numerous mathematical and statistical features as well as its uses. We investigate the goodness of fit of the power prakaamy distribution using real lifespan data. The fit shows good agreement with the power Lindley distribution with two parameters, the Shanker, the exponential, and the Lindley distributions. In this paper, we offer a brand-new distribution with three parameters called the weighted power Prakaamy (WPP) distribution. Para and Jan introduced a novel life time distribution called the Weighted Three parameter Pareto Type II distribution, which has applications in the medical sciences. We offer the novel distribution with the hope that it would outperform existing distributions in terms of output, dependability, and adaptability. In comparison to its sub-models, this distribution is more adaptable to modelling various diabetic dataset types due to its third parameter, which is weighted. The weighted power prakaamy distribution appears to be a more acceptable distribution that may be utilised in many cases with fitting survival data since it performs better at tolerating a wide range of (HR) function variants. By entering datasets for Type I, Type II, and gestational diabetic patients, we will analyse the goodness of fit when fitting the Weighted Power Prakaamy Distribution in contrast to the Lindley and Pranav distributions.

II. Weighted power prakaamy distribution

The PDF of power Prakaamy (PPD) is given by

$$f(x;\theta,\alpha)=\frac{\alpha\theta}{\theta^5+120}(1+x^{5\alpha})x^{\alpha-1}e^{-\theta x^\alpha}, x>0, \theta>0, \alpha>0 \quad (1)$$

The CDF of power prakaamy (PPD) is given by

$$F(x;\theta,\alpha) = 1-\left[\frac{1+\theta x^\delta\left(\begin{matrix}\theta^4x^{4\alpha}+5\theta^3x^{3\alpha}+20\theta^2x^{2\alpha}\\+60\theta x^\alpha+120\end{matrix}\right)}{\theta^5+120}\right]e^{-\theta x^\alpha}$$

$$\theta>0, \alpha>0 \quad (2)$$

In this case, the non-negative random variable (r.v) X and the pdf are considered (x). The weighted random field's non-negative (WF) is then expressed as w(x), in PDF (WR)

Variable X_w is given by $f_w(x)=\dfrac{w(x)f(x)}{E(w(x))}$,

$$x > 0$$

$Where\ w(x)\ to\ the\ non-negative(WF) weight\ function\ and$

$$E(w(x)) = \int w(x) f(x) dx < \infty.$$

In this work, the weighted Prakaamy distribution will be derived. It is evident that altering the weight function w(x) produced a number of weighted distributions. As a result, the distribution that is created has a PDF of where w and is known as a (WD) weighted distribution w(x) $=x_c$

$$f_w(x) = \frac{x^c f(x;\theta,\alpha)}{E(x^c)} \quad (3)$$

Where

$$E(x^c) = \int_0^\infty x^c f(x;\theta,\alpha)dx \quad (4)$$

$$E(x^c) = \int_0^\infty x^c \frac{\alpha\theta^6}{\theta^5+120}(1+x^{5\alpha})x^{\alpha-1}e^{-\theta x^\alpha}dx$$

$$E(x^c) = \int_0^\infty x^{\alpha+c-1}\frac{\alpha\theta^6}{\theta^5+120}(1+x^{5\alpha})e^{-\theta x^\alpha}dx$$

$$= \frac{\alpha\theta^6}{\theta^5+120}\int_0^\infty x^{\alpha+c-1}(1+x^{5\alpha})e^{-\theta x^\alpha}dx$$

$$= \frac{\alpha\theta^6}{\theta^5+120}\left[\int_0^\infty x^{\alpha+c-1}e^{-\theta x^\alpha}dx + \int_0^\infty x^{6\alpha+c-1}e^{-\theta x^\alpha}dx\right]$$

Put,

$$x^\alpha = t \Rightarrow x = t^{\frac{1}{\alpha}}\ Also\ \alpha x^{\alpha-1}dx = dt \Rightarrow dx = \frac{dt}{\alpha x^{\alpha-1}} \Rightarrow \frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}}$$

Equation (4) is simplified, and then the result of CDF of the (WPP) distribution is obtained as

$$\because E(x^c) = \frac{\alpha\theta^6}{\theta^5+120}\int_0^\infty t^{\frac{\alpha+c-1}{\alpha}}e^{-\theta t}\frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}} + \int_0^\infty t^{\frac{6\alpha+c-1}{\alpha}}e^{-\theta t}\frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}}$$

$$= \frac{\alpha\theta^6}{\theta^5+120}\left[\frac{1}{\alpha}\int_0^\infty t^{\frac{\alpha+c-1}{\alpha}}e^{-\theta t}\frac{dt}{t^{\frac{\alpha-1}{\alpha}}} + \frac{1}{\alpha}\int_0^\infty t^{\frac{6\alpha+c-1}{\alpha}}e^{-\theta t}\frac{dt}{t^{\frac{\alpha-1}{\alpha}}}\right]$$

$$= \frac{\alpha\theta^6}{\theta^5+120}\left[\frac{1}{\alpha}\int_0^\infty t^{\frac{\alpha+c-1}{\alpha}}e^{-\theta t}dt\, t^{-\left(\frac{\alpha-1}{\alpha}\right)} + \frac{1}{\alpha}\int_0^\infty t^{\frac{6\alpha+c-1}{\alpha}}e^{-\theta t}dt\, t^{-\left(\frac{\alpha-1}{\alpha}\right)}\right]$$

$$= \frac{\alpha\theta^6}{\theta^5+120}\left[\frac{1}{\alpha}\int_0^\infty t^{\frac{c}{\alpha}}e^{-\theta t}dt + \frac{1}{\alpha}\int_0^\infty t^{\frac{5\alpha+c}{\alpha}}e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^6}{\theta^5+120}\left[\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{c}{\alpha}+1\right)-1}e^{-\theta t}dt + \frac{1}{\alpha}\int_0^\infty t^{\left(\frac{5\alpha+c}{\alpha}+1\right)-1}e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^6}{\theta^5+120}\left[\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{c}{\alpha}+1\right)-1}e^{-\theta t}dt + \frac{1}{\alpha}\int_0^\infty t^{\left(\frac{5\alpha+c+\alpha}{\alpha}\right)-1}e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^6}{\theta^5+120}\left[\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{\alpha+c}{\alpha}\right)-1}e^{-\theta t}dt + \frac{1}{\alpha}\int_0^\infty t^{\left(\frac{6\alpha+c}{\alpha}\right)-1}e^{-\theta t}dt\right]$$

$\because$ It follows gamma distribution,

$$= \frac{\alpha\theta^6}{\theta^5+120}\left[\frac{1}{\alpha}\frac{\Gamma\frac{\alpha+c}{\alpha}}{\theta^{\frac{\alpha+c}{\alpha}}} + \frac{1}{\alpha}\frac{\Gamma\frac{6\alpha+c}{\alpha}}{\theta^{\frac{6\alpha+c}{\alpha}}}\right]$$

$$= \frac{\alpha\theta^6}{\theta^5+120}\left[\frac{\theta^5\Gamma\frac{\alpha+c}{\alpha}}{\alpha\theta^{\frac{\alpha+c}{\alpha}}} + \frac{\Gamma\frac{6\alpha+c}{\alpha}}{\alpha\theta^{\frac{\alpha+c}{\alpha}}}\right]\alpha\theta^{\frac{\alpha+c}{\alpha}}$$

$$= \frac{\alpha\theta^6}{\theta^5+120}\left[\frac{\theta^5\Gamma\frac{\alpha+c}{\alpha}\Gamma\frac{6\alpha+c}{\alpha}}{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}\right]$$

$$E(x^c) = \left[\frac{\theta^5\Gamma\frac{\alpha+c}{\alpha}\Gamma\frac{6\alpha+c}{\alpha}}{\alpha\theta^{\frac{c}{\alpha}}(\theta^5+120)}\right] \quad (5)$$

