

On $(gg)^*$ - Closed Sets in Topological Spaces

I. Christal Bai^{1*}, T.Shyla Isac Mary²

^{1,2}Department of Mathematics, Nesamony Memorial Christian College, Marthandam, Tamil Nadu, India.,
(Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, TamilNadu, India)

*Corresponding author: christalstalin@gmail.com

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Abstract- In this paper, we introduce a new class of closed sets called generalization of generalized star closed sets (briefly $(gg)^*$ - closed) in topological Spaces. A subset A of a topological space (X, τ) is called $(gg)^*$ - closed if U contains regular closure of A whenever U contains A and U is gg -open in (X, τ) . We studied the relation of this set with some of the other closed and generalized closed sets and some of the characteristics, of $(gg)^*$ - closed sets have been investigated and studied.

Keywords: $(gg)^*$ - closed set, gg - open, regular closure.

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I. INTRODUCTION

The concept of generalized closed sets [1] in Topological spaces was introduced by N. Levine in 1970. D. E. Cameron and M. Stone introduced regular semi open sets [2] and regular open sets [3] respectively. In 2017, Basavaraj M. Ittanagi and Govardhana Reddy introduced and studied generalization of generalized closed sets [4] in Topological spaces.

In this paper we introduce a new class of closed set called $(gg)^*$ - closed sets in Topological spaces. Section 1, gives the overall introduction to the paper, followed by section 2, where we recall some of the existing closed and open sets. Section 3, provides us with the introduction to the concept of $(gg)^*$ - closed set. In section 4, the independency of $(gg)^*$ - closed sets with some of the existing closed and generalized closed sets are studied and its outcome is shown in the form of a diagram. In section 5, some of the properties of $(gg)^*$ - closed sets are studied, analyzed and proved; which leads to section 6, the conclusion of the paper. After which, the references that were dealt with during the analyses are given at the end of the paper.

II. PRELIMINARIES

Throughout this paper (X, τ) represent the topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X , the closure of A and interior of A are denoted by $cl(A)$ and $int(A)$ respectively.

Definition 2.1 A subset A of a topological space (X, τ) is called a

- (1) generalized - closed set (briefly g - closed) [1] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (2) regular semi open [2] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.
- (3) regular open set [3] if $A = int(cl(A))$ and a regular closed set if $cl(int(A)) = A$.
- (4) generalization of generalized closed set (briefly gg -closed) [4] if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi - open in X .

- (5) semi - open set [5] if $A \subseteq cl(int(A))$ and a semi - closed set if $int(cl(A)) \subseteq A$.
- (6) pre - open set [6] if $A \subseteq int(cl(A))$ and pre - closed if $cl(int(A)) \subseteq A$.
- (7) semi - pre open set [7] if $A \subseteq cl(int(cl(A)))$ and semi pre - closed if $int(cl(int(A))) \subseteq A$.
- (8) β -open set [7] if $A \subseteq cl(int(cl(A)))$, whenever $A \subseteq U$ and U is open in X .
- (9) α - open set [8] if $A \subseteq int(cl(int(A)))$ and α - closed set if $cl(int(cl(A))) \subseteq A$.
- (10) t - set [9] iff $int(A) = int(cl(A))$.
- (11) generalized semi - pre closed (briefly gsp - closed) [10] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (12) generalized pre - closed set (briefly gp - closed) [11] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (13) generalized semi - closed set (briefly gs - closed) [12] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (14) α -generalized closed set (briefly ag - closed) [13] if $acl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (15) regular generalized closed set (briefly rg - closed) [14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular - open in X .
- (16) generalized pre - regular closed set (briefly gpr - closed) [15] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- (17) generalized semi - pre regular - closed set (briefly $gspr$ - closed) [16] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- (18) generalized star pre closed (briefly g^*p - closed) [17] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X .
- (19) weakly closed set (briefly w - closed) [18] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi- open in X .
- (20) tgr - closed set [19] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a t -set.
- (21) regular w -closed (briefly rw - closed) [20] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi - open in X .
- (22) regular generalized α - closed set (briefly $rg\alpha$ - closed) [21] if $acl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular α - open in X .
- (23) generalized α - closed set (briefly $g\alpha$ - closed) [22] if $acl(A) \subseteq U$ whenever $A \subseteq U$ and U is α - open in X .
- (24) Semi - generalized closed set (briefly sg - closed) [23] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X .

