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Further Results on Accurate Domination in Graphs

B. Basavanagoud[1*], Sujata Timmanaikar[2]

[1]Department of Mathematics, Karnatak University, Dharwad, India
[2]Department of Mathematics, Government Engineering College, Haveri-581 110, India

Corresponding Author: b.basavanagoud@gmail.com, Tel.: +91-9449177029

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Abstract— The accurate domination number of graph G denoted by $\gamma_a(G)$ is the cardinality of a smallest set D that is dominating set of G and no $|D|-$element subset of $V_G - D$ is a dominating set of G. In this paper, we characterized the graphs with equal accurate domination number and maximal domination number($\gamma_a(G)=\gamma_m(G)$). Further, we obtained various bounds for $\gamma_a(G)$ in terms of minimum(maximum)degree, vertex(edge)connectivity, vertex(edge)covering number, chromatic number and domination(connected domination)number.

Keywords— *Domination number, Accurate domination number, Maximal domination number.*

I. Introduction

All graphs considered here are finite, nontrivial, undirected with no loops and multiple edges. For graph theoretic terminology we refer to Harary [3].

Let $G=(V,E)$ be a graph with $|V|=p$ and $|E|=q$. Let $\Delta(G)(\delta(G))$ denote the *maximum (minimum)* degree. A set of vertices which covers all the edges of a graph G is called a *vertex cover* for G. The smallest number of vertices in any vertex cover for G is called its *vertex covering number* and is denoted by $\alpha_0(G)$. A set of vertices in G is *independent* if no two of them are adjacent. The largest number of vertices in such a set is called the *vertex independence number* of G and is denoted by $\beta_o(G)$. The *corona* of two graphs G_1 and G_2 is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2. *Pendant vertex* of G, that is the vertex of degree 1. A vertex v is called a *support vertex* if v is neighbor of a pendant vertex and $d_G(v)>1$. A vertex $v \in V(G)$ is said to be *cut vertex* if $G-v$ is disconnected graph.

A *proper coloring* of a graph $G=(V(G),E(G))$ is a function from the vertices of the graph to a set of *colors* such that any two adjacent vertices have different colors. The *chromatic number* $\chi(G)$ is the minimum number of colors needed in a proper coloring of a graph. We denote the *path* on p vertices by P_p and a *bipartite graph* G is a graph whose vertex set V can be partitioned into two subsets V_1 and V_2 such that every edge of G joins V_1 with V_2 and is denoted by $K_{p,q}$. The *vertex connectivity* $\kappa=\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph. The *edge connectivity* $\lambda=\lambda(G)$ of a graph G is the minimum number of edges whose removal results in a disconnected or trivial graph.

A subset $D \subseteq V(G)$ is a *dominating set* of G if every vertex of $V(G)\setminus D$ has a neighbor in D. The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G.

A dominating set D is said to be *connected dominating set* if $\langle D\rangle$ is connected. The *connected domination number* $\gamma_c(G)$ of G is the minimum cardinality of a minimal connected dominating set of G.

A dominating set D is said to be *maximal dominating set* if $V\setminus D$ is not a dominating set. The *maximal domination* *number* $\gamma_m(G)$ of G is the minimum cardinality of a maximal dominating set of G.

A dominating set D is an *accurate dominating set* such that no $|D|$-element subset of $V(G)\setminus D$ is a dominating set of G. The *accurate domination number* $\gamma_a(G)$ of G is the cardinality of a smallest accurate dominating set of G. The accurate domination in graphs was introduced by Kulli and Kattimani [9], and further studied in a number of papers. For a comprehensive survey of domination in graphs, see [1, 2, 4, 5, 7, 8].

In this paper we study graphs for which the accurate domination number is equal to maximal domination number. In particular we characterized the graphs for which $\gamma_a(G)=\gamma_m(G)$. Also we constructed bounds for accurate domination number.

II. Graphs with γ_a equal to γ_m

We are interested in determining structure of graphs for which the accurate domination number is equal to the maximal domination number. The question was posed in [9].

Problem 1: Characterize the graphs for which, $\gamma_a(G)=\gamma_m(G)$.

To solve problem 1 we start with trees.

We begin with the following already known auxiliary results and straightforward observations.