Eqn (3) is yield the PDF of the (WPP) distribution if equations (5) and (1) are substituted.

$$f_w(x) = \frac{x^c\frac{\alpha\theta^6}{\theta^5+120}(1+x^{5\alpha})x^{\alpha-1}e^{-\theta x^\alpha}}{\left[\frac{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}{\alpha\theta^{\frac{c}{\alpha}}(\theta^5+120)}\right]}$$

$$f_w(x) = x^c\frac{\alpha\theta^6}{\theta^5+120}(1+x^{5\alpha})x^{\alpha-1}e^{-\theta x^\alpha} \times \frac{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}{\alpha\theta^{\frac{c}{\alpha}}(\theta^5+120)}$$

$$f_w(x) = \frac{x^{\alpha+c-1}\alpha\theta^{\frac{c}{\alpha}+6}(1+x^{5\alpha})e^{-\theta x^\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}$$

$$= \frac{x^{\alpha+c-1}\alpha\theta^{\frac{6\alpha+c}{\alpha}}(1+x^{5\alpha})e^{-\theta x^\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}$$

$$f_w(x) = \frac{x^{\alpha+c-1}\alpha\theta^{\frac{6\alpha+c}{\alpha}}(1+x^{5\alpha})e^{-\theta x^\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}$$

$$f_w(x) = \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}x^{\alpha+c-1}(1+x^{5\alpha})e^{-\theta x^\alpha}$$

$$f_w(x;\alpha,\theta,c) = \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}x^{\alpha+c-1}(1+x^{5\alpha})e^{-\theta x^\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}} \quad (6)$$

Cumulative distribution function of weighted power prakaamy distribution can be obtained at

$$F_w(x) = \int_0^x f_w(x)\,dx$$

$$= \int_0^x \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}} x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}$$

$$= \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\int_0^x \alpha\theta^{\frac{6\alpha+c}{\alpha}} x^{\alpha+c-1}(1 + x^{5\alpha})\,e^{-\theta x^\alpha}dx$$

$$= \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha\theta^{\frac{6\alpha+c}{\alpha}}\int_0^x x^{\alpha+c-1}\,e^{-\theta x^\alpha}dx + \alpha\theta^{\frac{6\alpha+c}{\alpha}}\int_0^x x^{6\alpha+c-1}\,e^{-\theta x^\alpha}dx\right]$$

Put $\theta x^\alpha = t \Rightarrow x^\alpha = \frac{t}{\theta} \Rightarrow x = \left(\frac{t}{\theta}\right)^{\frac{1}{\alpha}}$ *Also* $\alpha\theta x^{\alpha-1}dx = dt = dx = \frac{dt}{\alpha\theta x^{\alpha-1}} \Rightarrow dx = \frac{dt}{\alpha\theta\left(\frac{t}{\theta}\right)^{\frac{\alpha-1}{\alpha}}}$

Equation (6) becomes Equation (7) after being simplified, given the cumulative distribution function of the weighted power prakaamy distribution.

$$= \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha\theta^{\frac{6\alpha+c-\alpha}{\alpha}}\int_0^{\theta x^\alpha}\left(\frac{t}{\theta}\right)^{\frac{c}{\alpha}} e^{-t}dt + \alpha\theta^{\frac{6\alpha+c-\alpha}{\alpha}}\int_0^{\theta x^\alpha}\left(\frac{t}{\theta}\right)^{\frac{5\alpha+c}{\alpha}} e^{-t}dt\right]$$

$$= \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha\theta^{\frac{5\alpha+c}{\alpha}}\int_0^{\theta x^\alpha}\left(\frac{t}{\theta}\right)^{\frac{c}{\alpha}} e^{-t}dt + \alpha\theta^{\frac{5\alpha+c}{\alpha}-1}\int_0^{\theta x^\alpha}\left(\frac{t}{\theta}\right)^{\frac{5\alpha+c}{\alpha}} e^{-t}dt\right]$$

$$= \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha\theta^{\frac{5\alpha+c}{\alpha}}\int_0^{\theta x^\alpha}\left(\frac{t}{\theta}\right)^{\left(\frac{\alpha+c}{\alpha}\right)-1} e^{-t}dt + \alpha\theta^{\frac{5\alpha+c}{\alpha}-1}\int_0^{\theta x^\alpha}\left(\frac{t}{\theta}\right)^{\left(\frac{6\alpha+c}{\alpha}\right)-1} e^{-t}dt\right]$$

$$= \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{\alpha\theta^{\frac{5\alpha+c}{\alpha}}}{\theta^{\frac{\alpha+c+\alpha}{\alpha}-1}}\int_0^{\theta x^\alpha}\left(\frac{t}{\theta}\right)^{\left(\frac{\alpha+c}{\alpha}\right)-1} e^{-t}dt + \frac{\alpha\theta^{\frac{5\alpha+c}{\alpha}-1}}{\theta^{\frac{6\alpha+c+\alpha}{\alpha}-1}}\int_0^{\theta x^\alpha}\left(\frac{t}{\theta}\right)^{\left(\frac{6\alpha+c}{\alpha}\right)-1} e^{-t}dt\right]$$

$$= \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{\alpha\theta^{\frac{5\alpha+c}{\alpha}}}{\theta^{\frac{c}{\alpha}-1}}\int_0^{\theta x^\alpha}\left(\frac{t}{\theta}\right)^{\left(\frac{\alpha+c}{\alpha}\right)-1} e^{-t}dt + \frac{\alpha\theta^{\frac{5\alpha+c}{\alpha}-1}}{\theta^{\frac{5\alpha}{\alpha}-1}}\int_0^{\theta x^\alpha}\left(\frac{t}{\theta}\right)^{\left(\frac{6\alpha+c}{\alpha}\right)-1} e^{-t}dt\right]$$

$$= \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha\theta^{\frac{5\alpha}{\alpha}}\int_0^{\theta x^\alpha} t^{\left(\frac{\alpha+c}{\alpha}\right)-1} e^{-t}dt + \alpha\int_0^{\theta x^\alpha}\left(\frac{t}{\theta}\right)^{\left(\frac{6\alpha+c}{\alpha}\right)-1} e^{-t}dt\right]$$

$$= \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha^5\int_0^{\theta x^\alpha} t^{\left(\frac{\alpha+c}{\alpha}\right)-1} e^{-t}dt + \alpha\int_0^{\theta x^\alpha} t^{\left(\frac{6\alpha+c}{\alpha}\right)-1} e^{-t}dt\right]$$

$$F_w(x;\alpha,\theta,c) = \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha^5\gamma\left(\frac{\alpha+c}{\alpha},\theta x^\alpha\right) + \alpha\gamma\left(\frac{6\alpha+c}{\alpha},\theta x^\alpha\right)\right] \quad (7)$$

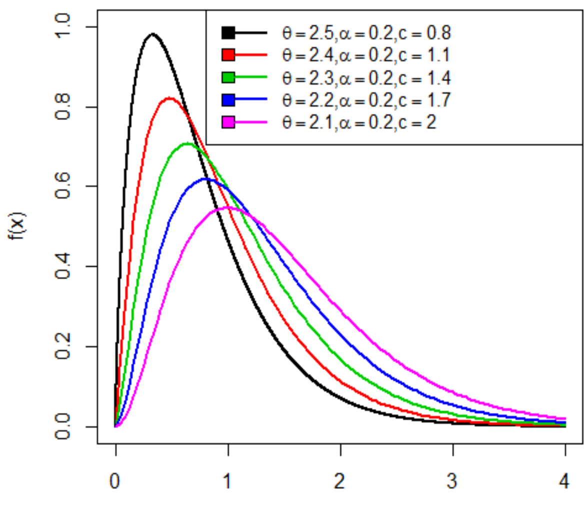

Fig.1:Pdf plot of WP PRAKAAMY distribution

III. Survival measures

The survival function (SF), hazard rate, and reverse hazard functions of the proposed weighted power prakaamy distribution.