- (25) R^* - closed set [24] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi - open in X .
- (26) $R^\#$ - closed set [25] if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is R^* -open in X .
- (27) βg^* - closed set [26] if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is β - open in X .
- (28) $r \wedge g$ - closed set [27] if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular - open in X .
- (29) g^{**} - closed set [28] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* - open in X .
- (30) g^* - closed set [29] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g - open in X .
- (31) generalized regular closed set (briefly gr - closed) [30] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (32) generalized regular star closed (briefly gr^* - closed) [31] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g - open in X .

The complements of the above closed sets are their open sets and vice versa.

Definition 2.2 [19] The regular closure of a subset $A \subseteq X$ is the set $rcl(A) = \bigcap \{ B \subseteq X : B \text{ is regular closed and } A \subseteq B \}$

III. GENERALIZATION OF GENERALIZED STAR - CLOSED SETS

Definition 3.1 A subset A of a topological space (X, τ) is called generalization of generalized star closed sets (briefly $(gg)^*$ -closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is gg - open.

Example 3.2 Let $X = \{a, b, c, d\}$, and $\tau = \{ \varnothing, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X \}$
 gg - open = $\{ \varnothing, \{a, c, d\}, \{c, d\}, \{a, d\}, \{a, c\}, \{a, b\}, \{d\}, \{b\}, \{c\}, \{a\}, X \}$
 $(gg)^*$ - closed = $\{ \varnothing, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X \}$.

Proposition 3.3 Every regular closed set is $(gg)^*$ - closed.

Proof: Let A be a regular closed set in X such that $A \subseteq U$ and U is gg - open. Then $rcl(A) = A$. Hence $rcl(A) \subseteq U$. Therefore A is $(gg)^*$ - closed.

Remark 3.4 The converse of the above proposition need not be true as shown in the following example.

Example 3.5 Let $X = \{a, b, c, d\}$, and $\tau = \{ \varnothing, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X \}$
 Then the set $\{a, b\}$ is $(gg)^*$ - closed but not regular closed.

Proposition 3.6

- (1) Every $(gg)^*$ - closed set is g - closed.
- (2) Every $(gg)^*$ - closed set is g^{**} -closed.
- (3) Every $(gg)^*$ - closed set is gsp - closed.
- (4) Every $(gg)^*$ - closed set is gp - closed.
- (5) Every $(gg)^*$ - closed set is gs - closed.

- (6) Every $(gg)^*$ - closed set is ag - closed.
- (7) Every $(gg)^*$ - closed set is rg - closed.
- (8) Every $(gg)^*$ - closed set is gpr - closed.
- (9) Every $(gg)^*$ - closed set is $gspr$ -closed.
- (10) Every $(gg)^*$ - closed set is g^*p - closed.
- (11) Every $(gg)^*$ - closed set is gr -closed.
- (12) Every $(gg)^*$ - closed set is gr^* -closed.

Proof:

