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A Deterministic Inventory Model with Weibull Deterioration and Quadratic Demand Rate under Trade Credit

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***Abstract*:** In this research paper, we have analysed a deterministic inventory model follows two parameter Weibull distribution for deteriorating items with Quadratic demand rate under trade credit. The study has two main purposes: firstly, to establish the mathematical model of an inventory system under the given condition, and secondly to demonstrate that the optimal solution not only exists but is also feasible. The derived model is then illustrated by a numerical example and its sensitivity analysis is carried out. A supplier offers a credit limit to the customers during which there is no interest charged but after the expiry of a prescribed time limit, the supplier will charge some interest. However, the customer usually has the reserve capital to make the payment at the beginning but decides to take the benefit of the credit limit.

Keywords: *Deterioration, Quadratic Demand, Two Parameter Weibull Distribution, Trade Credit*

I. INTRODUCTION

In the classical economic order quantity (EOQ) model, it was tacitly assumed that the customer must pay for the items as soon as the items are received. However, in practices or when the economy turns sour, the supplier frequently offers its customers a permissible delay in payments to attract new customer which is known as trade credit period. During this period, the retailer can earn revenues by selling items and by earning interests. Thus, it makes economic sense for the customer to delay the payment of the replenishment account up to the last day of the settlement period allowed by the supplier to the consumer. Similarly, for the supplier, it helps to attract a new customer as it can be considered as some sort of a loan. Furthermore, it helps in the bulk sale of goods and the existence of credit period serves to reduce the cost of holding stock to the user because it reduces the amount of capital invested in stock for the duration of the credit period. So, the concept of permissible delay in payments is beneficial for both the suppliers and the consumers.

Over the last two decades several researchers have studied inventory models of deteriorating items such as medicines, blood banks, green vegetables, fashion goods and highly volatile liquids like gasoline, alcohol, turpentine etc. which undergo physical depletion over time through the process of evaporation, wear and tear etc. Goyal [12] developed an EOQ model under conditions of permissible delay in payments. He ignored the difference between the selling price and the purchase cost and concluded that the economic replenishment interval and order quantity generally increases marginally under the permissible delay in payments.

Aggarwal and Jaggi [1] extended Goyal's model to the case of deterioration, Jamal et al. [16] generalized Aggarwal and Jaggi to the case of allowable shortage. Ghare and Schrader [11] developed a model for an exponentially decaying inventory. Covert and Philip [5] then extended the Ghare and Schrader's constant deteriorating rate to a two parameter Weibull distribution. Chen and Kang [7] proposed an integrated inventory model considering a permissible delay in payment and variant pricing strategy, M. Liang et.al.[8] developed an optimal order quantity under advance sales and permissible delays in payments, C. K. Jaggi [15] developed pricing and replenishment policies for imperfect quality deteriorating items under inflation and permissible delay in payments. Teng [23] provided an alternative conclusion from Goyal and mathematically proved that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. Chung-Yuan Dye [9] analysed a deterministic inventory model developed for deteriorating items with stock dependent demand and shortages. The conditions of permissible delay in payments were also taken into consideration. Chang et al. [10] then extended Teng's model and established an EOQ model for deteriorating items in which supplier provides a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. Teng et al. [20] assumed that the selling price is necessarily higher than the purchase cost and established an appropriate model for a retailer to determine its optimal price and lot size simultaneously when the supplier offers a permissible delay in payments. Venkateswarlu and Mohan [24] have developed inventory models for deteriorating items with time dependent quadratic demand and salvage value. Venkateswarlu and Mohan [25] studied an inventory model for time varying deterioration and price dependent quadratic demand with salvage value. Khatri and Gothi [17] developed an EPQ Model under Constant Amelioration Different Deteriorations with Exponential Demand Rate and Completely Backlogged Shortages. Mohan and Venkateswarlu [18] proposed an inventory model for time dependent quadratic demand with salvage considering deterioration rate is time dependent. Mohan and Venkateswarlu [19] developed an inventory model with quadratic demand, variable holding cost with salvage value using Weibull distribution deterioration rate. Ghosh and Chaudhuri [13] discussed an inventory model for a deteriorating item having an instantaneous supply, a quadratic time-varying demand considering shortages. In this model, the deterioration rate is considered as Weibull distribution deterioration of two parameters. Bhandari and Sharma [2] have studied a Single Period Inventory Problem with Quadratic Demand Distribution under the Influence of Marketing Policies. Bhojak and Gothi [3] developed an EOQ model with time – dependent demand and Weibully distributed deterioration. Bhojak and Gothi [4] have developed an EPQ model with time dependent holding cost and Weibully distributed deterioration under shortages. Chatterji and Gothi [6] have developed an inventory model for two – parameter Weibully deteriorated items with exponential demand rate and completely backlogged shortages. Gothi and Kirtan Parmar [14] have developed a deterministic inventory model by taking two parameter Weibull distribution to represent the distribution of time to deterioration and shortage are allowed and partially backlogged. Parmar, Aggarwal, and Gothi [21] developed an order level inventory model for deteriorating items under varying demand condition. Parmar and Gothi [22] developed a deterministic inventory model for deteriorating items where time to deterioration has Exponential distribution and time dependent quadratic demand.

In this research paper, a deterministic inventory model is developed by considering an EOQ model with two parameter Weibull distribution for deteriorating items with quadratic demand rate under trade credit. We employ Weibull distribution because it is widely used in reliability and survival analysis. In this model shortages are allowed to occur and they are completely backlogged. Numerical examples are provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is carried out further. The purpose of the study is to make the model more relevant and applicable in practice to minimize the total inventory cost.