3.1 Survival method

Reliability function is given the weighted power prakaamy distribution (WPPD), which is frequently referred to as the complement of the cdf.

$$s(x) = 1 - F_x(x)$$

$$s(x) = 1 - \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha^5\gamma\left(\frac{\alpha+c}{\alpha},\theta x^\alpha\right) + \alpha\gamma\left(\frac{6\alpha+c}{\alpha},\theta x^\alpha\right)\right]$$

3.1 Hazard function

The hazard rate the force of mortality it is given by

$$h(x) = \frac{f_w(x)}{1 - F_w(x)}$$

$$h(x) = \frac{\dfrac{\alpha\theta^{\frac{6\alpha+c}{\alpha}} x^{\alpha+c-1}(1+x^{5\alpha})\, e^{-\theta x^\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}}{\dfrac{\left[\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}\right] - \left[\begin{matrix}\alpha^5\gamma\left(\frac{\alpha+c}{\alpha},\theta x^\alpha\right)\\ +\alpha\gamma\left(\frac{6\alpha+c}{\alpha},\theta x^\alpha\right)\end{matrix}\right]}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}}$$

$$h(x) = \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}} x^{\alpha+c-1}(1+x^{5\alpha})\, e^{-\theta x^\alpha}}{\left[\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}\right] - \left[\begin{matrix}\alpha^5\gamma\left(\frac{\alpha+c}{\alpha},\theta x^\alpha\right) +\\ \alpha\gamma\left(\frac{6\alpha+c}{\alpha},\theta x^\alpha\right)\end{matrix}\right]}$$

3.2 Reverse hazard (RH) function

The weighted power prakaamy (WPP) distribution is given by

$$h(x) = \frac{f_w(x)}{F_w(x)}$$

$$h(x) = \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}} x^{\alpha+c-1}(1+x^{5\alpha})\, e^{-\theta x^\alpha}}{\left[\alpha^5\gamma\left(\frac{\alpha+c}{\alpha},\theta x^\alpha\right) + \alpha\gamma\left(\frac{6\alpha+c}{\alpha},\theta x^\alpha\right)\right]}$$

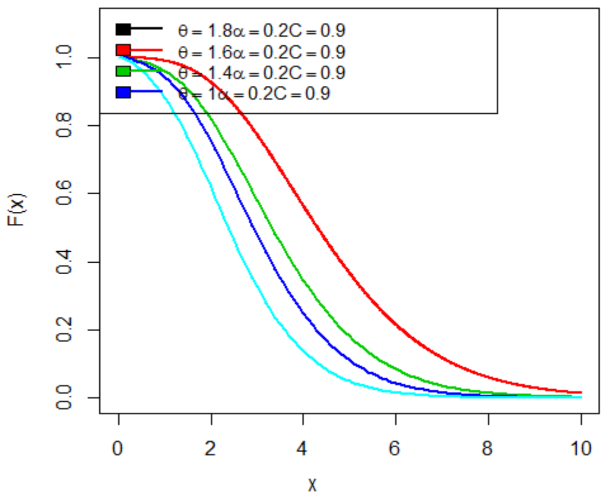

Fig.2. survival function of Weighted prakaamy distribution

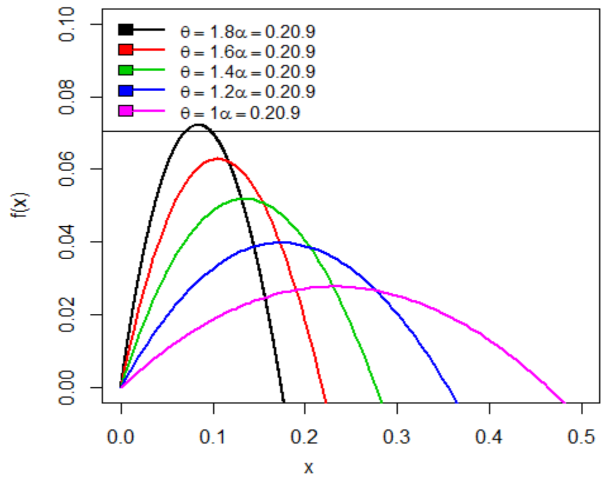

Fig.3. Showing Hazard function of WPP distribution

IV. Statistical possessions

We will look into a few statistical properties relating to the weighted power prakaamy (WPPD) distribution in this section.

4.1 Moments

If X represents the r.v of the weighted power prakaamy distribution by parameters, and c, then the r[th] instant E(Xr) around the source is.

$$\mu_r' = E[x^r] = \int_0^\infty x^r f_w(x)dx$$

$$= \int_0^\infty x^r \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}} x^{\alpha+c-1}(1+x^{5\alpha})\, e^{-\theta x^\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}} dx$$

$$= \int_0^\infty \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}} x^{\alpha+c+r-1}(1+x^{5\alpha})\, e^{-\theta x^\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}} dx$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\int_0^\infty x^{\alpha+c+r-1}\, e^{-\theta x^\alpha}dx + \int_0^\infty x^{6\alpha+c+r-1}\, e^{-\theta x^\alpha}dx\right] \qquad (8)$$

Put, $x^\alpha = t \Rightarrow x = \frac{t^{\frac{1}{\alpha}}}{\theta}$ $Also\ \alpha x^{\alpha-1}dx = dt \Rightarrow dx = \frac{dt}{\alpha x^{\alpha-1}} \Rightarrow dx = \frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}}$

After simplification Equation (8) become,

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\int_0^\infty t^{\frac{\alpha+c+r-1}{\alpha}} e^{-\theta t}\frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}} + \int_0^\infty t^{\frac{6\alpha+c+r-1}{\alpha}} e^{-\theta t}\frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}}\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\alpha}\int_0^\infty t^{\frac{c+r}{\alpha}} e^{-\theta t}\, dt + \frac{1}{\alpha}\int_0^\infty t^{\frac{5\alpha+c+r}{\alpha}} e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{c+r}{\alpha}+1\right)-1}e^{-\theta t}\,dt + \frac{1}{\alpha}\int_0^\infty t^{\left(\frac{5\alpha+c+r}{\alpha}+1\right)-1}e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{c+r+\alpha}{\alpha}\right)-1}e^{-\theta t}\,dt + \frac{1}{\alpha}\int_0^\infty t^{\left(\frac{5\alpha+c+r+\alpha}{\alpha}\right)-1}e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{\alpha+c+r}{\alpha}\right)-1}e^{-\theta t}\,dt + \frac{1}{\alpha}\int_0^\infty t^{\left(\frac{6\alpha+c+r}{\alpha}\right)-1}e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\alpha}\frac{\Gamma\frac{\alpha+c+r}{\alpha}}{\theta^{\frac{\alpha+c+r}{\alpha}}} + \frac{1}{\alpha}\frac{\Gamma\frac{6\alpha+c+r}{\alpha}}{\theta^{\frac{6\alpha+c+r}{\alpha}}}\right]$$

$$E(X)^r = \mu_r' = \frac{\theta^5\Gamma\frac{\alpha+c+r}{\alpha}+\Gamma\frac{6\alpha+c+r}{\alpha}}{\alpha\theta^{\frac{r}{\alpha}}\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}} \quad (9)$$