- (1) Let A be a $(gg)^*$ - closed set in X . Let U be an open set in X such that $A \subseteq U$. Since every open set is gg -open [4] and since A is $(gg)^*$ - closed, $rcl(A) \subseteq U$. But we have $cl(A) \subseteq rcl(A) \subseteq U$. Hence A is g -closed.
- (2) Let A be a $(gg)^*$ - closed set in X . Let U be a g^* -open set in X such that $A \subseteq U$. Since every g^* -open set is gg -open [4] and since A is $(gg)^*$ -closed, $rcl(A) \subseteq U$. But we have $cl(A) \subseteq rcl(A) \subseteq U$. Hence A is g^{**} -closed.
- (3) Let A be a $(gg)^*$ - closed set in X . Let U be an open set in X such that $A \subseteq U$. Since every open set is gg - open [4] and since A is $(gg)^*$ -closed, $rcl(A) \subseteq U$. But we have $spcl(A) \subseteq rcl(A) \subseteq U$. Hence A is gsp -closed
- (4) Let A be a $(gg)^*$ - closed set in X . Let U be an open set in X such that $A \subseteq U$. Since every open set is gg - open [4] and since A is $(gg)^*$ -closed, $rcl(A) \subseteq U$. But we have $pcl(A) \subseteq rcl(A) \subseteq U$. Hence A is gp -closed.
- (5) Let A be a $(gg)^*$ - closed set in X . Let U be an open set in X such that $A \subseteq U$. Since every open set is gg -open [4] and since A is $(gg)^*$ -closed, $rcl(A) \subseteq U$. But we have $scl(A) \subseteq rcl(A) \subseteq U$. Hence A is gs -closed.
- (6) Let A be a $(gg)^*$ - closed set in X . Let U be an open set in X such that $A \subseteq U$. Since every open set is gg -open [4] and since A is $(gg)^*$ -closed, $rcl(A) \subseteq U$. But we have $acl(A) \subseteq rcl(A) \subseteq U$. Hence $acl(A) \subseteq U$. Hence A is ag -closed.
- (7) Let A be a $(gg)^*$ -closed set in X . Let U be a regular open set in X such that $A \subseteq U$. Since every regular open set is gg - open [4] and A is $(gg)^*$ - closed, $rcl(A) \subseteq U$. But we have $cl(A) \subseteq rcl(A) \subseteq U$. Hence A is rg -closed.
- (8) Let A be a $(gg)^*$ - closed set in X . Let U be a regular open set in X such that $A \subseteq U$. Since every regular open set is gg - open [4] and A is $(gg)^*$ -closed, $rcl(A) \subseteq U$. But we have $pcl(A) \subseteq rcl(A) \subseteq U$. Hence A is gpr -closed.
- (9) Let A be a $(gg)^*$ - closed set in X . Let U be a regular open set in X such that $A \subseteq U$. Since every regular open set is gg - open [4] and A is $(gg)^*$ -closed, $rcl(A) \subseteq U$. But we have $spcl(A) \subseteq rcl(A) \subseteq U$. Hence A is $gspr$ -closed.
- (10) Let A be a $(gg)^*$ - closed set in X . Let U be a g - open set in X such that $A \subseteq U$. Since every g - open set is gg - open [4] and A is $(gg)^*$ -closed, $rcl(A) \subseteq U$. But we have

$pcl(A) \subseteq rcl(A) \subseteq U$. Hence A is g^*p - closed.

- (11) Let A be a $(gg)^*$ - closed set in X. Let U be an open set in X such that $A \subseteq U$. Since every open set is gg - open [4] and since A is $(gg)^*$ -closed, $rcl(A) \subseteq U$. Hence A is gr - closed.
- (12) Let A be a $(gg)^*$ - closed set in X. Let U be a g - open set in X such that $A \subseteq U$. Since every g - open set is gg - open [4] and since A is $(gg)^*$ -closed, $rcl(A) \subseteq U$. Hence A is gr^* - closed.

Remark 3.7 The converse of the above proposition need not be true as shown in the following example.

Example 3.8 Let $X = \{a,b,c,d\}$, and $\tau = \{ \varnothing, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X \}$. Then

- (1) $\{b\}$ is g - closed but not $(gg)^*$ -closed.
- (2) $\{b\}$ is g^{**} -closed but not $(gg)^*$ -closed.
- (3) $\{a\}$ is gsp -closed but not $(gg)^*$ -closed.
- (4) $\{a\}$ is gp -closed but not $(gg)^*$ -closed.
- (5) $\{d\}$ is gs -closed but not $(gg)^*$ -closed.
- (6) $\{b\}$ is ag -closed but not $(gg)^*$ -closed.
- (7) $\{a, c\}$ is rg - closed but not $(gg)^*$ -closed.
- (8) $\{c, d\}$ is gpr -closed but not $(gg)^*$ -closed.
- (9) $\{a,c,d\}$ is $gspr$ -closed but not $(gg)^*$ -closed.
- (10) $\{a\}$ is g^*p - closed but not $(gg)^*$ -closed.
- (11) $\{b\}$ is gr -closed but not $(gg)^*$ -closed.
- (12) $\{b\}$ is gr^* -closed but not $(gg)^*$ -closed.

IV. INDEPENDENCY OF $(gg)^*$ -CLOSED SETS WITH OTHER CLOSED SETS.

The following example shows that $(gg)^*$ -closed sets are independent from α -closed, regular semi -closed, $g\alpha$ -closed, $rg\alpha$ - closed, w -closed, rw - closed, sg - closed, pre - closed, g^*s - closed, R^* - closed, tgr - closed.

Example 4.1

Let $X = \{a, b, c, d\}$, and $\tau = \{ \varnothing, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X \}$.Then

- (1) $\{b,c\}$ is $(gg)^*$ - closed but not α -closed and $\{b\}$ is α -closed but not $(gg)^*$ - closed.
- (2) $\{b,c,d\}$ is $(gg)^*$ - closed but not $g\alpha$ -closed and $\{a\}$ is $g\alpha$ -closed but not $(gg)^*$ - closed.
- (3) $\{a,b\}$ is $(gg)^*$ - closed but not regular semi - closed and $\{c\}$ is regular semi -closed but not $(g g)^*$ - closed.