Proposition A [2]. For $p\geq 1$, $\gamma_a(P_p)=\left\lceil \frac{p}{3}\right\rceil$ unless $p\in\{2,4\}$ *when* $\gamma_a(P_p)=\left\lceil \frac{p}{3}\right\rceil+1$.

Proposition B [6]. For $p\geq 1$, $\gamma_m(P_p)=\left\lceil \frac{p}{3}\right\rceil+1$

Proposition C [6]. If $\gamma_c(G)>\frac{p}{2}$, then $\gamma_m(G)=\gamma_c(G)+1$, where $\gamma_c(G)$ is connected domination number of G.

Proposition D [6]. For any tree $\gamma_m(T)\leq m+1$. Furthermore, the bound is attained if and only if each cut vertex is adjacent to a pendant vertex, where m denotes the number of cut vertices of T.

Proposition E [9]. For any graph G, $\gamma_a(G)\leq\gamma_m(G)$. Furthermore equality holds if $G=C_p$.

Proposition 2.1 For Path P_p, ($p\notin\{2,4\}$), $\gamma_a(P_p)=\gamma_m(P_p)$.

Proof. Proof follows from the Propositions A and B.

Observation 2.2 *Every maximal dominating set is an accurate dominating set.*

But the converse of the above statement is not true. For example, $\gamma_a(K_{1,\ p-1})=1$ and $\gamma_m(K_{1,\ p-1})=2$.

Hence we arrive at the following inequality. For any graph G, $\gamma_a(G)\leq\gamma_m(G)$.

Proposition 2.3 *If* $G=H\circ K_1$ *is a corona graph then,* $\gamma_a(G)=\gamma_m(G)=\left\lceil\frac{p}{2}\right\rceil+1$.

Proof. Assume that G is a corona graph. If $G=K_1\circ K_1$ *or* $G=K_2\circ K_2$ then $G=P_2$ *or* P_4. By Proposition A, $\gamma_a(G)=\gamma_m(G)$. Hence we may assume that $G=H\circ K_1$, where H is any connected graph. Let $\{v_1,v_2,...,v_{p/2}\}=V(H)$ and let $F=\{v_{\frac{p}{2}+1},v_{\frac{p}{2}+2},...,v_p\}$ be new vertices attached to each $v_i, 1\leq i\leq p/2$. Clearly, either $|D|=F$ *or* $|D|=V(H)$ is a dominating set. Hence $D\cup\{v_i\}, 1\leq i\leq p/2$ *or* $D\cup\{v_i\}, p/2+1\leq i\leq p$ is an accurate dominating set. Further, $V-(D\cup\{v_i\})$ is not a dominating set, which implies that $D\cup\{v_i\}$ is a maximal dominating set. Hence

$$\begin{aligned}\gamma_a(G) &= |D\cup\{v_i\}|\\ &= p/2+1\\ &= \gamma_m(G).\end{aligned}$$

Corollary 2.4 1 *If* $G=T\circ K_1$, *where* T *is any nontrivial tree then* , $\gamma_a(G)=\gamma_m(G)$

Now we are in a position to give answer for an Open problem posed in [9].

Theorem 2.5 2 *If* D *be a dominating set of a graph* G, *then* $\gamma_a(G)=\gamma_m(G)$ *if and only if* $\gamma_a(G)=|D\cup\{v\}|$ *and V-* $(D\cup\{v\})$ *is not a dominating set.*

Proof. Assume that $\gamma_a(G)=\gamma_m(G)$. Let D and D' be minimal dominating and accurate dominating sets of G respectively. Then $|D'|\leq|D\cup\{v\}|=\gamma_m(G)$. Which implies,

$$\gamma_a(G)\leq|D\cup\{v\}| \quad (1)$$

Since $\gamma_a(G)+1\leq\gamma_m(G)$ that is $|D|+1\leq\gamma_m(G)=\gamma_a(G)$. Hence $|D|+1\leq=\gamma_a(G)$ in other words

$$|D\cup\{v\}|\leq\gamma_a(G) \quad (2)$$

Then from equations (1) and (2) $\gamma_a(G)=|D\cup\{v\}|$.

Conversely, suppose $\gamma_a(G)=|D\cup\{v\}|$ then the result follows from the above arguments.

III. Bounds for accurate domination number

In the following theorem we obtain bounds for $\gamma_a(G)$ in terms of $\gamma_c(G)$.