Putting r= 1,2,3,4 in (9) obtain the expected value of weighted power prakaamy (WPPD).

$$E(X) = \mu_1' = \frac{\theta^5\Gamma\frac{\alpha+c+r}{\alpha}+\Gamma\frac{6\alpha+c+r}{\alpha}}{\alpha\theta^{\frac{1}{\alpha}}\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}$$

$$E(X^1) = \mu_2' = \frac{\theta^5\Gamma\frac{\alpha+c+r}{\alpha}+\Gamma\frac{6\alpha+c+r}{\alpha}}{\alpha\theta^{\frac{2}{\alpha}}\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}$$

$$E(X^3) = \mu_3' = \frac{\theta^5\Gamma\frac{\alpha+c+r}{\alpha}+\Gamma\frac{6\alpha+c+r}{\alpha}}{\alpha\theta^{\frac{3}{\alpha}}\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}$$

$$E(X^4) = \mu_4' = \frac{\theta^5\Gamma\frac{\alpha+c+r}{\alpha}+\Gamma\frac{6\alpha+c+r}{\alpha}}{\alpha\theta^{\frac{4}{\alpha}}\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}$$

Variance = $\mu_2' - (\mu_1')^2$

$$variance = \frac{\theta^5\Gamma\frac{\alpha+c+r}{\alpha}+\Gamma\frac{6\alpha+c+r}{\alpha}}{\alpha\theta^{\frac{2}{\alpha}}\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}} - \left[\frac{\theta^5\Gamma\frac{\alpha+c+r}{\alpha}+\Gamma\frac{6\alpha+c+r}{\alpha}}{\alpha\theta^{\frac{1}{\alpha}}\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right]^2$$

$$S.D(\sigma) = \sqrt{\frac{\theta^5\Gamma\frac{\alpha+c+r}{\alpha}+\Gamma\frac{6\alpha+c+r}{\alpha}}{\alpha\theta^{\frac{2}{\alpha}}\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}} - \left[\frac{\theta^5\Gamma\frac{\alpha+c+r}{\alpha}+\Gamma\frac{6\alpha+c+r}{\alpha}}{\alpha\theta^{\frac{1}{\alpha}}\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right]^2}$$

4.2 Harmonic mean

In the harmonic mean of the weighted power prakaamy distribution can be obtained as

Harmonic mean = $E\left[\frac{1}{x}\right] = \int_0^\infty \frac{1}{x} f_w(x)dx$

$$= \int_0^\infty \frac{1}{x}\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}dx$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\int_0^\infty x^{\alpha+c-2}\,e^{-\theta t^\alpha}\,dx + \int_0^\infty x^{6\alpha+c-2}\,e^{-\theta t^\alpha}\,dx \quad (10)$$

Equation (10) becomes, we obtain the harmonic mean of weighted power prakaamy distribution as

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\alpha}\int_0^\infty t^{\frac{\alpha+c-2}{\alpha}}\,e^{-\theta t}\,\frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}} + \frac{1}{\alpha}\int_0^\infty t^{\frac{6\alpha+c-2}{\alpha}}\,e^{-\theta t}\frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}}\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\alpha}\int_0^\infty t^{\frac{\alpha+c-2-\alpha+1}{\alpha}}\,e^{-\theta t}\,dt + \frac{1}{\alpha}\int_0^\infty t^{\frac{6\alpha+c-2-\alpha+1}{\alpha}}\,e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{c-1}{\alpha}+1\right)-1}e^{-\theta t}\,dt + \frac{1}{\alpha}\int_0^\infty t^{\left(\frac{5\alpha+c-1}{\alpha}+1\right)-1}e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{c-1+\alpha}{\alpha}\right)-1}e^{-\theta t}\,dt + \frac{1}{\alpha}\int_0^\infty t^{\left(\frac{5\alpha+c-1+\alpha}{\alpha}\right)-1}e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{\alpha+c-1}{\alpha}\right)-1}e^{-\theta t}\,dt + \frac{1}{\alpha}\int_0^\infty t^{\left(\frac{6\alpha+c-1}{\alpha}\right)-1}e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\alpha}\frac{\Gamma\frac{\alpha+c-1}{\alpha}}{\theta^{\frac{\alpha+c-1}{\alpha}}} + \frac{1}{\alpha}\frac{\Gamma\frac{6\alpha+c-1}{\alpha}}{\theta^{\frac{6\alpha+c-1}{\alpha}}}\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{\Gamma\frac{\alpha+c-1}{\alpha}}{\alpha\theta^{\frac{\alpha+c-1}{\alpha}}} + \frac{\Gamma\frac{6\alpha+c-1}{\alpha}}{\alpha\theta^{\frac{6\alpha+c-1}{\alpha}}}\right]$$

$$= \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{\theta^5\Gamma\frac{\alpha+c-1}{\alpha} + \Gamma\frac{6\alpha+c-1}{\alpha}}{\alpha\theta^{\frac{-1}{\alpha}}}\right]$$

$$H.M = \left[\frac{\theta^5\Gamma\frac{\alpha+c-1}{\alpha} + \Gamma\frac{6\alpha+c-1}{\alpha}}{\alpha\theta^{\alpha}\left(\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}\right)}\right]$$

4.3 Moment generating function & characteristic function

How to get X's MGF is as follows: Let X serve as the weighted power prakaamy (WPP) distribution's r.v. variable.

$$M_X(t) = E[E^{tx}]$$

$$M_X(t) = \sum_{j=0}^{\infty}\frac{t^J}{J!}\left[\frac{\theta^5\Gamma\frac{\alpha+c+j}{\alpha} + \Gamma\frac{6\alpha+c+j}{\alpha}}{\alpha\theta^{\frac{j}{\alpha}}\left(\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}\right)}\right]$$

$$M_X(t) = \sum_{j=0}^{\infty}\frac{t^J}{J!}\left[\frac{\theta^5\Gamma\frac{\alpha+c+j}{\alpha} + \Gamma\frac{6\alpha+c+j}{\alpha}}{\alpha\theta^{\frac{J}{\alpha}}\left(\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}\right)}\right]$$

$$M_X(t) = \frac{1}{\left(\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}\right)}\sum_{j=0}^{\infty}\frac{t^j}{j!\,\theta_{\alpha}^{J}}\left[\theta^5\Gamma\frac{\alpha+c+j}{\alpha} + \Gamma\frac{6\alpha+c+j}{\alpha}\right]$$

Similarly characteristic function of weighted power prakaamy distribution can be obtained as

$$\phi_X(t) = M_X(it)$$

$$M_X(it) = \frac{1}{\left(\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}\right)}\sum_{j=0}^{\infty}\frac{it^J}{j!\,\theta^{\frac{J}{\alpha}}}\left[\theta^5\Gamma\frac{\alpha+c+j}{\alpha} + \Gamma\frac{6\alpha+c+j}{\alpha}\right]$$

V. Order statistics

If we suppose that X_1, X_2,..., Xn are the random samples and that $X_{(1)}$, $X_{(2)}$,..., $X_{(n)}$ are the order statistics of a random sample of size n selected from a continuous population with PDF function $f_x(x)$ and CDF $F_X(x)$, then the PDF of the rth order statistics X(r) is given by (x).

