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A Generalized Hybrid Steepest Descent Method for Variational Inequality Problem

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***Abstract*—**Variational inequalities are used as models for sovinga large number of problems in mathematical, physical, economics, optimization,finance and engineering. The fixed point formulation of any variational inequality problem can be formulated as a fixed point problem and is useful for existence of solution of the variational inequality problem as well as it also provides the facility to develop algorithms for approximation of solution of VI problem. A lot of research has been carried out to approximate solution of a variational inequality problem. In this paper, we propose to investigate a generalized hybrid steepest descent method and develop a convergence theory for solving variational inequality problem over the fixed point set of a mapping which is not necessarily Lipschitz continuous.Our result extends and generalizes many known results in recent history

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I. INTRODUCTION

Let H be a real Hilbert space. Let C be a nonempty closed convex subset of E and f : C→ C a mapping. A variational inequality problem over a nonempty closed convex subset D of C is formulated as finding an element $x^* \in D$ such that

$$\langle F(x^*), x - x^* \rangle \geq 0 \quad \text{…………………….(1.1)}$$

The problem (1.1) is denoted by $VIP_D(F,C)$. The solution set of variational inequality problem (1.1) is denoted by $\Omega[VIP_D(F,C)]$, i.e.,

$$\Omega[VIP_D(F, C)] = \{u \in D : \langle F(u), z - u\rangle \geq 0 \text{ for all } z \in D\}.$$

It is well known that convex minimization problem [4,9,11,13] of a differentiable convex function subject to a closed convex set $D \subseteq X$ of the form: find a point $x^* \in D$ such that

$$\psi(x^*) = \min\{\psi(x) : x \in D\}, \text{……………….(1.2)}$$

where $\psi: X \to R$ is a differentiable convex function, can be casted into the variational inequality problem over D :

find $u \in D$ such that $\langle \nabla\psi(u), z - u\rangle \geq 0$ for all $z \in D$,

where $\nabla\psi : X \to X$ is the gradient of ψ.

It is well known that if F is an η-strongly monotone and k−Lipschitz continuous, then, for $\mu \in (0, 2\eta/k^2)$,the mapping $P_C(I - \mu F)$ is contraction on C and hence $VIP_C(F,X)$ has a unique solution

$x^* \in C$ and the projection gradient method:

$$x_{n+1} = P_C(I - \mu F)x_n, n \in N \quad \text{……………………………(1.3)}$$

converges strongly to x^* (see [27, Theorem 46.C]). Note that computation of the metric projection, P_C, onto C is not necessarily easy. In order to reduce such difficulty, which is caused by the metric projection PC, in [26, Theorem 3.3, p. 486], Yamada introduced the following hybrid steepest descent method (for short, HSDM) for solving the variational inequality

$$VIP_{Fix(T)}(F,C): x_{n+1} = (I - \beta_n \mu F)Tx_n \ , n \in N \quad \text{……………………………..(1.4)}$$

Where $\{\beta_n\}$ is a sequence in $(0,1]$ and T is a nonexpansive mapping from X into itself with a nonempty fixed point set $Fix(T)$. Yamada [26, Theorem 3.3, p. 486] proved that the sequence $\{\beta_n\}$ defined by (1.4) converges strongly to a unique solution of $VIP_{Fix(T)}(F,X)$. There are many papers dealing with variational inequality problems when the constrained set D is a set of fixed point of a nonexpansive mapping or set of common fixed points of a family of nonexpansive mappings (see [7,6,8,16,18,19,23,25,28]).

It is well known that if $\psi : X \to R$ is a differentiable convex function such that $\nabla\psi$ is L-Lipschitz continuous for some $L > 0$, then $\nabla\psi$ is $1/L$-inverse strongly monotone. Hence $I-\gamma\nabla\psi$ is nonexpansive for $\gamma\in(0,2/L)$ (see [10]) and hence hybrid steepest descent method is applicable for solving the variational inequality problem $VIP_{Fix(I-\gamma\nabla\psi)}(F,X)$. The variational inequality problem $VIP_{Fix(I-\gamma\nabla\psi)}(F,X)$ is equally interesting when $\nabla\psi$ is uniformly continuous, but not necessarily L-Lipschitz continuous. In [12], Goldstein studied weak convergence of a hybrid steepest descent method when $\nabla\psi$ is uniformly continuous.

The problem is to find- Is it possible to develop a hybrid steepest descent method which converges strongly to a solution of the variational inequality problem $VIP_{Fix(I-\gamma\nabla\psi)}(F,X)$ for some $\gamma>0$?

The main purpose of this paper is to investigate a hybrid steepest descent method for solving the variational inequality problem $VIP_D(F,C)$ when constrained set D is a set of fixed points of a selfmapping on C which is more general than nonexpansive.A strong convergence theorem for strongly asymptotically nonexpansive mapping is established in Section 3.Our results are definitive and solves problem (1.1) in the mathematical theory of nearly Lipschitzian mappings and also improve several known results for the class of Lipschitzian type mappings in Hilbert spaces.