Theorem 3.1 3 *If* $\gamma_c(G)>\dfrac{p}{2}$ *then* $\gamma_a(G)\leq\gamma_c(G)+1$.

Proof. Let $\gamma_c(G) > \frac{p}{2}$. Then by Proposition C, $\gamma_m(G) \le \gamma_c(G)+1$. Hence the result follows from the fact that $\gamma_a(G) \le \gamma_m(G)$.

Proposition 3.2 4 *For any tree T, $\gamma_a(T) = r+1 = \gamma_m(T)$ if and only if every cut vertex is adjacent to an end vertex, where r is the number of cut vertices of a tree T.*

Proof. Let $F = \{v_1, v_2, v_3, ..., v_r\}$ be the set of cut vertices of a tree T such that $|F| = r$. Then each end vertex $v \in T$ together with F forms an accurate dominating set as well as maximal dominating set. Hence

$$\begin{aligned} \gamma_a(G) &= |F|+1 \\ &= r+1 \\ &= \gamma_m(T) \end{aligned}$$

Converse follows from Proposition D.

Proposition 3.3 5 *For any tree $T, \gamma_a(T) \le \beta_0(T)+1$, where β_0 is vertex independence number.*

Proof. Since tree T is a bipartite graph, we know that for any bipartite graph $G, \alpha_0(G) = \beta_0(G)$. Let S be maximum independent set of vertices in T. Then for any vertex $v \in S, V-S \cup \{v\}$ is a maximal dominating set of T. Hence $\gamma_m(T) \le \beta_0(T)+1$. Therefore, the result follows from the fact that $\gamma_a(T) \le \gamma_m(T)$.

Proposition 3.4 F*or any graph G,* $\gamma_a(G) \le p - \delta(G)+1$.

Proof. Let F be a vertex covering set of G, then $|F| < \frac{p}{2}$. Clearly F is an accurate dominating set such that $\gamma_a(G) \le \alpha_0(G)+1$. From [10], we have $\alpha_0(G) \le p - \delta(G)$ this implies that,

$$\begin{aligned} \gamma_a(G) &\le \alpha_0(G)+1 \\ &\le p-\delta(G)+1. \end{aligned}$$

Suppose $|F| = \frac{p}{2}$. Then for every vertex $v \in V-F$, $F \cup \{v\}$ is an accurate dominating set.

Proposition 3.56 *For any graph G, $\gamma_a(G) \le p - \kappa(G)+1$, where κ is the vertex connectivity of G.*

Proof. Let G be a graph with $V(G) = \{v_1, v_2, v_3, ..., v_n\}$. Let F' be the maximal independent dominating set of vertices of G. Then by Proposition 3.4

$$\begin{aligned} \gamma_a(G) &\le \alpha_0(G)+1 \\ &\le p-\beta_0(G)+1. \end{aligned}$$

Since $\alpha_0(G) \le p - \kappa(G)$ therefore,

$$\gamma_a(G) \le p-\kappa(G)+1.$$

Proposition 3.67 *For any graph G, $\gamma_a(G) \le p - \lambda(G)+1$, where λ is the edge connectivity of G.*

Proof. Let G be a graph with vertex covering set F. Since $|F| \le \frac{p}{2}$ and $F \cup \{v\}$, where $v \in V-F$ will form an accurate dominating set of G. Hence,

$$\begin{aligned} \gamma_a(G) &\le |F \cup \{v\}| \\ &= \alpha_0(G)+1 \end{aligned}$$

Since in [10], $\alpha_0(G) \le p - \lambda(G)$, therefore the result follows.

Proposition 3.7 8 *For any graph G, $\gamma_a(G) \le \kappa(G)+1$.*

Proof. Let G be a graph and D be a minimum dominating set of G. Let $\kappa(G)$ be a vertex connectivity of graph G. From [10] we know that $\gamma(G)+\kappa(G) \le p$, which implies that

$$\kappa(G) \le p-\gamma(G) \tag{3.1}$$

Since D is a dominating set then for any vertex $v \in D, (V-D) \cup \{v\}$ is an accurate dominating set of G. Thus,

$$\begin{aligned} \gamma_a(G) &\le |(V-D) \cup \{v\}| \\ &= p-\gamma(G)+1 \\ &\le \kappa(G)+1 \quad (by\ equation\ (2.1)) \end{aligned}$$