$$f_{w(r)}(x) = \frac{n!}{(r-1)!\,(n-r)!}f_w(x)[F_X(x)]^{r-1}[1 - F_X(x)]^{n-r} \qquad (11)$$

Equations (6) and (8) can be substituted for each other in equation (11), resulting in the PDF of the rth order statistics of the weighted power prakaamy distribution as

$$f_{w(r)}(x) = \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\right]\left[\frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha^5\gamma\left(\frac{\alpha+c}{\alpha},\theta x^{\alpha}\right) + \alpha\gamma\left(\frac{6\alpha+c}{\alpha},\theta x^{\alpha}\right)\right]^{r-1}\right]\left[1 - \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha^5\gamma\left(\frac{\alpha+c}{\alpha},\theta x^{\alpha}\right) + \alpha\gamma\left(\frac{6\alpha+c}{\alpha},\theta x^{\alpha}\right)\right]^{r-1}\right]^{n-r}$$

I.e. the pdf of higher order statistics X(n) of weighted power prakaamy distribution can be obtained as

$$f_{x(n)}(x) = \frac{n\alpha\theta^{\frac{6\alpha+c}{\alpha}}x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha^5\gamma\left(\frac{\alpha+c}{\alpha},\theta x^{\alpha}\right) + \alpha\gamma\left(\frac{6\alpha+c}{\alpha},\theta x^{\alpha}\right)\right]^{r-1}\right]^{n-1}$$

The pdf of first order statistic $X_{(1)}$ of weighted power prakaamy distribution can be obtained as

$$f_{x(1)}(x) = \frac{n\alpha\theta^{\frac{6\alpha+c}{\alpha}}x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[1 - \frac{1}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\left[\alpha^5\gamma\left(\frac{\alpha+c}{\alpha},\theta x^{\alpha}\right) + \alpha\gamma\left(\frac{6\alpha+c}{\alpha},\theta x^{\alpha}\right)\right]^{r-1}\right]^{n-1}$$

VI. Likelihood Ratio Test

The letters X_1, X_2,.... X_n stand for a random sample of size n taken from the weighted power prakaamy distribution. The theory is put to practise.

$H_0: f(x) = f(x;\theta,\alpha) again\ H_1: f(x)\ f_w(x;\theta,\alpha,c)$

You can tell whether the power prakaamy distribution or the weighted power prakaamy distribution comes from sample of size "n" by looking at the test statistic below.

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n}\frac{f_w(x;\theta,\alpha,c)}{f(x;\theta,\alpha)}$$

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n}\frac{\dfrac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}}{\dfrac{\alpha\theta^6}{\theta^5+120}(1+x^{5\alpha})x^{\alpha-1}e^{-\theta x^{\alpha}}}$$

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}} x^{\alpha+c-1}(1+x^{5\alpha})\, e^{-\theta x^{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}} \times \frac{\alpha\theta^6}{\theta^5+120}(1+x^{5\alpha})x^{\alpha-1}e^{-\theta x^{\alpha}}$$

$$= \prod_{i=1}^{n}\left[\frac{\alpha\theta^{\frac{c}{\alpha}} x_i{}^{c}(\theta^5+120)}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\right]$$

$$= \left[\frac{\alpha\theta^{\frac{c}{\alpha}} x_i{}^{c}(\theta^5+120)}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\right] \prod_{i=1}^{n} x_i{}^{c}$$

We should reject the null hypothesis, if

$$\Delta = \left[\frac{\alpha\theta^{\frac{c}{\alpha}}(\theta^5+120)}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\right] \prod_{i=1}^{n} x_i{}^{c} > K$$

Equivalently, we also reject null hypothesis, where

$$\Delta^* = \prod_{i=1}^{n} x_i{}^{c} > K\left[\frac{\alpha\theta^{\frac{c}{\alpha}}(\theta^5+120)}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\right]$$

$$\Delta^* = \prod_{i=1}^{n} x_i{}^{c} > K \; where \; k^* = k\left[\frac{\alpha\theta^{\frac{c}{\alpha}}(\theta^5+120)}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\right]^n$$

The distribution of 2log Δ is given as a χ^2 distribution with one d.f for large sample sizes n, and the chi-square distribution is also used to get the p value. Therefore, when the probability value is provided by, we reject the null hypothesis.

$$p(\Delta^* > \lambda^*) where\ \lambda^* - \prod_{i=1}^{n} x^i \text{ is less than a specified level of significance and } \prod_{i=1}^{n} x_i \text{ is this the observed value of the statistic } \Delta^*$$

VII. Bonferroni & Lorenz curves

In order to ascertain how wealth, income, and poverty are distributed, economists employ the Bonferroni and Lorenz curves, also referred to as income distribution curves. However, they are also used in other industries nowadays, such as reliability, insurance, and demography. The Bonferroni and Lorenz curves are given by.

$B(p) = \frac{1}{P\mu_1'}\int_0^q x f_w(x)\,dx$ And $L(p) = \frac{1}{\mu_1'}\int_0^q x f_w(x)\,dx$

Where

$$\mu_1' = \frac{\theta^5\Gamma\frac{\alpha+c+r}{\alpha} + \Gamma\frac{6\alpha+c+r}{\alpha}}{\alpha\theta^{\frac{1}{\alpha}}\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}$$

$$\because B(p) = \frac{\alpha\theta^{\frac{1}{\alpha}}\left(\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}\right)}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha} + \Gamma\frac{6\alpha+c+1}{\alpha}\right)} \int_0^q \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}} x^{\alpha+c-1}(1+x^{5\alpha})\, e^{-\theta x^{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}dx$$

$$B(p) = \frac{\alpha\theta^{\frac{1}{\alpha}}\left(\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}\right)}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha} + \Gamma\frac{6\alpha+c+1}{\alpha}\right)} \int_0^q \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}} x^{\alpha+c}(1+x^{5\alpha})\, e^{-\theta x^{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}dx$$

$$B(p) = \frac{\alpha\theta^{\frac{1}{\alpha}}\left(\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}\right)}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha} + \Gamma\frac{6\alpha+c+1}{\alpha}\right)} \frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\int_0^q x^{\alpha+c}(1+x^{5\alpha})\, e^{-\theta x^{\alpha}}dx$$

$$B(p) = \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha} + \Gamma\frac{6\alpha+c+1}{\alpha}\right)}\int_0^q x^{\alpha+c}(1+x^{5\alpha})\, e^{-\theta x^{\alpha}}dx$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha} + \Gamma\frac{6\alpha+c+1}{\alpha}\right)}\left[\int_0^q x^{\alpha+c}\, e^{-\theta x^{\alpha}}dx + \int_0^q x^{6\alpha}\, e^{-\theta x^{\alpha}}dx\right] \tag{12}$$