The paper is organized in follows, Section I contains the introduction of variational inequality problem and hybrid steepest descent method , Section II contain the related work or preliminaries required for obtaining our result, Section III contain our main result and strong convergence of the method is established , Section IV concludes research work with future directions

II. PRELIMINARIES

Let C be a nonempty subset of a normed space X and T $:C\to X$ a mapping. T is called an L-Lipschitzmapping if there exists $L \in[0,\infty)$ such that

$$\|Tx-Ty\|\le L\|x-y\| \text{ for all } x,y\in C.$$

The L-Lipschitz mapping T is called a non-expansive operator if L = 1 and contraction if $L \in[0,1)$. Wedenote by $B_r[x]$ the closed ball with center $x \in X$ and radius $r >0$ and by Fix(T) the set of fixed points of T.

A sequence $\{x_n\}$in C is said to be an approximating fixed point sequence if

$\lim_{n\to\infty} \|x_n- Tx_n\|=0$.

Recall that T is called a demiclosed mapping if

$(\{x_n\}$ in C, $x_n\to x$ weakly and $Tx_n\to y$ for some $x,y\in X \Rightarrow (x\in C$ and $Tx = y)$.

The following technical lemmas will be required.

Lemma 2.1. ([24]) Let $\{a_n\}$ be a sequence of nonnegative real numbers and let $\{b_n\}$be a sequence in R satisfying the following condition:

$$a_{n+1}\le(1-\beta_n)a_n+b_n \text{ for all } n \in N;$$

where $\{a_n\}$ is a sequence in (0,1]. If $\sum_{n=1}^{\infty}\beta_n=\infty$ and $\limsup_{n\to\infty}\left(b_n/\beta_n\right)\le 0$, then $\{a_n\}$ converges to zero.

Lemma 2.2.[2.2] Let $\{x_n\}$ and $\{z_n\}$be bounded sequences in a Banach space X and let $\{b_n\}$ be asequence in $[0,1]$ with $0<\liminf_{n\to\infty}\beta_n\le\limsup_{n\to\infty}\beta_n<1$.Suppose that

$$x_{n+1}=(1-\beta_n)\, x_n+\beta_n z_n \text{ for all } n \in N$$

and that $\limsup_{n\to\infty}(\|z_{n+1}-z_n\|-\|x_{n+1}-x_n\|)\le 0$. Then $\lim_{n\to\infty}\|z_n-x_n\|=0$.

Definition 2.1*:* A nearly Lipschitzian mapping T with sequence $\{(\eta(T^n),a_n\}$is said to be

(i) nearlynonexpansive if $\eta(T^n) = 1$ for all $n \in N$,

(ii) nearly asymptotically nonexpansive if $\eta(T^n)\ge 1$ for all $n \in N$ and $\lim_{n\to\infty}\eta(T^n) = 1$,

(iii) nearly uniformly k-Lipschitzian if $\eta(T^n)\le k$ for all $n \in N$ and for some $k \in[0,\infty)$,

(iv) nearly uniform k-contraction if $\eta(T^n)\le k <1$ for all $n\in N$.

By the definitions, we have the following implication:

contraction$\Rightarrow$nearly uniformly k-contraction$\Rightarrow$nearly nonexpansive:

Definition 2.2. Let C be a nonempty subset of a normed space X and fix a sequence $\{a_n\}$ in $[0,\infty)$ with $a_n\to 0$. A mapping $T : C \to C$ is said to be nearly asymptotically quasi-nonexpansive with respect to $\{a_n\}$ in $[0,\infty)$ with $a_n\to 0$ if $Fix(T) \neq 0$ and there exists a sequence $\{k_n\}$ in $[1,\infty)$ with $\lim_{n\to\infty}k_n=1$ such that

$$\|T^nx-p\| \le k_n(\|x-p\|+a_n) \text{ for all } x \in C,\ p \in Fix(T)$$

and $n \in N$.

Remark 2.1. Nearly asymptotically quasi-nonexpansive mappings are also called generalized asymptoticallyquasi-nonexpansive mappings (see [20]).

Remark 2.2. Every nearly asymptotically nonexpansive mapping with nonempty fixed point set is nearlyasymptotically quasi-nonexpansive mapping.

Lemma 2.3. (see[14, Corollary 4.3]) Let C be a nonempty closed convex subset of a uniformly convex Banach space X and $T : C \to C$ a demicontinuous nearly asymptotically nonexpansive mapping. Then $Fix(T)$ is closed and convex

III. MAIN RESULT

From Lemma 2.5 and [27] we see that the variationalinequality problem
$VIP_{Fix(T)}(F,C)$ has a unique solution $x^* \in (T)$.
We now introduce a hybrid steepest descent-like method for computation of unique solution $x^* \in F(T)$of the variational inequality problem $VIP_{Fix(T)}(F,C)$.