II. ASSUMPTIONS

The following assumptions are considered to develop the model:

1. $\theta(t) = \alpha\beta t^{\beta-1}$ is the two parameter Weibull deterioration rate, where $0 < \alpha << 1$, $\beta > 0$ are scale and shape parameters respectively.
2. The demand rate of the product is $D(t) = -(a+bt+ct^2)$ (where a, b, c > 0).
3. The inventory holding cost is linear function with time dependent and it is $C_h=h+rt$ ($h, r > 0$).
4. Shortages are allowed and are fully backlogged.
5. Replenishment is instantaneous.
6. Lead time is zero.
7. Delay in payment is allowed.
8. During time t_1, inventory is depleted due to deterioration and demand of the item at time t_1 in the inventory becomes zero and shortages start.

III. NOTATIONS

The mathematical model is developed using the following notations:

1. $\theta(t)$: Rate of deterioration per unit time.
2. p : Selling price per unit.
3. $D(t)$: Demand rate of the product.
4. C_p : Production cost per unit per unit time.
5. C_d : Deterioration cost per unit per unit time.
6. C_s : Shortage cost per unit per unit time.
7. A : Ordering cost per unit per time of an order.
8. T : The length of the cyclic period.
9. t_1 : The time at which inventory level reaches to zero.
10. Q : Order quantity per cycle.
11. I_e : Interest earned per rupee per unit time.
12. I_p : Interest charge per rupee per unit time($I_p > I_e$).
13. M : Permissible delay in settling the account.
14. TC_i : Total cost per unit time in the i^{th} case (i =1,2).

IV. MATHEMATICAL MODEL AND ANALYSIS.

The graphical representation of the mathematical model is shown in the Figure 1.

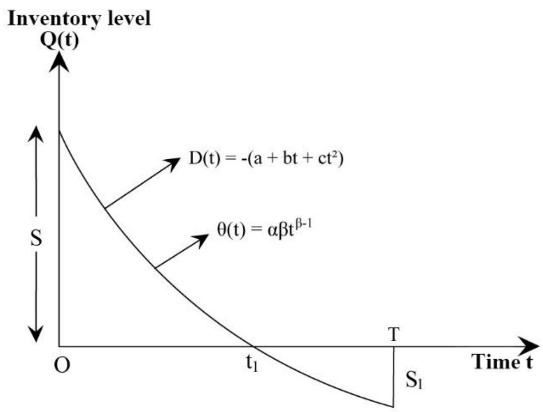


Figure 1: Graphical Representation of the Inventory System

The inventory model with the above described assumptions and notations is depicted in figure 1. The variation of inventory level *Q(t)* with respect to time t due to the combined effect of demand and deterioration is also shown. The Cycle starts with an inventory level of *S* units. At time t_1 inventory level becomes zero and shortages occur. During the period [0, T] the inventory level can be described by differential equations (1) and (2) pertaining to the situations as explained.

$$\frac{dQ(t)}{dt}+\alpha\beta t^{\beta-1}Q(t)=-\left(a+bt+ct^2\right); \quad (0\le t\le t_1) \qquad (1)$$

$$\frac{dQ(t)}{dt}=-\left(a+bt+ct^2\right); \qquad \left(t_1\le t\le T\right) \qquad (2)$$

The boundary conditions are $Q(0)$ = S, $Q(t_1) = 0$ and $Q(T) = -S_1$

Using the boundary condition $Q(0)$ = S the solution of equation (1) is

$$Q(t)=S\left(1-\alpha t^{\beta}\right)-\left[\left(at+\frac{bt^2}{2}+\frac{ct^3}{3}\right)+\alpha\beta\left(\frac{at^{\beta+1}}{\beta+1}+\frac{bt^{\beta+2}}{2(\beta+2)}+\frac{ct^{\beta+3}}{3(\beta+3)}\right)\right]; \quad (0\le t\le t_1) \qquad (3)$$

Using the boundary condition $Q(t_1) = 0$ in equation (3) we get

$$S=\frac{\left(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}\right)+\alpha\beta\left(\frac{at_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)}+\frac{ct_1^{\beta+3}}{3(\beta+3)}\right)}{1-\alpha t_1^{\beta}}; \quad (4)$$

Similarly, using the boundary condition $Q(t_1) = 0$ the solution of equation (2) is given by

$$Q(t)=-\left[a\left(t-t_1\right)+\frac{b}{2}\left(t^2-t_1^2\right)+\frac{c}{3}\left(t^3-t_1^3\right)\right]; \left(t_1\le t\le T\right) \qquad (5)$$

Substituting $Q(T) = -S_1$ in equation (5), we get

$$S_1=a\left(T-t_1\right)+\frac{b}{2}\left(T^2-t_1^2\right)+\frac{c}{3}\left(T^3-t_1^3\right); \qquad (6)$$

Order quantity Q per cycle is

$$Q=S+S_1$$

$$Q=\left\{\begin{array}{l}\dfrac{\left(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}\right)+\alpha\beta\left(\frac{at_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)}+\frac{ct_1^{\beta+3}}{3(\beta+3)}\right)}{1-\alpha t_1^{\beta}}\\ +\left[a(T-t_1)+\frac{b}{2}\left(T^2-t_1^2\right)+\frac{c}{3}(T^3-t_1^3)\right]\end{array}\right\}; \qquad (7)$$

V. COST COMPONENTS

On the basis of the assumptions and description of the model, the total cost TC consists of the following cost components:

1. Ordering Cost (OC)

The ordering cost (OC) over the period [0, T] is

$$OC=A \qquad (8)$$

2. Deterioration Cost (DC)

The deterioration cost due to deterioration over the period [0, T] is

$$DC=C_d\left[\int_0^{t_1}\alpha\beta t^{\beta-1}Q(t)\,dt\right]$$

$$DC=C_d\,\alpha\beta\left[\int_0^{t_1}t^{\beta-1}\left[S\left(1-\alpha t^{\beta}\right)-\left[\left(at+\frac{bt^2}{2}+\frac{ct^3}{3}\right)+\alpha\beta\left(\frac{at^{\beta+1}}{\beta+1}+\frac{bt^{\beta+2}}{2(\beta+2)}+\frac{ct^{\beta+3}}{3(\beta+3)}\right)\right]\right]dt\right]$$