After the simplification of equation (12), we obtain the income distribution curves as

$$\because B(p) = \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha} + \Gamma\frac{6\alpha+c+1}{\alpha}\right)}\left[\int_0^q t^{\frac{\alpha+c}{\alpha}}\, e^{-\theta t}\frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}} + \int_0^q t^{\frac{6\alpha+c}{\alpha}}\, e^{-\theta t}\frac{dt}{\alpha t^{\frac{\alpha-1}{\alpha}}}\right]$$

$$B(p) = \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha} + \Gamma\frac{6\alpha+c+1}{\alpha}\right)}\left[\frac{1}{\alpha}\int_0^q t^{\frac{\alpha+c}{\alpha}}\, e^{-\theta t}dt\, t^{-\left(\frac{\alpha+1}{\alpha}\right)} + \frac{1}{\alpha}\int_0^q t^{\frac{6\alpha+c}{\alpha}}\, e^{-\theta t}dt\, t^{-\left(\frac{\alpha+c}{\alpha}\right)}\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha} + \Gamma\frac{6\alpha+c+1}{\alpha}\right)}\left[\frac{1}{\alpha}\int_0^q t^{\frac{\alpha+c-\alpha+1}{\alpha}}\, e^{-\theta t}dt + \frac{1}{\alpha}\int_0^q t^{\frac{6\alpha+c+1}{\alpha}}\, e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha} + \Gamma\frac{6\alpha+c+1}{\alpha}\right)}\left[\frac{1}{\alpha}\int_0^q t^{\frac{c+1}{\alpha}}\, e^{-\theta t}dt + \frac{1}{\alpha}\int_0^q t^{\frac{5\alpha+c+1}{\alpha}}\, e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha} + \Gamma\frac{6\alpha+c+1}{\alpha}\right)}\left[\frac{1}{\alpha}\int_0^q t^{\left(\frac{c+1}{\alpha}+1\right)-1}\, e^{-\theta t}dt + \frac{1}{\alpha}\int_0^q t^{\left(\frac{5\alpha+c+1}{\alpha}+1\right)-1}\, e^{-\theta t}dt\right]$$

$$= \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha}+\Gamma\frac{6\alpha+c+1}{\alpha}\right)}$$
$$\left[\frac{1}{\alpha}\int_0^q t^{\left(\frac{c+1+\alpha}{\alpha}\right)-1} e^{-\theta t}dt + \frac{1}{\alpha}\int_0^q t^{\left(\frac{5\alpha+c+1+\alpha}{\alpha}\right)-1} e^{-\theta t}dt\right]$$
$$\left[\frac{1}{\alpha}\gamma\left(\frac{\alpha+c+1}{\alpha},\theta q\right)+\frac{1}{\alpha}\gamma\left(\frac{6\alpha+c+1}{\alpha},\theta q\right)\right]$$
$$= \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha}+\Gamma\frac{6\alpha+c+1}{\alpha}\right)}$$
$$\left[\frac{\gamma\left(\frac{\alpha+c+1}{\alpha},\theta q\right)}{\alpha}+\frac{\gamma\left(\frac{6\alpha+c+1}{\alpha},\theta q\right)}{\alpha}\right]$$
$$= \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha}+\Gamma\frac{6\alpha+c+1}{\alpha}\right)}$$
$$\left[\frac{\gamma\left(\frac{\alpha+c+1}{\alpha},\theta q\right)+\gamma\left(\frac{6\alpha+c+1}{\alpha},\theta q\right)}{\alpha}\right]$$

$\because B(p)$

$$= \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha}+\Gamma\frac{6\alpha+c+1}{\alpha}\right)}\left[\gamma\left(\frac{\alpha+c+1}{\alpha},\theta q\right)+\gamma\left(\frac{6\alpha+c+1}{\alpha},\theta q\right)\right] \quad (14)$$

$$L(p) = \frac{\alpha\theta^{\frac{6\alpha+c+1}{\alpha}}}{P\left(\theta^5\Gamma\frac{\alpha+c+1}{\alpha}+\Gamma\frac{6\alpha+c+1}{\alpha}\right)}\left[\gamma\left(\frac{\alpha+c+1}{\alpha},\theta q\right)+\gamma\left(\frac{6\alpha+c+1}{\alpha},\theta q\right)\right]$$

VIII. Entropies

Numerous disciplines, including probability and statistics, physics, communication theory, and economics, make use of the concept of entropy. Rudolf Clausius, a German physicist, coined the term "entropy" first (1865). Measures of entropy make it possible to assess the diversity, unpredictability, or randomness of a system. A state of chaos or a descent into disorder are some synonyms for entropy. An indicator of the degree of uncertainty is the entropy of the random variable X.

8.1 Renyi Entropy

Alfred Renyi made the initial proposal for entropy, also referred to as Renyi entropy (1957). As a measure of diversity, the Renyi entropy is a crucial topic in statistics and ecology. For the random variable X, the Renyi entropy of order is given by.

$$e(\beta) = \frac{1}{1-\beta}\log\left(\int f_w(x)^\beta dx\right) \text{ where } \beta>0 \,\&\, \beta\neq 1$$

$$e(\beta) = \frac{1}{1-\beta}\log\int_0^\infty\left[\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right]^\beta dx$$

$$= \frac{1}{1-\beta}\log\left[\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^\beta\int_0^\infty x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^\alpha}dx\right] \quad (15)$$

Using binomial expansion of

$$(1+x^{5\alpha})^\beta = \sum_{j=0}^\infty\binom{\beta}{j}x^{5\alpha j}$$

Equation (15) becomes

$$\because e(\beta) = \frac{1}{1-\beta}\log\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^\beta \sum_{j=0}^\infty\binom{\beta}{j}x^{5\alpha j}\int_0^\infty x^{\beta(\alpha+c-1)}\,e^{-\theta\beta x^\alpha}dx$$

$$= \frac{1}{1-\beta}\log\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^\beta\sum_{j=0}^\infty\binom{\beta}{j}\int_0^\infty x^{\beta(\alpha+c-1)}\,e^{-\theta\beta x^\alpha}dx \quad (16)$$

Equation (16) becomes

$$\because e(\beta) = \frac{1}{1-\beta}\log\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^\beta \times\sum_{j=0}^\infty\binom{\beta}{j}\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{\beta(\alpha+c-1)+5\alpha j}{\alpha}\right)}\,e^{-\theta\beta t}dx$$

$$= \frac{1}{1-\beta}\log\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^\beta\sum_{j=0}^\infty\binom{\beta}{j}\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{\beta(\alpha+c-1)+5\alpha j}{\alpha}\right)}\,e^{-\theta\beta t}dx$$

$$= \frac{1}{1-\beta}\log\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^\beta\sum_{j=0}^\infty\binom{\beta}{j}\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{\beta(\alpha+c-1)+5\alpha j-\alpha+1}{\alpha}\right)}\,e^{-\theta\beta t}dx$$

$$= \frac{1}{1-\beta}\log\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^\beta\sum_{j=0}^\infty\binom{\beta}{j}\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{\beta(\alpha+c-1)+5\alpha j-\alpha+1}{\alpha}+1\right)-1}\,e^{-\theta\beta t}dx$$

$$= \frac{1}{1-\beta}\log\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^\beta\sum_{j=0}^\infty\binom{\beta}{j}\frac{1}{\alpha}\int_0^\infty t^{\left(\frac{\beta(\alpha+c-1)+5\alpha j-\alpha+1+\alpha}{\alpha}\right)-1}\,e^{-\theta\beta t}dx$$

$$= \frac{1}{1-\beta}\log\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^{\beta}\sum_{j=0}^{\infty}\binom{\beta}{j}\frac{1}{\alpha}\int_0^{\infty} t^{\left(\frac{\beta(\alpha+c-1)+5\alpha j-1}{\alpha}\right)-1} e^{-\theta\beta t}\,dx$$

$$\because e(\beta) = \frac{1}{1-\beta}\log\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^{\beta}\sum_{j=0}^{\infty}\binom{\beta}{j}\frac{1}{\alpha}\frac{\Gamma\frac{\beta(\alpha+c-1)+5\alpha j-1}{\alpha}}{(\theta\beta)^{\frac{\beta(\alpha+c-1)+5\alpha j-1}{\alpha}}}$$