Proposition 3.8 9 *For any graph G, $\gamma_a(G) \le \chi(G)$, where $\chi(G)$ is the chromatic number of G.*

Proof. Let G be a graph. Let $F = \{c_1, c_2, c_3, ..., c_k\}$ be the color class required to color the graph G. Then the chromatic number of a graph G is $\chi(G) \le |F|$. From [10] $\gamma(G)+\chi(G) \le p+1$ implies that,

$$\gamma(G) \le p+1-\chi(G) \tag{3.2}$$

By Proposition (3.6),

$$\gamma_a(G) \leq p+1-\gamma(G)$$

$$\gamma_a(G) \leq \chi(G).$$

Proposition 3.9 10 *For any graph* G, $\gamma_a(G) \leq \beta_0(G)+1.$

Proof. Let G be a graph and let $F=\{v_1,v_2,v_3,...,v_k\}$ be maximum independent set of vertices of G such that $\beta_0(G)=|F|$. Let D be a minimum dominating set of G then for any vertex $v \in V-D$, $(V-D)\cup\{v\}$ is an accurate independent dominating set. Hence,

$$\gamma_a(G) \leq p-\gamma(G)+1 \quad (3.3)$$

From [10], we have the inequality $\beta_0(G)+\gamma(G) \leq p$ implies that,

$$\gamma(G) \leq p-\beta_0(G) \quad (3.4)$$

by equations (3.3) and (3.4) result follows.

Proposition 3.10 11 *For any graph* G, $\gamma_a(G) \leq p-\alpha_0(G)+2.$

Proof. Let $F=\{v_1,v_2,v_3,...,v_s\}$ be a minimum vertex covering set of G. Since every minimum vertex covering set is a dominating set of G, that is $|D| \leq \alpha_0(G)$ and $D\cup\{v\}$ is an accurate dominating set.

Therefore,

$$\gamma_a(G) \leq |D\cup\{v\}|$$
$$\leq \alpha_0(G)+1.$$

We have,

$$\gamma_a(G) \leq \gamma(G)+1. \quad (3.5)$$

From [10], we have the inequality

$$\gamma(G)+\alpha_0(G) \leq p+1.$$

Hence,

$$\gamma(G) \leq p-\alpha_0(G)+1.$$

Then equation (3.5) becomes,

$$\gamma_a(G) \leq p-\alpha_0(G)+2.$$

Proposition 3.11 12 *For any graph* G, $\gamma_a(G) \leq \gamma(G)+p-\Delta(\overline{G})-1.$

Proof. Let D be a dominating set of G and v be a vertex of minimum degree that is $\delta(G)=deg v.$ Then either $v \in D$ or some vertex u adjacent to v belongs to $D.$ Thus $D\cup N[v]$ is a maximal dominating set of $G.$ Hence,

$$\gamma_m(G) \leq \gamma(G)+\delta(G).$$

Since $\delta(G)+\Delta(\overline{G})=p-1$ and also we know that,

$$\gamma_a(G) \leq \gamma_m(G)$$
$$\leq \gamma(G)+\delta(G)$$
$$\leq \gamma(G)+p-\Delta(\overline{G})-1.$$

Proposition 3.12 13 *For any graph* G, $\gamma_a(G) \leq \gamma(G)+\lambda(G)+\frac{p}{2}-1.$

Proof. Let D be a minimal dominating set of G and $v \in V(G)$ is a vertex of minimum degree. Then clearly $\gamma_m(G) \leq \gamma(G)+\delta(G).$ Further, from [10] we have $\delta(G)-\lambda(G) \leq \frac{p}{2}-1$ implies that

$$\delta(G) \leq \lambda(G)+\frac{p}{2}-1.$$

Since

$$\gamma_a(G) \leq \gamma_m(G)$$
$$\leq \gamma(G)+\delta(G)$$
$$\leq \gamma(G)+\lambda(G)+\frac{p}{2}-1.$$

Proposition 3.13 14 *For any graph* G, $\gamma_a(G) \leq \gamma(G)+\kappa(G)+\frac{p}{2}-1.$

