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Flexible Fuzzy Softification of Group Structures

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Abstract: In this paper, we introduce the concept of upper t-subgroups of a flexible fuzzy soft intersection group (FFSIG) and investigate various structures of flexible fuzzy soft intersection groups related to upper t- subgroups with suitable example.

Keywords: soft set, fuzzy soft set, soft int-group, upper t-subgroup, flexible fuzzy set, inclusion, relative complement.

I. INTRODUCTION

The algebraic structures of soft set theory have also been studied extensively. Aktas and Cagman [2] introduced the basic concepts of soft groups, soft subgroups, normal soft subgroups and soft homomorphism and discussed their basic properties. Feng et.al [5, 6] considered the algebraic structure of semi ring and introduced the notion of soft semi ring. Some basic algebraic properties of soft semi ring and some related notions such as soft ideals, idealistic soft semi rings and soft semi ring homomorphism were defined and investigated with illustrative examples .Jun [8] applied the notion of soft sets to the theory of BCK/BCI- algebras and introduced the notion of soft BCK/ BCI- algebras, soft sub algebras and then derived their basic properties. Jun and Park [9] introduced the concept of soft Hilbert algebra, soft Hilbert abysmal algebra, soft Hilbert deductive algebra and investigated their properties. Jun [7] also in another paper, introduced the notion of soft p- ideals, p- idealistic soft BCI- algebras and discussed their basic properties. Based on the work of [5,6], introduced the basic notions of soft rings as a parameterized family of subrings of a ring over a ring with some illustrative examples. The notions of soft subrings, soft ideal of a soft ring, idealistic soft rings and soft ring homomorphism were introduced with some corresponding example. They made a theoretical study of the algebraic structures of soft sets such as lattice structures and introduced the concept of soft equality relation and also discussed its related properties. It was proved that soft equality relation is a congruence relation with respect to some operations. In relation to the work of [1], introduced some new operations on soft ring such as extended sum, restricted sum, extended product, restricted product and established some of their basic properties. Ali et.al[3] defined some algebraic structures such as semi groups, semi rings and lattices associated with soft sets and completely described the distributive and absorption laws on operations of soft sets. MV- algebras and BCK-algebras associated with soft set, with a fixed set of parameters were also studied. Atagun and Sezgin [15,16] introduced and studied some sub-structures such as soft subrings and soft ideals of a ring, soft subfield of a field and soft sub module of a module with several illustrative examples. Some related properties on operations of restricted intersection, product and sum for these soft sub- structures were established and investigated with examples. By introducing the concept of normalistic soft group, normalistic soft group homomorphism and establishing that the normalistic soft group isomorphism is an equivalence relation on normalistic soft groups which defined in [2]. Jun et.al [9] discussed the notion of positive implicative ideals of BCK-algebras based on soft set theory and their basic properties. On flexible fuzzy subgroups with flexible fuzzy order discussed by [17]. In this paper, we introduce the concept of upper t-subgroups of a flexible fuzzy soft intersection group (FFSIG) and analyse various structures of flexible fuzzy soft intersection groups related some softification of upper t-subgroups.

II. PRELIMINARIES

Definition 2.1[Molodtsov]: A pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U.Note that a soft set (F, A) can be denoted by F_A .

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In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A , F_B , F_C respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A , G_A , H_A respectively.

Definition 2.2[Molodtsov]: The relative complement of the soft set F_A over U is denoted by F_A^r , where $F_A^r: A \to P(U)$ is a mapping given as $F_A^r(a) = U \setminus F_A(a)$, for all $a \in A$.

Definition2.3[Molodtsov]: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$. The restricted intersection of F_A and G_B is denoted by $F_A \sqcup G_B$, and is defined as $F_A \sqcup G_B = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.

Definition 2.4[Molodtsov] : Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cup G(c)$.

Definition 2.5[Zadeh]: A function 'f' is defined from a universe X to a closed interval [0, 1] is called a fuzzy set (i,e) a mapping $f: X \rightarrow [0,1]$.

Definition2.6: Let F_A be a soft set over U and Abe a subset of U. Then upper α -inclusion of F_A , denoted by $F_A^{\supseteq \alpha}$, is defined as $F_A^{\supseteq \alpha} = \{x \in A/F(x) \supseteq \alpha\}$. Similarly,

 $F_A^{\subseteq \alpha} = \{x \in A \mid F(x) \subseteq \alpha\}$ is called the lower α -inclusion of F_A .