Since α is very very small, ignoring higher power of α, we get

$$DC = C_d \alpha\beta \left[\frac{\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \alpha\beta\left(\frac{at_1^{\beta+1}}{\beta+1} + \frac{bt_1^{\beta+2}}{2\beta+4} + \frac{ct_1^{\beta+3}}{3\beta+9} \right) \right) t_1^{\beta}}{\left(1-\alpha t_1^{\beta}\right)\beta} - \frac{at_1^{\beta+1}}{\beta+1} - \frac{bt_1^{\beta+2}}{2(\beta+2)} - \frac{ct_1^{\beta+3}}{3(\beta+3)} \right]; \quad (9)$$

3. Inventory Holding Cost (IHC)

The inventory holding cost over the period [0, T] is

$$IHC = \int_0^{t_1} (h + rt) Q(t)\, dt$$

Using the expression of $Q(t)$ from (3) and S from (4), we get

$$IHC = \left\{ \begin{array}{l} h \left(\dfrac{\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \alpha\beta\left(\frac{at_1^{\beta+1}}{\beta+1} + \frac{bt_1^{\beta+2}}{2\beta+4} + \frac{ct_1^{\beta+3}}{3\beta+9} \right) \right)\left(t_1 - \frac{at_1^{\beta+1}}{\beta+1} \right)}{1-\alpha t_1^{\beta}} - \frac{at_1^2}{2} - \frac{bt_1^3}{6} - \frac{ct_1^4}{12} \right. \\ \left. \quad -\alpha\beta\left(\frac{at_1^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{bt_1^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{ct_1^{\beta+4}}{3(\beta+3)(\beta+4)} \right) \right) \\ +r \left(\dfrac{\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \alpha\beta\left(\frac{at_1^{\beta+1}}{\beta+1} + \frac{bt_1^{\beta+2}}{2\beta+4} + \frac{ct_1^{\beta+3}}{3\beta+9} \right) \right)\left(\frac{t_1^2}{2} - \frac{a t_1^{\beta+2}}{\beta+2} \right)}{1-\alpha t_1^{\beta}} - \frac{at_1^3}{3} - \frac{bt_1^4}{8} - \frac{ct_1^5}{15} \right. \\ \left. \quad -\alpha\beta\left(\frac{at_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{bt_1^{\beta+4}}{2(\beta+2)(\beta+4)} + \frac{ct_1^{\beta+5}}{(\beta+3)(\beta+5)} \right) \right) \end{array} \right\}; \quad (10)$$

4. Shortage Cost (SC)

The shortage cost over the period [0, T] is

$$SC = -C_S \int_{t_1}^{T} Q(t)\, dt$$

Using the expression of $Q(t)$ from (5), we get

$$SC = C_s \left(\frac{a\left(T^2 - t_1^2\right)}{2} + \frac{b\left(T^3 - t_1^3\right)}{6} + \frac{c\left(T^4 - t_1^4\right)}{12} - \left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} \right)\left(T - t_1\right) \right); \quad (11)$$

5. Production Cost (PC)

Substituting the values of S and S_1 from equation (4) and (6) the production cost over the period [0, T] is

$$PC = C_p \left[S + S_1 \right]$$

$$PC = C_p \left[\frac{\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \alpha\beta\left(\frac{at_1^{\beta+1}}{\beta+1} + \frac{bt_1^{\beta+2}}{2\beta+4} + \frac{ct_1^{\beta+3}}{3\beta+9} \right) \right)}{1-\alpha t_1^{\beta}} + a\left(T - t_1\right) + \frac{b\left(T^2 - t_1^2\right)}{2} + \frac{c\left(T^3 - t_1^3\right)}{3} \right]; \quad (12)$$

Now, there are two distinct cases for the period M of permissible delay in Payments.

Case I: $M \le t_1$ (Payment at or before total depletion of inventory)

The graphical representation for case I is shown in the Figure 2

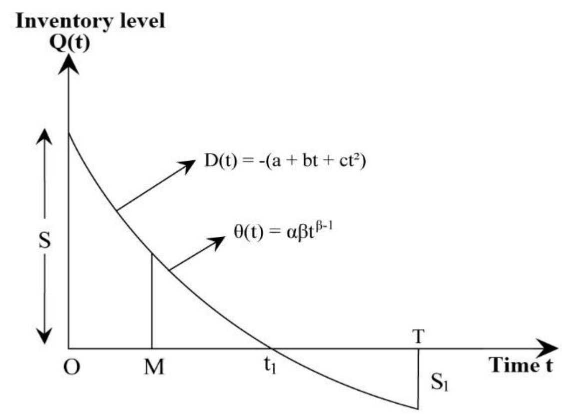


Figure 2: Graphical Representation of the Inventory System when $M \le t_1$

In this case, the credit time expires on or before the inventory depleted completely to zero. The interest payable per cycle for the inventory not being sold after the due date M is interest payable in the time horizon when $M < t \le t_1$

Interest Payable is given by

$$IP_1 = C_p I_p \int_M^{t_1} Q(t)\, dt$$

$$IP_1 = C_p I_p \left(\frac{\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \alpha\beta\left(\frac{at_1^{\beta+1}}{\beta+1} + \frac{bt_1^{\beta+2}}{2\beta+4} + \frac{ct_1^{\beta+3}}{3\beta+9} \right) \right)\left(t_1 - M - \frac{\alpha\left(t_1^{\beta+1} - M^{\beta+1}\right)}{\beta+1} \right)}{1-\alpha t_1^{\beta}} - \frac{a\left(-M^2 + t_1^2\right)}{2} - \frac{b\left(-M^3 + t_1^3\right)}{6} - \frac{c\left(-M^4 + t_1^4\right)}{12} - \alpha\beta\left(\frac{a\left(t_1^{\beta+2} - M^{\beta+2}\right)}{(\beta+1)(\beta+2)} + \frac{b\left(t_1^{\beta+3} - M^{\beta+3}\right)}{2(\beta+2)(\beta+3)} + \frac{c\left(t_1^{\beta+4} - M^{\beta+4}\right)}{3(\beta+3)(\beta+4)} \right) \right); \quad (13)$$