8.2 Tsallis Entropy

In 1988, Tsallis proposed an entropic expression with an index q that results in non-extensive statistics. The so-called non-extensive statistical mechanics is based on Tsallis entropy.

$$S_\lambda = \frac{1}{\lambda-1}\left[1-\int_0^{\infty} f_w(x)^{\lambda}dx\right]$$

$$= \frac{1}{\lambda-1}\left[1-\int_0^{\infty}\left[\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right]^{\lambda}dx\right]$$

$$= \frac{1}{\lambda-1}\left[1-\left[\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right]^{\lambda}\int_0^{\infty} x^{\lambda(\alpha+c-1)}\,e^{-\lambda\theta x^{\alpha}}\,(1+x^{5\alpha})^{\lambda}dx\right] \quad (17)$$

Using binomial expansion of

$$(1+x^{5\alpha})^{\lambda} = \sum_{j=0}^{\infty}\binom{\lambda}{j}1^{\lambda-j}x^{5\alpha j}$$

$$= \sum_{j=0}^{\infty}\binom{\lambda}{j}x^{5\alpha j}$$

Equation (16) becomes

$$\because S_\lambda = \left[\frac{1}{\lambda-1}\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}x^{5\alpha j}\int_0^{\infty} x^{\lambda(\alpha+c-1)}\,e^{-\lambda\theta x^{\alpha}}\,dx\right]$$

Equation (17) becomes

$$\because s_\lambda = \frac{1}{\lambda-1}\left[1-\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\int_0^{\infty} x^{\lambda(\alpha+c-1)+5\alpha j}\,e^{-\lambda\theta x^{\alpha}}\,dx\right] \quad (18)$$

$$= \frac{1}{\lambda-1}\left[1-\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\frac{1}{\alpha}\int_0^{\infty} t^{\frac{\lambda(\alpha+c-1)+5\alpha j}{\alpha}}\,e^{-\lambda\theta t}\,t\quad t^{-\left(\frac{\alpha-1}{\alpha}\right)}\right]$$

$$= \frac{1}{\lambda-1}\left[1-\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\frac{1}{\alpha}\int_0^{\infty} t^{\left(\frac{\lambda(\alpha+c-1)+5\alpha j-\alpha+1}{\alpha}+1\right)-1}\,e^{-\lambda\theta t}d\,t\right]$$

$$= \frac{1}{\lambda-1}\left[1-\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\frac{1}{\alpha}\int_0^{\infty} t^{\left(\frac{\lambda(\alpha+c-1)+5\alpha j-\alpha+1+\alpha}{\alpha}\right)-1}\,e^{-\lambda\theta t}d\,t\right]$$

$$= \frac{1}{\lambda-1}\left[1-\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\frac{1}{\alpha}\int_0^{\infty} t^{\left(\frac{\lambda(\alpha+c-1)+5\alpha j+1}{\alpha}\right)-1}\,e^{-\lambda\theta t}d\,t\right]$$

$$\because s_\lambda = \frac{1}{\lambda-1}\left[1-\left(\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\frac{1}{\alpha}\int_0^{\infty}\frac{\Gamma\frac{\lambda(\alpha+c-1)+5\alpha j+1}{\alpha}}{(\lambda\theta)^{\frac{\lambda(\alpha+c-1)+5\alpha j+1}{\alpha}}}\,e^{-\lambda\theta t}d\,t\right]$$

IX. Maximum likelihood estimation & fishers information matrix

In this section, we will look at the parameter estimate of the weighted power Prakaamy distribution using the Fisher's Information matrix and the maximum likelihood estimation approach. The probability function can be expressed as follows, presuming that $X_1, X_2, \ldots, X_n$ is a random sample of size n selected from the weighted power prakaamy distribution.

$$L(x) = \prod_{i=1}^{n} f_w(x)$$

$$= \prod_{i=1}^{n}\left[\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right]$$

$$= \left[\frac{\alpha\theta^{\frac{6\alpha+c}{\alpha}}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right]^{n}\prod_{i=1}^{n}\left[x^{\alpha+c-1}(1+x^{5\alpha})\,e^{-\theta x^{\alpha}}\right]$$

$$= \frac{\alpha\theta^{n\left(\frac{6\alpha+c}{\alpha}\right)}}{\left(\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}\right)^{n}}$$

$$\prod_{i=1}^{n}\left[x^{\alpha+c-1}(1+x^{5\alpha})\, e^{-\theta x^{\alpha}}\right]$$

The log likelihood function is given by

$$\log l = n\left(\frac{6\alpha+1}{\alpha}\right)\log - n\log\left[\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}\right] + (\alpha+c - 1)\sum_{i=1}^{\infty}\log(1+x^{5\alpha}) - \theta\sum_{i=1}^{n}x_i^{\alpha} \quad (19)$$

Equation (19) can be differentiated with respect to θ, α and c to provide to greatest probability estimate, which must satisfy the normal equations.

$$\frac{\partial\log l}{\partial\theta} = \frac{n\alpha}{\alpha\theta}\left(\frac{6\alpha+c}{\alpha}\right) - n\frac{5\theta^4\Gamma\frac{6\alpha+c}{\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\sum_{i=1}^{n}x_i^{\alpha} = 0$$

$$\frac{\partial\log l}{\partial\theta} = \frac{n\alpha}{\alpha\theta}\left(\frac{\alpha(6)+(6c+c)1}{\alpha^2}\right) - n\,\Psi\theta^5\left(\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}\right) + \sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}\frac{x_i^{5\alpha}}{(1+x_i^{5\alpha})} - \theta\sum_{i=1}^{n}x_i^{\alpha}\log x_i = 0$$

$$\frac{\partial\log l}{\partial\theta} = \frac{n\alpha}{\alpha\theta}\left(\frac{6\alpha-6\alpha-c}{\alpha^2}\right) - n\,\Psi\left[\theta^5\left(\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}\right)\right] + \sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}\frac{x_i^{5\alpha}}{(1+x_i^{5\alpha})} - \theta\sum_{i=1}^{n}x_i^{\alpha}\log x_i = 0$$

$$\frac{\partial\log l}{\partial\theta} = -\frac{n\theta\alpha}{\alpha^3\theta} - n\,\Psi\left[\theta^5\left(\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}\right)\right] + \sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}\frac{x_i^{5\alpha}\log x_i}{(1+x_i^{5\alpha})} - \theta\sum_{i=1}^{n}x_i^{\alpha}\log x_i = 0$$

$$\frac{\partial\log l}{\partial\theta} = -\frac{n}{\alpha^3\theta}\theta\alpha - n\,\Psi\left[\theta^5\left(\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}\right)\right] + \sum_{i=1}^{n}\log x_i = 0$$

$The\ implementation\ of\ the\ digamma\ function\ (.)$. Due to the intricacy of the preceding likelihood equations, it is quite challenging to algebraically solve the system of nonlinear equations. As a result, we employ a numerical strategy like the Newton-Raphson method to estimate the necessary distribution parameters.