Definition2.7 [Naim Cagman]:Let U be an initial universe, E be the set of all parameters and $A \subseteq E$. A pair (F, A) is called a flexible fuzzy soft set over U where F: $A \rightarrow \tilde{P}(U)$ is a mapping from A into $\tilde{P}(U)$, where $\tilde{P}(U)$ denotes the collection of all subsets of U.

Definition 2.8: Let G be a group and δ be a flexible fuzzy soft over X. If for all $x, y \in G$

- (i) $\max{\delta(xy)} \le \min{\delta(x) \cap \delta(y)}$ and
- (ii) $\max{\{\delta(x^{-1})\}} \le \min{\{\delta(x)\}}$, then the flexible fuzzy soft set δ is called a flexible fuzzy soft int-subgroup of X and denoted by $\delta \subsetneq G$.

Example 2.9: Let $G = Z_4$ be the set of parameters and $U = \{1, -1, i, -i\}$ be the universal set of a flexible fuzzy soft set over U is constructed by

 $\delta_{\rm G}(0) = \{ 1, -1, i, -i \}$

 $\delta_G(1) = \delta_G(3) = \{i\}$ $\delta_G(2) = \{1, i\}$. Clearly δ_G is flexible fuzzy soft intersection group over U. Here

Im $(\delta_G) = \{ \{i\}, \{1,i\}, \{1,-1,i,-i\}\}$, thus all the upper t-sub groups of δ_G are $\delta_G^{\supseteq\{i\}} = \mathbb{Z}_4$, $\delta_G^{\supseteq\{1,i\}} = \{ 0,2\}, \delta_G^{\supseteq\{1,-1,i,-i\}} = \{0\}$.

Now, let define a flexible fuzzy soft set over U such that

 $\Delta_G \{ 0 \} = \{ 1, -1, i \}, \ \Delta_G \{ 1 \} = h_G(3) = \{ -1 \}, \ \Delta_G \{ 2 \} = \{ -1, i \}.$

Obviously, Δ_G is a FFSI – group over U. Too and the family of upper t – subgroups of Δ_G are $\Delta_G \stackrel{\geq \{-1\}}{=} \mathbb{Z}_4$, $\Delta_G \stackrel{\geq \{-1,i\}}{=} \{0,2\}$,

 $\Delta_G \cong \{1,-1,i\} = \{0\}$. It is seen that two FFSI – groups δ_G and h_G have the same family of upper t-subgroups, however δ_G is not soft equal to Δ_G .

III. PROPERTIES OF FLEXIBLE FUZZY SOFT INTERSECTION GROUPS

Proposition 3.1: Let S_G be the class of FFSI – groups of a group G over U. If define a relation R on S_G by $\delta_G R \Delta_G$, then its upper t-subgroups on the relation R is an equivalence relation.

Note 3.2: Example 2.9, it is shown that Δ_G and δ_G may be such that $\delta_G R \Delta_G$ but δ_G and Δ_G need not be soft equal. The equivalence relation defined in this proposition 3.1 S_G into equivalence classes.

Let $\delta_G \in S_G$ and $|\delta_G|$ denote the equivalence class contained δ_G . If the group G is finite, then the number of possible distinct upper t-subgroups are finite, as each upper t-subgroups is a subgroup of G in the usual sense. All these remarks lead us

to the conclusion that the number of possible chains of upper t-subgroups, we have the following that the number of equivalence is finite, although S_G is an infinite family when U is infinite.

Corollary 3.3: If G is a finite group, then the number of distinct equivalence classes in S_G under the definition of equivalence relation defined in proposition 3.1 is finite.

Moreover, S_G can be written as a disjoint Union

 $S_{G} = |\delta_{G}^{-1}|U| |\delta_{G}^{-2}|U| |\delta_{G}^{-3}|U....U| |\delta_{G}^{-k}| \text{ where } |\delta_{G}^{-i}| \text{ denotes } 1 \leq i \leq k \text{ on all distinct equivalence classes. Here, again note that } |\delta_{G}^{-i}| \text{ has an infinite number of FFSI } - \text{ groups when } U \text{ is finite.}$

Proposition 3.4: Let Δ_G and δ_G be two FFSI- groups of a finite group G having the identical family of upper t-subgroups and

the sets $\text{Im}(\delta_G)$ and $\text{Im}(\Delta_G$)be ordered by combination.