Interest earned per cycle IE_1 is the interest earned during the positive inventory level and is given by

$$IE_1 = pI_e \int_0^M \left(a+bt+ct^2\right)t\,dt$$

$$IE_1 = pI_e\left(\frac{aM^2}{2}+\frac{bM^3}{3}+\frac{cM^4}{4}\right); \qquad (14)$$

Total inventory cost per unit time is given by

$$TC_1 = \frac{1}{T}\left[OC+SC+IHC+IP_1-IE_1+PC+DC\right]$$

$$TC_1=\frac{1}{T}\left[\begin{array}{l}
A+C_S\left(\begin{array}{l}\dfrac{a\left(T^2-t_1^2\right)}{2}+\dfrac{b\left(T^3-t_1^3\right)}{6}+\dfrac{c\left(T^4-t_1^4\right)}{12}\\ -\left(at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}\right)\left(T-t_1\right)\end{array}\right)+\\
h\left(\begin{array}{l}\dfrac{\left(\begin{array}{l}at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}\\ +\alpha\beta\left(\dfrac{at_1^{\beta+1}}{\beta+1}+\dfrac{bt_1^{\beta+2}}{2\beta+4}+\dfrac{ct_1^{\beta+3}}{3\beta+9}\right)\end{array}\right)\left(t_1-\dfrac{at_1^{\beta+1}}{\beta+1}\right)}{1-\alpha t_1^{\beta}}\\ -\dfrac{at_1^2}{2}-\dfrac{bt_1^3}{6}-\dfrac{ct_1^4}{12}-\alpha\beta\left(\dfrac{at_1^{\beta+2}}{(\beta+1)(\beta+2)}+\dfrac{bt_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\dfrac{ct_1^{\beta+4}}{3(\beta+3)(\beta+4)}\right)\end{array}\right)\\
+r\left(\begin{array}{l}\dfrac{\left(at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}+\alpha\beta\left(\dfrac{at_1^{\beta+1}}{\beta+1}+\dfrac{bt_1^{\beta+2}}{2\beta+4}+\dfrac{ct_1^{\beta+3}}{3\beta+9}\right)\right)\left(\dfrac{t_1^2}{2}-\dfrac{at_1^{\beta+2}}{\beta+2}\right)}{1-\alpha t_1^{\beta}}\\ -\dfrac{at_1^3}{3}-\dfrac{bt_1^4}{8}-\dfrac{ct_1^5}{15}-\alpha\beta\left(\dfrac{at_1^{\beta+3}}{(\beta+1)(\beta+3)}+\dfrac{bt_1^{\beta+4}}{2(\beta+2)(\beta+4)}+\dfrac{ct_1^{\beta+5}}{(\beta+3)(\beta+5)}\right)\end{array}\right)\\
+C_pI_p\left(\begin{array}{l}\dfrac{\left(at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}+\alpha\beta\left(\dfrac{at_1^{\beta+1}}{\beta+1}+\dfrac{bt_1^{\beta+2}}{2\beta+4}+\dfrac{ct_1^{\beta+3}}{3\beta+9}\right)\right)\left(t_1-M-\dfrac{\alpha\left(t_1^{\beta+1}-M^{\beta+1}\right)}{\beta+1}\right)}{1-\alpha t_1^{\beta}}\\ -\dfrac{a\left(-M^2+t_1^2\right)}{2}-\dfrac{b\left(-M^3+t_1^3\right)}{6}-\dfrac{c\left(-M^4+t_1^4\right)}{12}\\ -\alpha\beta\left(\dfrac{a\left(t_1^{\beta+2}-M^{\beta+2}\right)}{(\beta+1)(\beta+2)}+\dfrac{b\left(t_1^{\beta+3}-M^{\beta+3}\right)}{2(\beta+2)(\beta+3)}+\dfrac{c\left(t_1^{\beta+4}-M^{\beta+4}\right)}{3(\beta+3)(\beta+4)}\right)\end{array}\right)\\
-pI_e\left(\dfrac{aM^2}{2}+\dfrac{bM^3}{3}+\dfrac{cM^4}{4}\right)\\
+C_p\left[\begin{array}{l}\dfrac{\left(at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}+\alpha\beta\left(\dfrac{at_1^{\beta+1}}{\beta+1}+\dfrac{bt_1^{\beta+2}}{2\beta+4}+\dfrac{ct_1^{\beta+3}}{3\beta+9}\right)\right)}{1-\alpha t_1^{\beta}}\\ +a\left(T-t_1\right)+\dfrac{b\left(T^2-t_1^2\right)}{2}+\dfrac{c\left(T^3-t_1^3\right)}{3}\end{array}\right]\\
+C_d\alpha\beta\left[\begin{array}{l}\dfrac{\left(at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}+\alpha\beta\left(\dfrac{at_1^{\beta+1}}{\beta+1}+\dfrac{bt_1^{\beta+2}}{2\beta+4}+\dfrac{ct_1^{\beta+3}}{3\beta+9}\right)\right)t_1^{\beta}}{\left(1-\alpha t_1^{\beta}\right)\beta}\\ -\dfrac{at_1^{\beta+1}}{\beta+1}-\dfrac{bt_1^{\beta+2}}{2(\beta+2)}-\dfrac{ct_1^{\beta+3}}{3(\beta+3)}\end{array}\right]
\end{array}\right];$$