- (4) $\{b,c\}$ is $(gg)^*$ - closed but not $rg\alpha$ -closed and $\{a, c, d\}$ is $rg\alpha$ -closed but not $(gg)^*$ - closed.
- (5) $\{b,d\}$ is $(gg)^*$ - closed but not w -closed and $\{b\}$ is w -closed but not $(gg)^*$ - closed.
- (6) $\{b,c\}$ is $(gg)^*$ - closed but not rw - closed and $\{c, d\}$ is rw - closed but not $(gg)^*$ - closed.
- (7) $\{b,c,d\}$ is $(gg)^*$ - closed but not sg - closed and $\{a,d\}$ is sg - closed but not $(gg)^*$ - closed.
- (8) $\{b,c,d\}$ is $(gg)^*$ - closed but not g^*s - closed and $\{d\}$ is g^*s - closed but not $(gg)^*$ - closed.
- (9) $\{a,b\}$ is $(gg)^*$ - closed but not pre- closed and $\{c\}$ is pre- closed but not $(gg)^*$ - closed.
- (10) $\{b,d\}$ is $(gg)^*$ - closed but not R^* - closed and $\{a,c,d\}$ is R^* - closed but not $(gg)^*$ - closed.
- (11) $\{b,c\}$ is $(gg)^*$ - closed but not tgr - closed and $\{c, d\}$ is tgr - closed but not $(gg)^*$ - closed.

Remark 4.2

From the above discussions and known results the relationship between $(gg)^*$ -closed sets and other existing generalizations of closed sets are implemented in Figure: 1

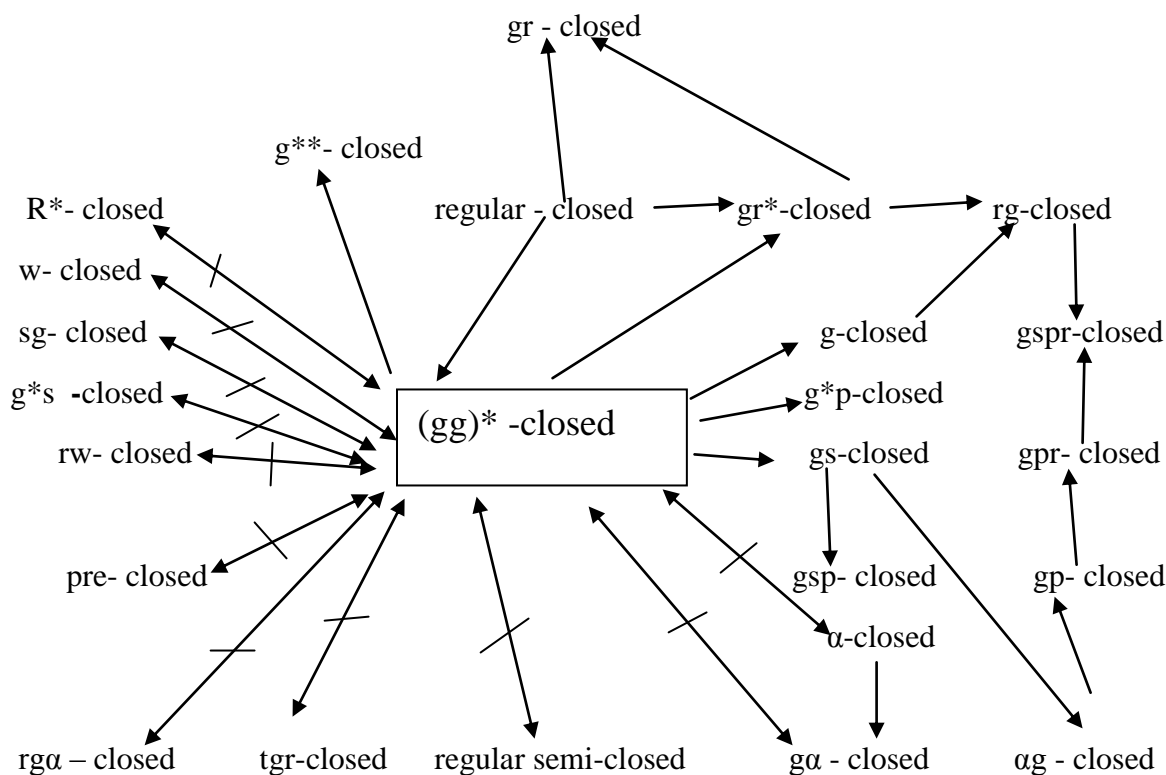


Figure: 1

In the above figure $A \longrightarrow B$ means the set A implies the set B but not conversely and $A \longleftarrow\!| \! \longrightarrow B$ means the set A and B are independent of each other.