If Im(δ_G) = { t₀,t₁,t₂,....,t_m} and Im (Δ_G) = { s₀,s₁,....,s_n}, then

- (i) m = n
- (ii) $\delta_G^{\supseteq ti} = \delta_G^{\supseteq si}, 0 \le i \le m,$
- (iii) If $x \in G$ such that $\delta_G(x) = t_i$, then $\Delta_G(x) = s_i$, $0 \le i \le m$.

Proof: (i) Since Δ_G and δ_G have the identical family of upper t – subgroups , it follows that m = n.

(ii) Since $t_0 \ge t_1 \ge t_2 \ge \dots \ge t_m$ and $s_o \ge s_1 \ge s_2 \ge \dots \ge s_n$, by proposition 3.1 that two chains of upper t – subgroups are

 $\delta_{G}^{\supseteq t_{o}} \leq \delta_{G}^{\supseteq t} \leq \delta_{G}^{\supseteq t} \leq \delta_{G}^{\supseteq t} \leq \dots \leq \delta_{G}^{\supseteq t} = G,$

$$\Delta_G \stackrel{\supseteq l_o}{=} \leq \Delta_G \stackrel{\supseteq t}{=} 1 \leq \Delta_G \stackrel{\supseteq t}{=} 2 \leq \dots \leq \Delta_G \stackrel{\supseteq l}{=} n = G$$

Since the two upper t- subgroups are identical, it is obvious that

$$\delta_{\mathbf{G}}^{\supseteq t_o} = \Delta_{\mathbf{G}}^{\supseteq t_o} = \{\mathbf{e}\}.$$

Let $\delta_G^{\supseteq t_1} = \Delta_G^{\supseteq t_1}$, for some j > 0.

Suppose that $\delta_G^{\supseteq t} = \Delta_G^{\supseteq s} j$ for some j > 1.

Again $\Delta_G \stackrel{\supseteq s}{=} i = \Delta_G \stackrel{\supseteq t}{=} i$ for some $t_1 \ge t_i$.

It is obvious that $\mathbf{t_i} \neq \mathbf{t_1}$. Thus $\delta_G \stackrel{\supseteq t}{=} \Delta_G \stackrel{\supseteq s}{=} i \subset \Delta_G \stackrel{\supseteq s}{=} j$, so $\delta_G \stackrel{\supseteq t}{=} i = \Delta_G \stackrel{\supseteq s}{=} j$. Now $\Delta_G \stackrel{\supseteq s}{=} j = \delta_G \stackrel{\supseteq t}{=} i \subset \delta_G \stackrel{\supseteq t}{=} i_{,\text{ so }} \Delta_G \stackrel{\supseteq s}{=} j \subset \delta_G \stackrel{\supseteq t}{=} i$ Note that, $\delta_G \stackrel{\supseteq t}{=} i \subset \delta_G \stackrel{\supseteq t}{=} i$

Contradicts one another, because the combination are both proper combination, so $\delta_G \stackrel{\supseteq i}{=} i \subset \Delta_G \stackrel{\supseteq s}{=} i$. The rest of the proof follows by induction method on i. Finally, it is obtained that $\delta_G \stackrel{\supseteq t}{=} i = \Delta_G \stackrel{\supseteq s}{=} j$, $0 \le i \le m$.

Let $x \in G$ such that $\delta_G(x) = t_i$ and $\Delta_G(x) = s_j$, by previous theorem (ii), $\delta_G^{\supseteq i} i = \Delta_G^{\supseteq s_j}$, $0 \le i \le m$. Thus $x \in \Delta_G^{\supseteq s} i$. Implies that