(15)

Case II: $t_1 < M < T$ (Payment at or after depletion of Inventory)

The graphical representation for case II is shown in the Figure 3

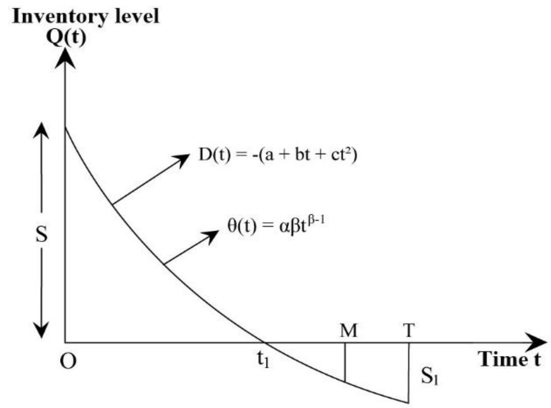


Figure 3: Graphical Representation of the Inventory System when $t_1 < M < T$

In this case, the interest payable per cycle is Zero i.e., $IP_2=0$ because the supplier can be paid in full time M, the permissible delay therefore, the interest earned per cycle is the interest earned during the positive inventory period plus the interest earned from the case invested during the given time period after the inventory exhausted at time t_1, and interest earned is given by

$$IE_2 = pI_e\left[\int_0^{t_1} t\left(a+bt+ct^2\right)dt+\left(M-t_1\right)\int_0^{t_1}\left(a+bt+ct^2\right)dt\right]$$

$$IE_2 = pI_e\left[\left(M\left(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}\right)-\frac{at_1^2}{2}-\frac{bt_1^3}{6}-\frac{ct_1^4}{12}\right)\right]; \quad (16)$$

Total inventory cost per unit time is given by

$$TC_2 = \frac{1}{T}\left[OC+SC+IHC+IP_2-IE_2+PC+DC\right]$$

$$TC_2=\frac{1}{T}\left[\begin{array}{l}
A+C_s\left(\begin{array}{l}\dfrac{a\left(T^2-t_1^2\right)}{2}+\dfrac{b\left(T^3-t_1^3\right)}{6}+\dfrac{c\left(T^4-t_1^4\right)}{12}\\ -\left(at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}\right)\left(T-t_1\right)\end{array}\right)\\
+h\left(\begin{array}{l}\dfrac{\left(at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}+\alpha\beta\left(\dfrac{at_1^{\beta+1}}{\beta+1}+\dfrac{bt_1^{\beta+2}}{2\beta+4}+\dfrac{ct_1^{\beta+3}}{3\beta+9}\right)\right)\left(t_1-\dfrac{at_1^{\beta+1}}{\beta+1}\right)}{1-\alpha t_1^{\beta}}\\ -\dfrac{at_1^2}{2}-\dfrac{bt_1^3}{6}-\dfrac{ct_1^4}{12}-\alpha\beta\left(\dfrac{at_1^{\beta+2}}{(\beta+1)(\beta+2)}+\dfrac{bt_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\dfrac{ct_1^{\beta+4}}{3(\beta+3)(\beta+4)}\right)\end{array}\right)\\
+r\left(\begin{array}{l}\dfrac{\left(at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}+\alpha\beta\left(\dfrac{at_1^{\beta+1}}{\beta+1}+\dfrac{bt_1^{\beta+2}}{2\beta+4}+\dfrac{ct_1^{\beta+3}}{3\beta+9}\right)\right)\left(\dfrac{t_1^2}{2}-\dfrac{at_1^{\beta+2}}{\beta+2}\right)}{1-\alpha t_1^{\beta}}\\ -\dfrac{at_1^3}{3}-\dfrac{bt_1^4}{8}-\dfrac{ct_1^5}{15}-\alpha\beta\left(\dfrac{at_1^{\beta+3}}{(\beta+1)(\beta+3)}+\dfrac{bt_1^{\beta+4}}{2(\beta+2)(\beta+4)}+\dfrac{ct_1^{\beta+5}}{(\beta+3)(\beta+5)}\right)\end{array}\right)\\
-pI_e\left(M\left(at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}\right)-\dfrac{at_1^2}{2}-\dfrac{bt_1^3}{6}-\dfrac{ct_1^4}{12}\right)\\
+C_p\left[\begin{array}{l}\dfrac{\left(at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}+\alpha\beta\left(\dfrac{at_1^{\beta+1}}{\beta+1}+\dfrac{bt_1^{\beta+2}}{2\beta+4}+\dfrac{ct_1^{\beta+3}}{3\beta+9}\right)\right)}{1-\alpha t_1^{\beta}}\\ +a\left(T-t_1\right)+\dfrac{b\left(T^2-t_1^2\right)}{2}+\dfrac{c\left(T^3-t_1^3\right)}{3}\end{array}\right]\\
+C_d\alpha\beta\left[\begin{array}{l}\dfrac{\left(at_1+\dfrac{bt_1^2}{2}+\dfrac{ct_1^3}{3}+\alpha\beta\left(\dfrac{at_1^{\beta+1}}{\beta+1}+\dfrac{bt_1^{\beta+2}}{2\beta+4}+\dfrac{ct_1^{\beta+3}}{3\beta+9}\right)\right)t_1^{\beta}}{\left(1-\alpha t_1^{\beta}\right)\beta}\\ -\dfrac{at_1^{\beta+1}}{\beta+1}-\dfrac{bt_1^{\beta+2}}{2(\beta+2)}-\dfrac{ct_1^{\beta+3}}{3(\beta+3)}\end{array}\right]
\end{array}\right]; \tag{17}$$