We use the asymptotic normality findings to determine the confidence interval. Thinking that $\hat{\lambda} = (\hat{\theta},\hat{\alpha},\hat{c})$ $denote\ the\ MLE\ of\ \lambda = \theta,\alpha,c\ we\ can\ state\ the\ result\ as,$

$$\sqrt{n}(\hat{\lambda}-\lambda) \rightarrow N_3(0, I^{-1}(\lambda))$$

$The\ Fisher's\ Information\ matrix\ is\ represented\ by\ I(l).$

$$I(\lambda) = -\frac{1}{n}\begin{bmatrix} E\left(\frac{\partial^2\log l}{\partial\theta^2}\right) & E\left(\frac{\partial^2\log l}{\partial\theta\partial\alpha}\right) & E\left(\frac{\partial^2\log l}{\partial\theta\partial c}\right) \\ E\left(\frac{\partial^2\log l}{\partial\alpha\partial\theta}\right) & E\left(\frac{\partial^2\log l}{\partial\alpha^2}\right) & E\left(\frac{\partial^2\log l}{\partial\alpha\partial c}\right) \\ E\left(\frac{\partial^2\log l}{\partial c\partial\theta}\right) & E\left(\frac{\partial^2\log l}{\partial c\partial\alpha}\right) & E\left(\frac{\partial^2\log l}{\partial c^2}\right)\end{bmatrix}$$

$$E\left(\frac{\partial^2\log l}{\partial c^2}\right) = -\frac{n\alpha^2\left(\frac{6\alpha+c}{\alpha}\right)}{(\alpha\theta)^2} - n\left[\frac{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}\left(20\theta^3\Gamma\frac{\alpha+c}{\alpha}\right) - 5\theta^4\Gamma\frac{\alpha+c}{\alpha}\left(5\theta^4\Gamma\frac{\alpha+c}{\alpha}\right)}{\left[\theta^5\left(\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}\right)\right]^2}\right]$$

$$E\left(\frac{\partial^2\log l}{\partial c^2}\right) = \frac{3n\alpha^2\theta^2c}{(\alpha^3\theta)^2} - n\Psi^1\theta^5\left(\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}\right) + \sum_{i=1}^{n}\frac{(1+x_i^{5\alpha})x_i^{5\alpha}(\log x_i)^2 - x_i^{5\alpha}\log x_i\,(x_i^{5\alpha}\log x_i)}{(1+x_i^{5\alpha})^2} - \theta\sum_{i=1}^{\infty}x_i^{5\alpha}(\log x_i)^2$$

$$E\left(\frac{\partial^2\log l}{\partial c^2}\right) = -n\Psi^1\theta^5\left(\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}\right)$$

$$E\left(\frac{\partial^2\log l}{\partial\theta\partial c}\right) = E\left(\frac{\partial^2\log l}{\partial c\partial\theta}\right) = \frac{n}{\alpha}\left(\theta^5\frac{\alpha}{\alpha\theta}\right) - n\Psi\left[\frac{5\theta^4\Gamma\frac{6\alpha+c}{\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha}+\Gamma\frac{6\alpha+c}{\alpha}}\right]$$

$$E\left(\frac{\partial^2 \log l}{\partial\theta\partial c}\right) = \frac{n\alpha}{\alpha^2\theta} - n\Psi\left[\frac{5\theta^4\Gamma\frac{6\alpha+c}{\alpha}}{\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}}\right]$$

$$E\left(\frac{\partial^2 \log l}{\partial\alpha\partial c}\right) = E\left(\frac{\partial^2 \log l}{\partial c\partial\alpha}\right) = \frac{-n\alpha}{\alpha^2\theta} - n\Psi^1\left[\theta^5\Gamma\frac{\alpha+c}{\alpha} + \Gamma\frac{6\alpha+c}{\alpha}\right]$$

X. Data analysis

In this section, the effectiveness of the recently constructed model is evaluated using the Type I, Type 2 and Gestational Diabetes data sets. We also demonstrate that the Pranav, Lindley, and exponential distributions are outperformed by the known distribution. The data set is shown in Table 1.

Table 1: Diabetes data sets

Age: 41, 68, 35, 40, 70, 27, 16, 26, 36, 45, 12, 38, 46, 46, 30, 49, 54, 44, 36, 36, 33, 44, 66, 53, 16, 22, 24, 41, 68, 35, 40, 70, 27, 16, 26, 36, 45, 12, 38, 46, 46, 30, 49, 54, 44, 36, 36, 33, 44, 66, 53, 16, 22, 24, 38.
Insulin : 0, 0, 0, 94, 168, 0, 88, 0, 543, 0, 0, 0, 0, 846, 175, 0, 230, 0, 83, 96, 235, 0, 0, 0, 146, 54, 0, 0, 192, 0, 0, 207, 70, 0, 0, 240, 82, 0, 0, 0, 0, 0, 36, 125, 71, 0, 0, 110, 82, 36, 76, 64, 0, 0, 0, 0, 36, 135, 495.
DPF:0.627,0.351,0.672,0.167,2.288,0.201,0.248,0.134,0.158,0.232,0.191,0.537,1.441,0.398,0.587,0.484,0.551,0.254,0.183,0.529,0.704,0.388,0.451,0.263,0.254,0.205,0.257,0.487,0.245,0.337,0.546,0.841,0.267.
BMI:33.6,23.6,23.3,28.3,43.5,21.3,26,35.4,30,27.9,45.2,43.5,0,29.8,23.3,28.3,43.5,21.3,26,35.4,30,33.6,23.6,23.3,28.3,43.5,21.3,26,35.6,21.3,26,35.4,30,27.9,45.2,43.5,0,29.8,23.3,31.4,24.5,19.7,23.5
SkinThickness:33.3,23.5,28.7,24.9,42.6,29.2,28.4,35.2,29.3,35.3,27.4,33.4,23.4,35.4,39.6,36,36.7,24.9,42.6,29.2,28.4,35.2,29.3,35.3,27.4,33.4,23.4,35.4,39.6,36,36.7,33.3,23.5,28.7,24.9,42.6,29.2,28,35.2

Table: 2

Disease	long-term complications	Affecting age group (%)	Worldwide Affecting (%)	World wide Previous year Death survey
Type 1	Ketoacidosis	10-14	11% (2021)	6.7 million deaths (2021)
	Diabetic food disease			
Type 2	Microvascular	45-65	More than 90% (2021)	
	Macrovascular			
GDM	Hypoglycemia	30-34	16.5% (2021)	
	Hyperglycemia			

Both the estimation of model comparison criterion values and the estimation of unknown parameters are performed using the R software package. The performance of the aforementioned models is evaluated using the AICC (corrected Akaike information criterion), BIC, and AIC metrics with that of the weighted Hamza distribution, including the Pranav, Lindley, Exponential, length-biased Shanker, and length-biased Devya distributions. Among the distributions mentioned above, one is deemed to be superior if it has lower values for AIC, AICC, BIC, and -2log L.

$$AIC = -2\ln l + 2k, BIC = -2\ln l + k\ln n,$$

$$AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

If the sample size is n and there are k parameters.

Table 3: MLEs and criteria for comparison

Distribution	Estimates	-2log L	AIC	BIC	AICC
Weighted power prakaamy	θ=0.6291444 α=0.2372561	169.7310	171.5603	173.0987	167.3241
Weighted power Lindley	θ= 0.23391471	271.9396	273.8396	275.6033	269.6504
Weighted power Pranav	θ= 0.12530893	292.9248	294.9248	296.9240	290.4532

XI. Conclusion

In this research, we suggested and discussed the features of a weighted power prakaamy distribution. Because the form and scale parameters are adjustable, the Weighted Power Prakaamy Distribution can be applied in a variety of situations. The earliest use of maximum likelihood estimation was in the field of frequent estimation. The distribution's efficacy was shown by the analysis of data from a diabetic patient's case. Mostly type II diabetes 95% worldwide affected and then mostly diabetes affected age group type I 10-14, Type II 45, gestational diabetes is 30-34. The weighted power prakaamy distribution, as compared to the Lindley, Pranav, and weighted power prakaamy distributions, indicates greater match, according to the data.

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