 $\Delta_{G} (\mathbf{x})_{= s_{j}} \text{ such that } s_{j} \ge s_{i}. \text{ So } \Delta_{G} \stackrel{\supseteq s}{}^{j} \le \Delta_{G} \stackrel{\supseteq s}{}^{i} i.$ Similarly, by previous theorem (ii) $\Delta_{G} \stackrel{\supseteq s}{}^{j} = \Delta_{G} \stackrel{\supseteq s}{}^{i} i.$ Therefore, since $\mathbf{x} \in \Delta_{G} \stackrel{\supseteq s}{}^{i} i, \mathbf{x} \in \delta_{G} \stackrel{\supseteq s}{}^{j} and so,$ $\delta_{G} (\mathbf{x}) = t_{i} \ge t.$ It follows that $\delta_{G} \stackrel{\supseteq t}{}^{i} i \le \delta_{G} \stackrel{\supseteq t}{}^{j} i$ However, by theorem (ii), $\delta_{G} \stackrel{\supseteq t}{}^{i} i = \Delta_{G} \stackrel{\supseteq s}{}^{i} i and \delta_{G} \stackrel{\supseteq t}{}^{j} = \Delta_{G} \stackrel{\supseteq s}{}^{j}.$ So, $\Delta_{G} \stackrel{\supseteq s}{}^{i} i = \delta_{G} \stackrel{\supseteq t}{}^{j} i = \Delta_{G} \stackrel{\supseteq s}{}^{j} j,$ Thus, $\Delta_{G} \stackrel{\supseteq s}{}^{i} i < \Delta_{G} \stackrel{\supseteq s}{}^{j} j,$ which contradicts the fact that $\Delta_{G} \stackrel{\supseteq s}{}^{j} j < \Delta_{G} \stackrel{\supseteq s}{}^{i} i,$ if we do not have $\Delta_{G} \stackrel{\supseteq s}{}^{j} i = \Delta_{G} \stackrel{\supseteq s}{}^{i} i$

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We know that $\Delta_G \stackrel{\supseteq s}{=} j = \Delta_G \stackrel{\supseteq s}{=} i$ if and only if $s_j = s_i$. Thus $\delta_G(x) = t_i$, and $\Delta_G(x) = s_j = si$. Hence the proof.

Proposition 3.5:Let ${}^{\delta_{G}}$ and ${}^{\Delta_{G}}$ be two FFSI – groups of a finite group G such that their family of upper t – subgroups are identical and their image sets are both ordered by combination. Thus ${}^{\delta_{G}}=\Delta_{G}$, then $\operatorname{Im}(\delta_{G})=\operatorname{Im}(\Delta_{G})$.

Proof: Let $\delta_G = {}^{\Delta_G} \cdot \operatorname{Then} \operatorname{Im}(\delta_G) = \operatorname{Im}(\Delta_G)$ is obvious. Conversely, suppose that $\operatorname{Im}(\delta_G) = \operatorname{Im}(\Delta_G)$. Let $\operatorname{Im}(\delta_G) = \{ t_0, t_1, t_2, \dots, t_n \}$ and $\operatorname{Im}(\Delta_G) = \{ s_0, s_1, s_2, \dots, s_n \}$ such that $t_2 \ge t_2 \ge t_2$ and $s_1 \ge s_2 \ge s_2$.

$$\begin{split} & \text{Im} \ (\Delta_G) = \{ \ s_o, s_1, s_2, \dots, s_n \ \} \ \text{such that} \ t_0 \geq t_1 \geq \dots, \geq t_n \text{and} \ s_o \geq s_1 \geq s_2, \dots, \geq s_n. \\ & \text{Let} \ s_o \in \text{Im}(\delta_G) \ . \text{Thus} \ s_o = t_{\alpha o} \text{for some} \ \alpha_o. \\ & \text{Let} \ t_{\alpha o} \neq t_o. \ \text{It follows that} \ t_{\alpha o} \not\supseteq \ t_o. \\ & \text{Since} \ t_o \ \text{is the maximal element of the chain.} \\ & \text{Now, let} \ s_1 \in \text{Im}(\delta_G) \ \text{and} \ s_o s_1 = t_{\alpha 1} \ \text{for some} \ t_1. \\ & \text{Since} \ \ S_o \not\supseteq \ \ S_1, \ \text{it implies that} \ t_{\alpha_o} \supset t_{\alpha 1} \ . \\ & \text{Similarly,} \end{split}$$

 $t\alpha_{o} \supset t\alpha_{1} \supset t\alpha_{2} \supset \dots \supset t\alpha_{r}$, where $s_{0} = t\alpha_{o} \not\supseteq_{t_{o}}$.

This means that there does not exist only $s_i \in Im(\Delta_G)$ such that $t_o = s_i$.

But the contradicts the fact that $Im(\delta_G) = Im(\Delta_G)$

Hence we must have $s_o = t_o$. Similarly, one can obtain that $s_i = t_i$, $0 \le i \le r$. By proposition 3.4 (iii), $\Delta_G(l_i)$, for all $i \in G$. Hence $\delta_G = \Delta_G$. Hence the proof.