The average total cost of the system for the is given by

$$TC=\begin{cases}TC_1 & ;M\le t_1\\ TC_2 & ;t_1<M\le T\end{cases} \tag{18}$$

We consider two different cases of the average total cost for a different permissible in payments. TC_1 and TC_2 can be calculated by substituting the values of equations (13) (14) and (16) in equations (15) and (17) respectively and hence average total cost TC is estimated. Hence, TC remains a function of T and t_1 only, which are decision variables. T^* and t_1^* are the optimum values of T and t_1 respectively, which minimize the cost function TC_i (where i=1, 2) and they are the solution of equations.

$$\frac{\partial TC_i}{\partial t_1}=0 \quad and \quad \frac{\partial TC_i}{\partial T}=0 \quad (where\ i=1,\ 2) \tag{19}$$

Such that

$$\left.\frac{\partial^2 TC}{\partial t_1^2}\right|_{t_1=t_1^*,T=T^*}>0 \qquad \& \qquad \left.\begin{vmatrix}\dfrac{\partial^2 TC}{\partial t_1^2} & \dfrac{\partial^2 TC}{\partial t_1\partial T}\\ \dfrac{\partial^2 TC}{\partial t_1\partial T} & \dfrac{\partial^2 TC}{\partial T^2}\end{vmatrix}\right|_{t_1=t_1^*,T=T^*}>0 \tag{20}$$

The optimum values t_1^* and T^* can be obtained by using appropriate software.
The above – developed model is illustrated by means of the following numerical example for each case.

A numerical example for all the cases is calculated followed by its sensitivity analysis. It consists of determining all the cost components for a particular set of parameters for each different case to illustrate the model.

VI. NUMERICAL EXAMPLE

Example I: To illustrate the above inventory model, the following parametric values are assumed: a=5, b=12, c=1.2, A=2500, p=2, h=0.9, r=0.7, α=0.0001, β=4.5, C_d=4, C_p=8, C_s=12, I_p=10, I_e=6 and M=0.4 (With appropriate units of measurement)
We obtain the optimal values t_1^*= 0.7368629627 units, T^*= 3.054230960 units, PC=661.1022131 units, IHC = 3.303014043 units, DC = 0.0001757930339 units, SC=844.3071131 units and optimal total cost per unit time TC_1 = 1329.181795 units.

Example II: To illustrate the above inventory model, the following parametric values are assumed: a=125, b=12, c=1.2, A=100, p=2, h=0.9, r=0.7, α=0.0001, β=1.2, C_d=10, C_p=8, C_s=8, I_p=12, I_e=4, and M=0.4 (With appropriate units of measurement)
We obtain the optimal value of t_1^*=0.3976257383 units, T^*=0.4393722862 units, PC=448.9306876 units, IHC=10.07105267 units, DC=0.007673018248 units, SC=0.907231448 units and optimal total cost per unit time TC_2 = 1087.527003 units.

VII. SENSITIVITY ANALYSIS

Sensitivity Analysis works on the principle of **"Ceteris Paribus"** i.e. Observing the change in the value of total cost with respect to one parameter keeping the rest of the parameters constant. It is a technique used to determine how different values of independent variables impact a particular dependent variable and the magnitude of impact under a given set of assumptions.

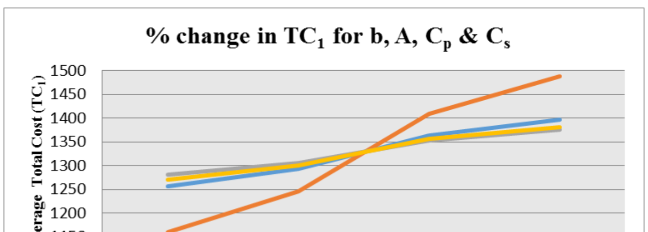
Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. In this section, we study the sensitivity of total cost TC per time unit for all two cases of total cost with respect to the changes in the values of the parameters a, b, c, A, p, h, r, α, β, C_d, C_p, C_s, M, I_p and I_e.

This analysis is performed by considering 10% and 20% increase and a decrease in the values of each one of the above parameters keeping all other remaining parameters fixed. The results are presented in the form of a table.

Graphical Presentation of the Sensitivity Analysis is required to facilitate the process of drawing a conclusion regarding the magnitude of influence each parameter individually holds on the average total cost. It is especially valuable in presenting data in a simple, clear and effective manner as well as making trends easily recognizable.

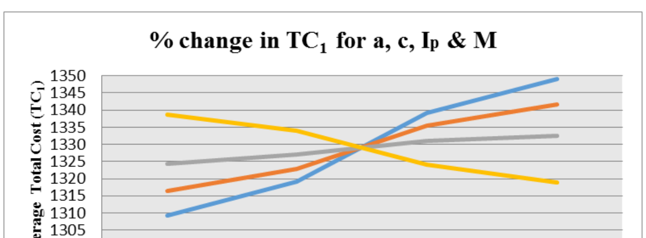
VIII. GRAPHICAL REPRESENTATION OF VARIOUS PARAMETERS

Graphical representation of the various parameters is shown in the following figures:



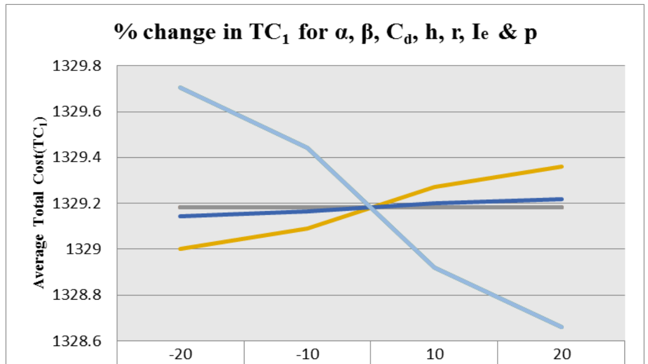
	-20	-10	10	20
b	1256.437058	1293.604939	1363.332146	1396.19616
A	1159.143293	1245.826198	1409.66719	1487.640787
Cp	1280.60186	1305.279831	1352.470426	1375.257167
Cs	1269.893625	1300.585256	1355.953063	1381.116274

