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On Asymptotic Testing of the Parameters of Two Pareto Distributions

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***Abstract*—**Considering two independent Pareto distributions we have obtained the asymptotic tests for the parameters in this article. After getting the distribution of the sum of the transformed variables, the conditional distribution of complete sufficient statistic of interest has been obtained by fixing the sum of the transformed variables. This conditional distribution is used to obtain the tests for testing. (i) one parameter is larger than the other and (ii) equality of both parameters. Further the power functions of the asymptotic tests have been derived and some power points have been presented.

Keywords*—Asymptotic Distributions, Asymptotic tests, ParetoDistributions, Conditional tests*

I. INTRODUCTION

Let U and V be independently distributed Pareto distributions with probability density function (p.d.f)

$$f_U(u;\theta_1)=\begin{cases}\dfrac{c\theta_1^c}{u^{c+1}};\ \theta_1<u<b,\ \ c>0, known\\ 0; \qquad\qquad\qquad\qquad otherwise\end{cases} \qquad (1.1)$$

$$f_V(v;\theta_2)=\begin{cases}\dfrac{c\theta_2^c}{v^{c+1}};\ \theta_2<v<b,\ \ c>0, known\\ 0; \qquad\qquad\qquad\qquad otherwise\end{cases} \qquad (1.2)$$

The Uniformly Most Powerful (UMP) test for testing one-sided and two-sided hypothesis of uniform distribution (and hence generalized uniform) is available in the literature (see Lehmann). Uniform distribution being a particular case of non-regular family of distributions, Bhatt [7] has obtained UMP test for testing this general family. Also Bhatt and Patel [8], have obtained UMPI test and confidence bounds. Generally for regular family adequate amount of literature is available for testing such as Engelhardt at el [2], Keating at el [4], Handa et al [6], Al-Shah at el [5], Bayond at el [1], all have studied comparison of two regular distributions, but Bhatt and Patel [9], have studied power function distributions. In this article we have studied the asymptotic tests for testing the parameters of two Pareto distributions. Asymptotic UMPI tests have been derived in section 3 after obtaining necessary asymptotic conditional distributions in section 2. The power functions has been obtained in section 4 and has been tabulated for some points of the power function.

II. ASYMPTOTIC CONDITIONAL DISTRIBUTIONS

Let $U_{1,}$ $U_{2,}$... U_n and $V_{1,}V_{2,}$... V_n be two independent random samples from (1.1) and (1.2) respectively. Let U_{n_1} and V_{n_2} lowest order statistic for random samples on U and V respectively.

Since $F_U(u)=1-(\frac{\theta_1}{u})^c$ and $F_V(v)=1-(\frac{\theta_2}{v})^c$ are distribution functions of U and V respectively. The corresponding p.d.f. of U_{n1} and V_{n2} are

$$f_{U_{n_1}}(u)=n_1[\frac{\theta_1}{u}]^{c(n_1+1)}\frac{c}{u} \qquad (2.1)$$

and

$$f_{V_{n_2}}(v)=n_2[\frac{\theta_2}{v}]^{c(n_2+1)}\frac{c}{v} \qquad (2.2)$$

Making transformations

$$S_1=n_1c\left(1-\left(\frac{\theta_1}{u_{n_1}}\right)^{c+1}\right)$$

and

$$S_2 = n_2 c\left(1-\left(\frac{\theta_2}{u_{n_2}}\right)^{c+1}\right)$$

(2.1) and (2.2) transforms to

$$f_{S_1}(s_1) = \left(1-\frac{s_1}{cn_1}\right)^{cn_1-1} ds_1 \qquad (2.3)$$

and

$$f_{S_2}(s_2) = \left(1-\frac{s_2}{cn_2}\right)^{cn_2-1} ds_2 \qquad (2.4)$$

respectively. Taking limit as $n_1 \to \infty$ and $n_2 \to \infty$ S_1 and S_2 are asymptotically distributed as exponential E(0,1), where p.d.f. of E(μ,σ)

$$f_X(x;\mu,\sigma) = \begin{cases}\frac{1}{\sigma}e^{-\frac{1}{\sigma}(x-\mu)}, & \mu < x < b, 0 < \sigma \\ 0, & otherwise\end{cases} \qquad (2.5)$$