Algorithm 3.1.Hybrid steepest descent-like method

Step 0: Choose $x_1 \in C$ and $\beta_1 \in (0, 1]$ arbitrarily.
Step 1: Given $x_n \in C$; choose $\beta_n \in (0, 1]$ and define $x_{n+1} \in C$ by
$x_{n+1} := (1-\lambda_n)y_n + \lambda_n T^n(y_n)$,
$y_n = x_n - \beta_n \mu F(x_n)$ for all $n \in N$. (3.2)

We study convergence analysis of Algorithm 3.1 under the following assumptions:

(A1) $\lim_{n\to\infty}\beta_n = 0$ and $\sum_{n=1}^{\infty}\beta_n = \infty$;

(A2) $a \le \lambda_n \le b$ for all $n \in N$ and for some $a, b \in (0,1)$;

(A3) $\sum_{n=1}^{\infty}(\eta(T^n)-1) < \infty$ and $\lim_{n\to\infty}\frac{a_n}{\beta_n} = 0$ with a constant $K \in [0,\infty)$ such that $a_n/\beta_n \le K$ for all $n \in N$.

Theorem 3.1. Let C be a nonempty closed convex subset of a real Hilbert space X and $T : C \to C$ a uniformly continuous strongly asymptotically nonexpansive mapping with sequence $\{(\gamma(T^n),u_n)\}$ such that $Fix(T) \neq \phi$. Let $F : C \to X$ be γ-strongly monotone and L-Lipschitz continuous for some positive constants γ and L and let $\mu \in (0,2\gamma / L^2)$ such that $(I-\beta\mu F)(C) \subseteq C$ for all $\beta \in (0,1]$. Let $\{x_n\}_{n\in N}$ be a sequence in C generated by Algorithm 3.1, where $\{\lambda_n\}$ is a sequence in (0,1). Assume that assumptions (A1) - β(A3) hold. Then we have the following:

(a) $\Omega[VIP_{Fix(T)}(F,C)] = \{x^*\}$
(b) The orbit $\{x_n\}$ of the Algorithm 3.1 is well defined in the closed convex set $Br[x^*] \cap C$, where r is a positive constant such that
$\max\{||x_1-x^*||, 1/\tau(\mu||F(x^*)||+K)\} \le r$.
(c) If the following assumption holds:

(A4),
$\lim_{n\to\infty}\| T^n(x_n) - T^{n+1}(x_n)\| = 0$ and $\lim_{n\to\infty}\| T^n(x_n) - T^{n+1}(x_n)\| = 0$
then $\{x_n\}$converges strongly to x^*.

Proof: (a) It follows from *Remark 3.1*
(b) For every $n \in N$, we have

$$\begin{aligned}||y_n-x^*|| &= ||x_n - \beta_n\mu F(x_n) - x^*||\\ &= ||(I-\mu\beta_n F)(x_n) - (I-\mu\beta_n F)(x^*) - \mu\beta_n F(x^*)||\\ &\le ||(I-\mu\beta_n F)(x_n) - (I-\mu\beta_n F)(x^*)|| + \beta_n\mu\, ||F(x^*)||\\ &\le (1-\beta_n\tau)\, ||x_n - x^*|| + \beta_n\mu\, ||F(x^*)||.\end{aligned}$$

Thus,

$$\begin{aligned}||x_{n+1}-x^*|| &= ||(1-\lambda_n)y_n + \lambda_n T^n(y_n) - x^*||\\ &\le (1-\lambda_n)\, ||y_n - x^*|| + \lambda_n ||T^n(y_n) - x^*||\\ &\le (1-\lambda_n)\, ||y_n - x^*|| + \lambda_n\gamma(T^n)(||y_n-x^*||) + u_n)\\ &\le \gamma(T^n)\,[||y_n-x^*||) + u_n]\\ &\le \gamma(T^n)[(1-\beta_n\tau)||x_n-x^*|| + \beta_n\mu||F(x^*)|| + u_n]\\ &\le \gamma(T^n)[(1-\beta_n\tau)\,||x_n-x^*|| + \beta_n\tau(\mu/\tau\, ||F(x^*)|| + K/\tau)]\\ &\le \gamma(T^n)\max\{||x_n-x^*||, 1/\tau(\mu||F(x^*)|| + K)\}.\end{aligned}$$