Figure 4: Graphical Representation of b, A, C_p & C_s



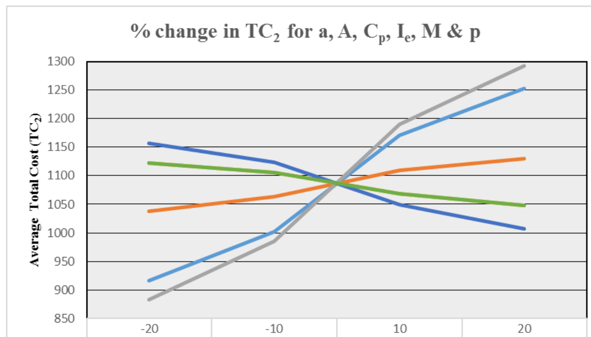
	-20	-10	10	20
a	1309.309659	1319.259383	1339.076522	1348.944311
c	1316.317094	1322.807638	1335.444521	1341.601468
Ip	1324.403759	1327.047764	1330.939462	1332.411667
M	1338.672798	1334.024982	1324.141437	1318.902106

Figure 5: Graphical Representation of a, c, I_p & M



	-20	-10	10	20
α	1329.181094	1329.181351	1329.181865	1329.182122
β	1329.182391	1329.182089	1329.181518	1329.181259
Cd	1329.181784	1329.181789	1329.181801	1329.181807
h	1329.001806	1329.091913	1329.271454	1329.360889
r	1329.145007	1329.16341	1329.200198	1329.218509
Ie	1329.703254	1329.442539	1328.921023	1328.660235
p	1329.703254	1329.442539	1328.921023	1328.660221

Figure 6: Graphical Representation of α, β, C_d, h, r, I_e & p

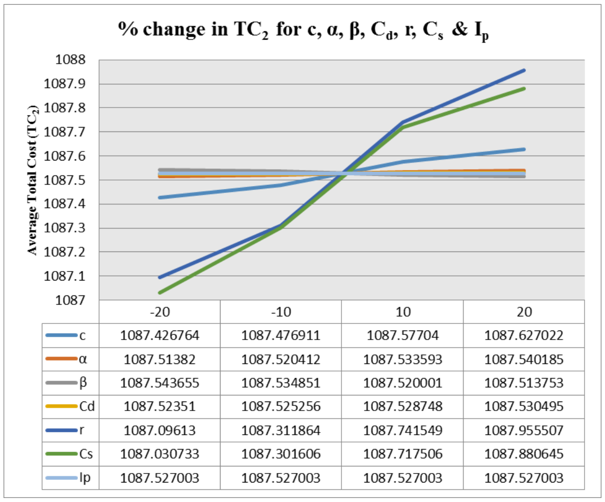


	-20	-10	10	20
a	916.130305	1002.572414	1171.047002	1253.162945
A	1037.676454	1063.787003	1109.453465	1129.934371
Cp	883.1408639	985.3428065	1189.69388	1291.843845
Ie	1122.714025	1105.65196	1068.248323	1047.6749530
M	1156.267784	1123.102266	1049.112867	1007.206089
p	1122.714025	1105.65196	1068.248323	1047.6749530

Figure 7: Graphical Representation of a, A, C_p, I_e, M & p

	-20	-10	10	20
b	1084.062658	1085.804774	1089.22987	1090.923228
h	1083.326872	1085.438381	1089.593110	1091.6370660

Figure 8: Graphical Representation of b & h



	-20	-10	10	20
c	1087.426764	1087.476911	1087.57704	1087.627022
α	1087.51382	1087.520412	1087.533593	1087.540185
β	1087.543655	1087.534851	1087.520001	1087.513753
Cd	1087.52351	1087.525256	1087.528748	1087.530495
r	1087.09613	1087.311864	1087.741549	1087.955507
Cs	1087.030733	1087.301606	1087.717506	1087.880645
Ip	1087.527003	1087.527003	1087.527003	1087.527003

Figure 9: Graphical Representation of c, α, β, Cd, r, C

IX. CONCLUSIONS

From **Figure – 4**, **5** and **6,** it is observed that the total cost per unit is highly sensitive to changes in the values of b, A, C_p, C_s and moderately sensitive to changes in the values of a, c, I_p, M and less sensitive to changes in the values of α, β, C_d, h, r, I_e, p.

From the **Figure – 7, 8** and **9,** it is observed that the total cost per time is highly sensitive to changes in the values of a, A, C_p, I_e, M, p and moderately sensitive to changes in the values of b, h and less sensitive to changes in the values of c, α, β, C_d, r, C_s, I_p.

From **Table – 1**, it is observed that as the values of the parameters a, b, c, α, A, C_p, C_d, h, r, C_s, I_p increase the average total cost also increases and for the values of the parameters β, I_e, M and p decrease the average total cost also decreases.

From **Tale – 2,** it is observed that as the values of the parameters a, b, c, α, A, C_p, C_d, h, r, C_s, I_p increase the average total cost also increases and for the values of the parameters β, I_e, M and p decrease the average total cost also decreases.