Since U and V are independently distributed S_1 and S_2 are independently distributed. As S_1 and S_2 are independently distributed as E(0,1), $W_1 = \eta_1 S_1$, $W_2 = \eta_2 S_2$ and $T = W_1 + W_1$ then the conditional distribution of W_1 given T has p.d.f. as

$$f_{W_1/t}(w_1/t) = \frac{\epsilon_1 e^{-\epsilon_1 w_1}}{1-e^{-\epsilon_1 t}};\ w_1 < t \qquad (2.6)$$

Where

$$\epsilon_1 = \frac{1}{\eta_1} - \frac{1}{\eta_2}, \eta_1 = \frac{\theta_1}{cn_1}, \eta_2 = \frac{\theta_2}{cn_2} \qquad (2.7)$$

which can be shown by taking the joint p.d.f. of W_1 *and* W_1 and making the transformation $T = W_1 + W_1$, so that we get the marginal distribution of T as

$$f_T(t) = \frac{e^{-\frac{t}{\eta_1}-\frac{t}{\eta_2}}}{\eta_1-\eta_2};0 < t, \eta_1 \neq \eta_2$$

Using this marginal distribution the conditional distribution of W_1 given $T = t$ has p.d.f. given in (2.6)

III. ASYMPTOTIC TESTS

Someone may be interested in testing the hypothesis that one parameter is larger than the other, that is

$$H_1 : \theta_2 > \theta_1$$

or may be interested in testing equality of parameters, that is

$$H_2 : \theta_2 = \theta_1$$

It is easy to see that H_1 is equivalent to H_1'

$$H_1': \epsilon_1 > \epsilon_{10}$$

where $\epsilon_{10} = \frac{cn\theta_0}{\theta_1\theta_2}, n_1 = n_2 = n, \theta_0 = \theta_2 - \theta_1$

and H_2 is equivalent to H_2'

$$H_2': \epsilon_1 = 0 , n_1 = n_2 = n$$

Using the monotonicity of likelihood, the usual asymptotic level α UMPI test for H_1 is

$$\phi_1 = \phi(w_1/t) = \begin{cases}1, & if\ w_1 > c^*\ for\ fixed\ t \\ 0, & otherwise\end{cases} \qquad (3.1)$$

where c is given by

$$c^* = \frac{1}{\epsilon_{10}}\log[\frac{1}{1-\alpha(1-e^{-\epsilon_{10}t})}], \text{ for fixed } s_2 \text{ and fixed } \epsilon_{10} \qquad (3.2)$$

Further the asymptotic UMPI level α test for testing H_2' is

$$\phi_2 = \phi(w_1/t) = \begin{cases}0, & if\ c_1^* < w_1 < c_2^*\ for\ fixed\ t \\ 1, & otherwise\end{cases} \qquad (3.3)$$

where $c_1^* and\ c_2^*$ are given by

$$(1-\alpha)[1-e^{-\epsilon_{10}t}] = e^{-\epsilon_{10}c_1^*} - e^{-\epsilon_{10}c_2^*} \qquad (3.4)$$

and

$$\epsilon_{10}(1-e^{-\epsilon_{10}t}) + e^{-\epsilon_{10}c_2^*}(\epsilon_{10}c_2^*+1) - e^{-\epsilon_{10}c_1^*}(\epsilon_{10}c_1^*+1) = \alpha \qquad (3.5)$$

IV. THE POWER FUNCTIONS

In this section we discuss the asymptotic power points.

The asymptotic UMPI test at level α for testing $H_1 : \theta_2 > \theta_1$ has been obtained in (3.1) and its power function is $\beta_{\phi_1}(\epsilon_1)$,

$$\beta_{\phi_1}(\epsilon_1) = \int_{c^*}^{\infty} \frac{\epsilon_1 e^{-\epsilon_1 w_1}}{1 - e^{-\epsilon_1 t}} \mathrm{dw}_1 ;$$

where c^* is given by (3.2)

For some selected values the power points are given in table (4.1). Also the asymptotic UMPI test at level α for the test given in (3.3) and its power function $\beta_{\phi_2}(\epsilon_1)$,

$$\beta_{\phi_1}(\epsilon_1) = 1 - \int_{c_1^*}^{c_2^*} \frac{\epsilon_1 e^{-\epsilon_1 w_1}}{1 - e^{-\epsilon_1 t}} \mathrm{dw}_1 ;$$

where the constants $c_1^* and\ c_2^*$ are given by solving the equation (3.4) and (3.5) respectively. For some values the power points are given in table (4.2).