V. CHARACTERISTICS OF $(gg)^*$ - CLOSED SETS

Theorem 5.1 The Union of any two $(gg)^*$ -closed sets of X is $(gg)^*$ -closed.

Proof: Let A and B be the $(gg)^*$ -closed sets in X . Let U be a gg - open set in X such that $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $(gg)^*$ -closed sets in X , $rcl(A) \subseteq U$ and $rcl(B) \subseteq U$. We have by [19], $rcl(A \cup B) = rcl(A) \cup rcl(B) \subseteq U$. This implies $rcl(A \cup B) \subseteq U$. Hence $A \cup B$ is $(gg)^*$ -closed.

Remark 5.2 Intersection of two $(gg)^*$ -closed sets need not be $(gg)^*$ -closed as shown in the following example.

Example 5.3 Let $X = \{a, b, c, d\}$, and $\tau = \{ \emptyset, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X \}$.

Let $A = \{a, b\}$ and $B = \{b, c\}$ be two $(gg)^*$ -closed sets in X
Then $A \cap B = \{b\}$ is not a $(gg)^*$ -closed set.

Theorem 5.4 A subset A of X is $(gg)^*$ -closed set in X if and only if $rcl(A) - A$ contains no non-empty gg - closed set.

Proof : Let F be a non-empty gg -closed set in X such that $F \subseteq rcl(A) - A$. That is $F \subseteq rcl(A) \cap [(A)^c]$. Therefore $F \subseteq rcl(A)$ and $F \subseteq A^c$ and so $A \subseteq F^c$. Now since A is $(gg)^*$ -closed, and F^c is gg -open, $rcl(A) \subseteq F^c$. This implies $F \subseteq [rcl(A)]^c$. Also we have $F \subseteq rcl(A)$. Therefore $F \subseteq rcl(A) \cap [rcl(A)]^c = \emptyset$. This is a contradiction. Therefore $rcl(A) - A$ contains no non-empty gg -closed set. Conversely, suppose that $rcl(A) - A$ contains no non-empty gg - closed set. Suppose $rcl(A)$ is not contained in U . Let U be a gg -open set in X such that $A \subseteq U$. Then $rcl(A) \cap U^c$ is a non - empty gg -closed set and contained in $rcl(A) - A$. Which is a contradiction. Hence A is a $(gg)^*$ -closed set.

Theorem 5.5 Let $A \subseteq B \subseteq rcl(A)$ and A is $(gg)^*$ -closed set in X , then B is also $(gg)^*$ -closed.

Proof: Let U be a gg -open set in X such that $A \subseteq U$. Now if $A \subseteq B \subseteq rcl(A)$, then $rcl(A) \subseteq rcl(B) \subseteq rcl(A)$. Therefore $rcl(B) = rcl(A)$. Since A is $(gg)^*$ -closed, $rcl(A) \subseteq U$. Therefore $rcl(B) = rcl(A) \subseteq U$. Hence B is $(gg)^*$ -closed.

Theorem 5.6 If A is gg -open subset of X and $(gg)^*$ -closed set in X . Then A is a regular closed set.

Proof: Since A is gg -open subset of X and $(gg)^*$ - closed, $rcl(A) \subseteq A$. But $A \subseteq rcl(A)$. Therefore $A = rcl(A)$. Hence A is regular closed.

Theorem 5.7 Let $A \subseteq B \subseteq X$, where B is gg -open and $(gg)^*$ -closed in X . If A is $(gg)^*$ -closed in B . Then A is $(gg)^*$ -closed in X .