Note3.6: Since all the subgroups of G, in general, do not form a chain. We can conclude that not all subgroups of G are upper t-subgroups of a given FFSI-group whose image set form a chain. Therefore, it turns out to be an interesting problem to find FFSI-group whose image sets form a chain and which accommodates as many subgroups of G as possible in the chain of upper t- subgroups of the FFSI group.

Theorem 3.7 : Any subgroup H of a group G can be realized as an upper t- subgroups of some FFSI – group over U.

Proof: Let δ_{G} be a FFSI – set over U defined by

$$\begin{split} \delta_{G}(x) &= \begin{cases} t, & \text{if } x \in H \\ = & \Phi, & \text{if } x \notin H. \end{cases} & \text{Then } \delta_{G} & \text{is a FFSI} - \text{group over } U. \\ & \text{Let } a, & b \in G. \end{split}$$

Case (i) : Suppose $a \in H$ and $b \in H$, then $ab \in H$. It follows that max $\{\delta_G(ab)\} = t$ and $\delta_G(a) = \delta_G(b) = t$. Thus,max $\{\delta_G(ab)\} \leq \min \{\delta_G(a), \delta_G(b)\}$. And also if $a \in H$, then so is a^{-1} , thus $\delta_G(a) = \delta_G(a^{-1}) = t$. Case (ii) : Now, suppose $a \in H$ and $b \notin H$ then $ab \notin H$. It follows that $\delta_G(a) = t$ and $\delta_G(b) = \delta_G(ab) = \Phi$. Therefore,max $\{\delta_G(ab)\} \geq \min \{\delta_G(a), \delta_G(b)\}$. Furthermore $\delta_G(a) = \delta_G(a^{-1})$ if $a \in H$ or $a \notin H$. Case (iii) : Now, suppose that $a \notin H$ and $b \notin H$. Then either $a, b \notin H$. It is easy to show that in any cases, max $\{\delta_G(ab)\} \leq \min \{\delta_G(a), \delta_G(b)\}$.

And $\delta_G(a) = \delta_G(a^{-1})$.

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Hence δ_G is a FFSI – group over U. Therefore, for this FFSI group, $\delta_G^{\geq t} = H$.

Note 3.8: It is known that if δ_G is a FFSI group over U, then $\delta_G(e) \ge \delta_G(x)$ for all $x \in G$.

Let $\delta_G(e) \ge t_e$. Then it turns out to be an interesting case to investigate the upper t_e subgroup of $\delta_G^{\supseteq t} e$ of G. Because it $x \in \delta_G^{\supseteq t} e$, then $\delta_G(x) \ge t = \delta_G(e)$ and it appears that only $e \in \delta_G^{\supseteq t} e$. But that is not always the case as seen in the following example.

Example 3.9: Consider the FFSI – group in theorem 3.7. Assume that $H \neq \{e\}$ and $H \neq G$. It is known that δ_G is a FFSI – group over U and Im (δ_G) = { Φ , t }.

Thus , two upper t – subgroups are $\delta_{G} \stackrel{\supseteq \phi}{=} G$ and $\delta_{G} \stackrel{\supseteq t}{=} \{ \ x \in G \ / \ \delta_{G}(x) \ge t \ \} = H.$

Since $e \in H$, $\delta_G(e) = t$; but $\delta_G^{\supseteq t} = H$, which is not equal to "e".

Definition 3.10: Let δ_G be a FFSI – group over U. Then e-set of δ_G , denoted by $G_{\delta G}$, is defined as $G_{\delta G} = \{ x \in G / \delta_G(x) = \delta_G(e) \}$

Theorem 3.11: Let δ_G be a FFSI – group over U. If $\delta_G^{(e)} = t_e$, then $\delta_G^{\supseteq t}_e = G_{\delta G}$.