Proof. Let $F=\{v_1,v_2,v_3,...,v_r\}$ be a minimum vertices required to result in a disconnected graph. Therefore $\kappa(G)=|F|.$ But in [10] we have the inequality $\delta(G)-\kappa(G) \leq \frac{p}{2}-1$ implies that

$$\delta(G) \leq \kappa(G)+\frac{p}{2}-1.$$

Since,

$$\gamma_a(G) \leq \gamma_m(G)$$
$$\leq \gamma(G)+\delta(G)$$
$$\leq \gamma(G)+\kappa(G)+\frac{p}{2}-1.$$

Proposition 3.14 15 *For any graph* G, $\gamma_a(G) \geq \gamma(G)+2\alpha_1(G)-p+1.$

Proof. Let $F'=\{e_1,e_2,e_3,...,e_k\}$ be minimum edge covering set of $G,$ that is $\alpha_1(G)=|F'|.$ Since

$2\alpha_1(G)-\delta(G)\leq p-1$ implies

$\delta(G)\geq 2\alpha_1(G)-p+1.$ By Proposition E we have,

$$\begin{aligned}\gamma_a(G) &\leq \gamma_m(G)\\ &\leq \gamma(G)+\delta(G)\\ &\geq \gamma(G)+2\alpha_1(G)-p+1.\end{aligned}$$

IV. CONCLUSION

In this paper we characterised the graphs for which the accurate domination number is equal to maximal domination number. In particular, we characterized the graphs for which $\gamma_a(G)=\gamma_m(G).$ Also we constructed bounds for accurate domination number. Further, this parameter can be used to study chemical properties of octane isomers.

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REFERENCES

[1] B. Basavanagoud and Sujata Timmanaikar, Accurate independent domination in graphs, Int. J. Math. Combin. (2), (2018) 87-96.
[2] J. Cyman, M.A.Henning and J. Topp, *On Accurate Domination in Graphs*, arXiv:1710.03308vl [math.CO] 9 Oct 2017.
[3] F. Harary, *Graph Theory*, Addison-Wesley, Reading, Mass, (1969).
[4] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker, Inc., New York, (1998).
[5] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, *Domination in Graphs- Advanced Topics*, Marcel Dekker, Inc., New York, (1998).
[6] V. R. Kulli and B. Janakiram, *The Maximal Domination Number of a Graph*, Graph Theory Notes of New York XXXIII, 11-13 (1997).
[7] V. R. Kulli, *Theory of Domination in Graphs*, Vishwa International Publications, Gulbarga, India (2010).
[8] V. R. Kulli, *Advances in Domination Theory-I*, Vishwa International Publications, Gulbarga, India (2012).
[9] V. R. Kulli and M.B. Kattimani, *Accurate Domination in Graphs*, In V.R.Kulli, ed., Advances in Domination Theory-I,Vishwa International Publications, Gulbarga, India, 1-8 (2012).
[10] Shaoji Xu, *Relation between parameters of a graph*, Discr. Math., 89, 65--88 (1991).

AUTHORS PROFILE

B. Basavanagoud is a professor in the department of Mathematics, Karnatak University, Dharwad-580 003, Karnataka state, India. He was chairman of the department for two terms, i.e., 2010-2012 and 2016-2018. He obtained his Ph. D degree from Gulbarga University, Kalaburgi, Karnataka, India, under the supervision of Prof. V. R. Kulli. He visited Taiwan on invitation to deliver an invited talk in Indo-Taiwan conference on Discrete Mathematics. He chaired a session and delivered a contributed talk in prestigious International Congress of Mathematicians (ICM) held at Hyderabad in the year 2010. He has more than 30 years of teaching experience, completed 7 research projects and organized 3 international conferences/ workshops. He has guided 12 students for their M. Phil, 10 students for their Ph. D. At present 5 students are working for their Ph. D. He has delivered more than 40 invited /contributed talks and has more than 140 research publications in reputed national/international journals. At present he is an Academic Council member of Karnatak University Dharwad (2017-2019). He is also life member for several academic bodies.

Sujata Timmanaikar received her M. Sc. degree from Karnatak University, Dharwad in 2008 and M. Phil degree in 2010 under the guidence of Dr. B. Basavanagoud, Department of studies in Mathematics, Karnatak University, Dharwad, Karnataka, India. Currently she is working as Assistant Professor in Government engineering college, Haveri. She is pursuing Ph. D(Part time) in Graph Theory under the guidance of Dr. B. Basavanagoud, Department of studies in Mathematics, Karnatak University, Dharwad, Karnataka, India. She has published ten research papers in international/national.