Proof: Let U be a gg -open set in X such that $A \subseteq U$. Since $A \subseteq U \cap B$, $U \cap B$ is gg -open in B and A is $(gg)^*$ -closed in B , $rcl(A) \subseteq U \cap B$. Now $rcl(A) \cap B \subseteq U \cap B$. Since $A \subseteq B$, $rcl(A) \subseteq rcl(B)$. Since B is gg -open and $(gg)^*$ - closed in X , by theorem 5.6, B is regular closed. Therefore $rcl(B) = B$. This implies $rcl(A) \subseteq B$. Thus $rcl(A) = rcl(A) \cap B \subseteq U \cap B \subseteq U$. Hence A is $(gg)^*$ -closed in X .

Theorem 5.8 For every point x of the space X the set $X - \{x\}$ is either $(gg)^*$ -closed (or) gg -open.

Proof: Suppose that $X - \{x\}$ is not gg -open. Then X is the only gg -open set containing $X - \{x\}$. That is $X - \{x\} \subseteq X$. This implies $rcl(X - \{x\}) \subseteq rcl(X) \subseteq X$. Therefore $X - \{x\}$ is a $(gg)^*$ - closed set in X .

Theorem 5.9

- (1) If A is β -open and βg^* -closed set in X . Then A is $(gg)^*$ -closed.
- (2) If A is R^* -open and $R^\#$ - closed set in X . Then A is $(gg)^*$ -closed.

(3) If A is regular open and $r \wedge g$ -closed set in X . Then A is $(gg)^*$ -closed.

Proof:

(1) Let A be a β -open and βg^* -closed set in X . Let U be any gg -open set in X such that $A \subseteq U$.

By Definition 2.1(27), $gcl(A) \subseteq A$. But we have $rcl(A) \subseteq gcl(A) \subseteq A$. Therefore $rcl(A) \subseteq U$. Thus we get A is $(gg)^*$ -closed.

(2) Let A be a R^* -open and $R^\#$ -closed set in X . Let U be any gg -open set in X such that

$A \subseteq U$. By Definition 2.1(26), $gcl(A) \subseteq A$. But we have $rcl(A) \subseteq gcl(A) \subseteq A$. Hence $rcl(A) \subseteq U$. Thus we get A is $(gg)^*$ -closed.

(3) Let A be a regular open and $r \wedge g$ -closed set in X . Let U be any gg -open set in X such that

$A \subseteq U$. By Definition 2.1(28), $gcl(A) \subseteq A$. But we have $rcl(A) \subseteq gcl(A) \subseteq A$. Hence $rcl(A) \subseteq U$. Thus we get A is $(gg)^*$ -closed.

Theorem 5.10

Let A be a regular semi-open set.

(a) If A is gg -closed then A is $(gg)^*$ -closed.

(b) If A is R^* -closed then A is $(gg)^*$ -closed.

Proof:

(a) Let A be a regular semi-open set and gg -closed set in X . Let U be any gg -open set in X such that $A \subseteq U$. By Definition 2.1(4), $gcl(A) \subseteq A$. But we have $rcl(A) \subseteq gcl(A) \subseteq A$. Hence $rcl(A) \subseteq U$. Thus we get A is $(gg)^*$ -closed.

(b) Let A be a regular semi-open set and R^* -closed set in X . Let U be any gg -open set in X such that $A \subseteq U$. By Definition 2.1(25), $gcl(A) \subseteq A$. But we have $rcl(A) \subseteq gcl(A) \subseteq A$.

Hence $rcl(A) \subseteq U$. Thus we get A is $(gg)^*$ -closed.

VI. CONCLUSION

The class of $(gg)^*$ -closed sets in topological spaces is defined using regular closure and gg -open sets. We have studied the relation of this set with some other closed sets and some of the properties are investigated.

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AUTHORS PROFILE

I.Christal Bai is a Research scholar in Department of Mathematics, Nesamony Memorial Christian College, Marthandam, Tamil Nadu, India . She pursued her M.Sc., and M. Phil., from Manonmaniam Sundaranar University in 2008, and 2009. She has one year teaching experience as Lecturer in Pon Jesly College of Engineering, Nagercoil, India. Her area of interest is Topology.



T .Shyla Isac Mary pursued M. Sc., M.Phil., and Ph. D from Manonmaniam Sundaranar University in 1999, 2001 and 2012. She is currently working as Assistant Professor in Department of Mathematics from Nesamony Memorial Christian College, Marthandam, India since 2003. She has published 25 papers in reputed national and international journals. Her area of research is Topology. Under her guidance two scholars are awarded Ph.D and at present 5 scholars are doing Ph.D. She has 15 years of teaching experience.