Inductively, we have

$$||x_n-x^*|| \le \prod_{i=1}^{n-1}\gamma(T^i)r, n \ge 2$$

Since $\sum_{n=1}^{\infty}(\gamma(T^n)-1) < \infty$, it follows that $\prod_{i=1}^{n-1}\gamma(T^n) = 1$.Thus, $\{x_n\}$is in $Br[x^*] \cap C$.Note

$$\begin{aligned}||y_n-x^*|| &\le (1-\beta_n\tau)\, ||x_n - x^*|| + \beta_n\mu\, ||F(x^*)||\\ &\le r + \mu\, ||F(x^*)||.\end{aligned}$$

(c) Note

$$\begin{aligned}x_{n+1} &= (1-\lambda_n)y_n + \lambda_n T^n(y_n)\\ &= (1-\lambda_n)(x_n - \beta_n\mu F(x_n)) + \lambda_n T^n(y_n)\\ &= (1-\lambda_n)x_n + \lambda_n z_n\end{aligned}$$

Where $z_n = \frac{1}{\lambda_n}\left[T^n(y_n) - \beta_n(1-\lambda_n)\mu F(x_n)\right]$.

Note

$$z_{n+1} - z_n = T^{n+1}(y_{n+1}) - \frac{\mu\beta_{n+1}(1-\lambda_{n+1})}{\lambda_{n+1}} F(x_{n+1})$$

$$-[T^n(y_n) - \frac{\mu\beta_n(1-\lambda_n)}{\lambda_n} F(x_n)]$$

=

$$T^{n+1}(y_{n+1}) - T^{n+1}(y_n) + T^{n+1}(y_n) - T^n(y_n) + \frac{\mu\beta_n(1-\lambda_n)}{\lambda_n} F(x_n) - \frac{\mu\beta_{n+1}(1-\lambda_{n+1})}{\lambda_{n+1}} F(x_{n+1}).$$

Set $\gamma_T = \sup_{n\in N}\gamma(T^n)$. Hence

$$\|z_{n+1} z_n\|$$

$$\leq \gamma(T^{n+1})(\| y_{n+1} - y_n \| + u_{n+1}) + \| T^{n+1}(y_n) - T^n(y_n) \|$$

$$+ \frac{\mu\beta_n(1-\lambda_n)}{\lambda_n} \| F(x_n) \| + \frac{\mu\beta_{n+1}(1-\lambda_{n+1})}{\lambda_{n+1}} \| F(x_{n+1}) \|$$

$$\leq \| y_{n+1} - y_n \| + \| T^{n+1}(y_n) - T^n(y_n) \| + (\gamma(T^{n+1}) - 1) \| y_{n+1} - y_n \| + u_{n+1}\gamma_T$$

$$+ \frac{\mu\beta_n(1-\lambda_n)}{\lambda_n} \| F(x_n) \| + \frac{\mu\beta_{n+1}(1-\lambda_{n+1})}{\lambda_{n+1}} \| F(x_{n+1}) \|$$

$$\leq \| x_{n+1} - x_n \| + \| T^{n+1}(y_n) - T^n(y_n) \| + \mu\beta_n \| F(x_n) \| + \mu\beta_{n+1} \| F(x_{n+1}) \| + (\gamma(T^{n+1}) - 1) K_1$$

$$+ u_{n+1}\gamma_T + \frac{\mu\beta_n(1-\lambda_n)}{\lambda_n} \| F(x_n) \| + \frac{\mu\beta_{n+1}(1-\lambda_{n+1})}{\lambda_{n+1}} \| F(x_{n+1}) \|$$

For some $K_1>0$. It follows that

$$\limsup_{n\to\infty}(\| z_{n+1} - z_n \| - \| x_{n+1} - x_n \|) \leq 0.$$

From Lemma 2.2 , we obtain that

$$\lim_{n\to\infty}(\| x_n - z_n \|) = 0 \quad \text{.....(3.3)}$$

From (3.2) and (3.3), we have

$$\| x_{n+1} - x_n \| = \lambda_n \| z_n - x_n \| \leq | z_n - x_n \| \to 0 \text{ as } n \to \infty.$$

Note

$$\| x_n - T(x_n) \| \leq \| x_n - T^n(x_n) | + \| T^n(x_n) - T^{n+1}(x_n) \| \; \omega_T (\| T^n x_n - x_n \|)$$

Which gives us that $||x_n - T(x_n)|| \to 0$ as $n\to\infty$. Since T is uniformly continuous, we have $||x_n - T^m(x_n)|| \to 0$ as $n\to\infty$ for all $m \in N$.

Finally, we conclude that $x_n \to x^*$ as $n\to\infty$.

This completes the proof.

IV. CONCLUSION

For finding a solution of a variational inequality problem with a strongly monotone mapping over the set of fixed points of a strongly asymptotically nonexpansive mapping on Hilbert spaces, we have given a steepest descent method.Its strong convergence has been proved without the Lipschitz continuity of the mapping in the framework of Hilbert spaces.

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