Table : 4.1

	n	θ1	θ2	∈	α=0.05	α=0.1
					c* = 0.59914	c* = 0.46052
∈₁₀=5,t=10.4	100	10	20	0.25	0.929963	0.962758
				1.25	0.472872	0.562343
				2.25	0.259739	0.354813
				3.25	0.142669	0.223872
				5	0.05	0.1
					c* = 0.59925	c* = 0.46062
∈₁₀=5,t=1.5	100	10	20	1	0.706965	0.812089
				2	0.314774	0.418874
				3	0.167529	0.253925
				4	0.091214	0.158813
				5	0.05	0.1
					c* = 0.61625	c* = 0.47764
∈₁₀=5,t=0.5	100	10	20	1.5	0.751968	0.925781
				2.5	0.300258	0.42463
				3.5	0.140004	0.227438
				4.5	0.069818	0.130285
				5	0.05	0.1
					c* = 0.29952	c* = 0.23205
∈₁₀=10,t=10.4	300	15	20	0.5	0.865667	0.896195
				2.5	0.472871	0.562341
				5.5	0.192501	0.281838
				7.5	0.105737	0.177828
				10	0.05	0.1
					c* = 0.29952	c* = 0.00201
∈₁₀=10,t=1.5	300	15	20	1.25	0.812213	0.885724
				3.25	0.380622	0.476792
				5.25	0.207551	0.298652
				7.25	0.113962	0.188368
				10	0.05	0.1
					c* = 0.30024	c* = 0.23093
∈₁₀=10,t=0.5	300	15	20	2	0.867774	0.996811
				4	0.347989	0.459175
				6	0.1737	0.26328
				8	0.092226	0.160575
				10	0.05	0.1
					c* = 0.19772	c* = 0.15356
∈₁₀=15,t=10.4	450	15	30	0.75	0.861245	0.891616
				3.75	0.472871	0.562341
				6.75	0.259739	0.354813
				9.75	0.142669	0.223872
				15	0.05	0.1
					c* = 0.19772	c* = 0.15356
∈₁₀=15,t=1.5	450	15	30	2.25	0.660642	0.94467
				5.25	0.350549	0.377758
				7.25	0.235059	0.241426
				10.25	0.129111	0.129834
				15	0.05	0.1
					c* = 0.19772	c* = 0.1535
∈₁₀=15,t=0.5	450	15	30	2.75	0.772716	0.8774225
				5.75	0.336046	0.4383228
				7.75	0.217163	0.310683
				10.75	0.117338	0.19283106
				15	0.05	0.1

Table 4.1 shows some power points for testing

$$H_1 : \theta_2 > \theta_1$$

Table : 4.2

	n	θ1	θ2	∈	α=0.05	α=0.1
					c*₁= 0.54466	c*₁= 0.4397
					c*₂= 1.2478	c*₂= 1.0151
∈₁₀=0.9,u=0.4	117	30	39	0.9	0.049128	0.1
				2.5	0.66437	0.598108
				5	0.926315	0.878917
				7.5	0.982385	0.96163
				10	0.995612	0.987501
				12.5	0.998888	0.995875
				15	0.999716	0.998631
					c*₁= 0.31031	c*₁= 0.3154
					c*₂= 2.0408	c*₂= 1.8612
∈₁₀=0.9,u=1.1	117	30	39	0.9	0.05	0.1
				2.5	0.514708	0.524632
				5	0.787244	0.792664
				7.5	0.902415	0.906084
				10	0.955087	0.957324
				12.5	0.979324	0.980603
				15	0.990482	0.991184
					c*₁= 0.02622	c*₁= 0.065344
					c*₂= 3.65634	c*₂= 3.26639
∈₁₀=0.9,u=5	117	30	39	0.9	0.05	0.1
				2.5	0.063562	0.150997
				5	0.122891	0.278717
				7.5	0.167709	0.387426
				10	0.23068	0.479751
				12.5	0.2795	0.558161
				15	0.325222	0.624753

Table 4.2 shows some power points for testing

$$H_2 : \theta_2 = \theta_1$$

V. CONCLUSION and Future Scope

In this research article we have tried to give the asymptotic tests for testing the parameters of two Pareto distributions. Attempt has also been made to apply the results to find the power points from the power functions. From table 4.1, one observes that power goes on decreasing as the difference between the parameters goes on increasing. It can also be observed from table 4.2 that as the difference between two parameters increases the power also goes on increasing contrary to the previous case. One can extend this theory to two-parameter truncated case also.

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