 $\begin{array}{ll} Proof: \, \delta_G \stackrel{\supseteq t}{}_e & = \{ \ x \in G \ / \ \delta_G^{(x)} \ge t_e \ \} \\ & = \{ \ x \in G \ / \ \delta_G^{(x)} = t_e \ \} \end{array}$

 $\begin{array}{ll} \text{Since } t_e \!\! \geq & \!\! \delta_G{}^{(x)} & \text{for all } x \in G \; . \\ & \!\! \delta_G \stackrel{\supseteq t}{}_e \;\! = \; \{ \; x \!\! \in \!\! G \; / \; \delta_G{}^{(x)} \!\! = \!\! \delta_G{}^{(e)} \; \} = G_{\delta G}. \end{array}$

Note 3.12: Let δ_G be a FFSI – group over U and $(t_o, t_1, t_2, t_3, \dots, t_n) \in Im(\delta_G)$ which satisfying that $t_o \ge t_1 \ge t_2 \ge t_3 \ge \dots \ge t_n$. Then the family of upper – t – subgroups from a chain , which denoted by $C(\delta_G) = \delta_G \supseteq^t 0 < \delta_G \supseteq^t 1 < \dots < \delta_G \supseteq^t n$.

Not to our surprise, only some of the upper t-subgroups of δ_G form a chain. Since all the subgroups of G, but in general, does not form a chain that is, it makes no sense to hope all the upper t – subgroups form a chain.

In the connection, see example 2.9, $\{0, 3\} \not\subseteq \{0, 2, 4\}$ and $\{0, 2, 4\} \not\subseteq (0, 3\}$.

Of course if the number of the Im (δ_G) forms a chain, so does the upper t – subgroups of δ_G . For further detail, refer to the following theorem.

Theorem 3.13: Let G be a finite group.Let δ_G be a FFSI – group over U, I be an arbitrary finite index set and G ($\delta_G^{\supseteq t}$) = { $\delta_G^{\supseteq t}$ } = { $\delta_G^{\supseteq t}$ }

(i) There exists a unique $i_e \in I$ such that $t_{ie} \ge t_i$, for all i.

(ii)
$$G_{\delta G} = \bigcap_{i \in I} \delta_G^{\supseteq ti} = \delta_G^{\supseteq t_{ie}}$$

(iii)
$$G = \bigcup_{i \in I} \delta_G^{\supseteq i}$$

- (iv) If the members of Im (δ_G) forms a chain, so do is G $(\delta_G \supseteq ti)$. **Proof:**
- (i) Since δ_G^(e)∈Im (δG), there exists a unique i_e∈ I such that δ_G^{(e)=tie}. We know that δ_G^(e)≥δ_G^(x) for all x ∈ G. It follows that t_{ie} ≥δ_G^(x), for all x∈G. Thus t_{ie} ≥ t_i, for all i∈ I.
 (ii) Since in theorem 3.12, it is proved that

Since in theorem 3.12, it is proved that

$$G_{\delta G} = -\delta_G^{\supseteq t_{ie}}, \text{ where } \delta_{G^{(e)}} = t_{ie}.$$
It is only shows that $G \delta_G^{\supseteq t_{ie}} = \bigcap_{i \in I} \delta_G^{\supseteq t_i}$. Since $t_{ie} \ge t_i$ for all $i \in I$,

 $\begin{array}{l} \mathbf{G}_{\delta \mathbf{G}} \stackrel{\supseteq t_{k}}{=} \leq \mathbf{G}_{\delta \mathbf{G}} \stackrel{\supseteq t_{k}}{=} , \text{ for all i.} \\ \mathbf{Thus} \ \mathbf{G}_{\delta \mathbf{G}} \stackrel{\supseteq t_{k}}{=} \subseteq \bigcap_{i \in I} \delta_{\mathbf{G}} \stackrel{\supseteq t_{i}}{=} , \text{ for all i} \quad \epsilon \bigcap_{i \in I} \delta_{\mathbf{G}} \stackrel{\supseteq t_{i}}{=} , \text{ then } \mathbf{x} \in \delta_{\mathbf{G}} \stackrel{\supseteq t_{k}}{=} , \text{ for all } \mathbf{i} \in \mathbf{I}. \end{array}$

IV. CONCLUSION

we conclude that not all subgroups of G are upper t-subgroups of a given FFSI-group whose image set form a chain. Therefore, it turns out to be an interesting problem to find FFSI-group whose image sets form a chain and which accommodates as many subgroups of G as possible in the chain of upper t- subgroups of the FFSI group. In this connection ,any subgroup H of a subgroup G can be realized as an upper –subgroups of some FFSI-group over U.

V. FUTURE WORK

To extend our work, further research can be done to study the properties of multi- fuzzy soft int-group in other algebraic structures such as modules, rings and fields.

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