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Sensitivity Analysis

Table – 1: Partial Sensitivity Analysis Based on Numerical Example – 1

Parameters	**% Change**	t_1	T	TC_1
a	-20	0.739564422	3.075239335	1309.309659
	-10	0.738210114	3.064699313	1319.259383
	10	0.735529364	3.043834536	1339.076522
	20	0.734202787	3.033508876	1348.944311
b	-20	0.758072800	3.219453674	1256.437058
	-10	0.746978282	3.133013821	1293.604939
	10	0.727595857	2.982031815	1363.332146
	20	0.719058273	2.915539038	1396.196160
c	-20	0.742471702	3.097903808	1316.317094
	-10	0.739615702	3.075656578	1322.807638
	10	0.734212761	3.033571618	1335.444521
	20	0.731652157	3.013627318	1341.601468
α	-20	0.736878169	3.054233424	1329.181094
	-10	0.736878169	3.054232311	1329.181351
	10	0.736858592	3.054230084	1329.181865
	20	0.736852067	3.054228972	1329.182122
β	-20	0.736857949	3.05423061	1329.182391
	-10	0.736860124	3.054230705	1329.182089
	10	0.736866252	3.054231333	1329.181518
	20	0.736869824	3.054231792	1329.181259
A	-20	0.707280087	2.823851873	1159.143293
	-10	0.722644729	2.943496149	1245.826198
	10	0.750121383	3.157506128	1409.66719
	20	0.762562485	3.254428859	1487.640787
C_p	-20	0.816609665	3.119987400	1280.60186
	-10	0.772865631	3.086063492	1305.279831
	10	0.706725378	3.024054595	1352.470426
	20	0.681138022	2.995239937	1375.257167
C_d	-20	0.736863184	3.054230994	1329.181784
	-10	0.736863073	3.054230977	1329.181789
	10	0.736862852	3.054230943	1329.181801
	20	0.736862742	3.054230926	1329.181807
h	-20	0.738296345	3.054319050	1329.001806
	-10	0.737578957	3.054274940	1329.091913
	10	0.736148358	3.054187109	1329.271454
	20	0.735435140	3.054143386	1329.360889
r	-20	0.737275837	3.054275377	1329.145007
	-10	0.737069283	3.05425315	1329.16341
	10	0.736656874	3.054208806	1329.200198
	20	0.736451016	3.054186688	1329.218509
C_s	-20	0.693793920	3.225115524	1269.893625
	-10	0.716095832	3.134319990	1300.585256
	10	0.756285065	2.982864769	1355.953063
	20	0.774516419	2.918727163	1381.116274
I_p	-20	0.807996850	3.064374343	1324.403759

	-10	0.769026176	3.058722847	1327.047764
	10	0.709864409	3.050580172	1330.939462
	20	0.686877546	3.047557697	1332.411667
I_e	-20	0.736950303	3.054911242	1329.703254
	-10	0.736906638	3.054571139	1329.442539
	10	0.736819278	3.053890706	1328.921023
	20	0.736775583	3.053550376	1328.660235
M	-20	0.668014920	3.051575712	1338.672798
	-10	0.702444421	3.052905264	1334.024982
	10	0.771270268	3.055551707	1324.141437
	20	0.805666057	3.056866409	1318.902106
p	-20	0.736950303	3.054911242	1329.703254
	-10	0.736906638	3.054571139	1329.442539
	10	0.736819278	3.053890706	1328.921023
	20	0.736775583	3.053550376	1328.660221

Table 1: **Partial Sensitivity Analysis Based on Numerical Example – 1**

Sensitivity Analysis				
Table – 2: Partial Sensitivity Analysis Based on Numerical Example – 2				
Parameters	**% Change**	t_1	T	TC_2
a	-20	0.431025176	0.507657533	916.1303050
	-10	0.413431066	0.471819539	1002.572414
	10	0.383240662	0.409661532	1171.047002
	20	0.369996586	0.382176241	1253.162945
b	-20	0.40040288	0.446662183	1084.062658
	-10	0.398995095	0.442969247	1085.804774
	10	0.396293048	0.435866942	1089.229870
	20	0.394995375	0.432449137	1090.923228
c	-20	0.397769514	0.439750767	1087.426764
	-10	0.397697535	0.43956131	1087.476911
	10	0.397554122	0.439183695	1087.57704
	20	0.397482686	0.438995533	1087.627022
α	-20	0.397642316	0.439375816	1087.51382
	-10	0.397634027	0.439374051	1087.520412
	10	0.39761745	0.439370522	1087.533593
	20	0.397609161	0.439368757	1087.540185
β	-20	0.397622505	0.439383859	1087.543655
	-10	0.397623774	0.439377342	1087.534851
	10	0.397628235	0.43936845	1087.520001
	20	0.397631127	0.439365627	1087.513753
A	-20	0.361357634	0.362878791	1037.676454
	-10	0.380405400	0.403054504	1063.787003
	10	0.41344845	0.472739506	1109.453465
	20	0.428157445	0.50375554	1129.934371
C_p	-20	0.400946009	0.446351971	883.1408639
	-10	0.399265977	0.44282024	985.3428065
	10	0.396023734	0.436004828	1189.69388
	20	0.394458488	0.432714759	1291.843845
C_d	-20	0.397630081	0.439373176	1087.52351
	-10	0.39762791	0.439372731	1087.525256
	10	0.397623567	0.439371841	1087.528748
	20	0.397621396	0.439371396	1087.530495
h	-20	0.402105632	0.439771601	1083.326872
	-10	0.399852689	0.439569915	1085.438381
	10	0.39542431	0.439178597	1089.593110
	20	0.393247946	0.438988733	1091.6370660
r	-20	0.398468428	0.439760011	1087.0961300
	-10	0.39804607	0.439565485	1087.311864
	10	0.397207415	0.439180401	1087.741549
	20	0.39679108	0.438989816	1087.955507
C_s	-20	0.396486082	0.44708954	1087.030733

	-10	0.397109825	0.442859209	1087.301606
	10	0.398059607	0.436448395	1087.717506
	20	0.39842959	0.433961207	1087.880645
I_p	-20	0.397625738	0.439372286	1087.527003
	-10	0.397625738	0.439372286	1087.527003
	10	0.397625738	0.439372286	1087.527003
	20	0.397625738	0.439372286	1087.527003
I_e	-20	0.425793932	0.495377228	1122.714025
	-10	0.410936892	0.467170842	1105.65196
	10	0.385458855	0.411536418	1068.248323
	20	0.374110603	0.38324008	1047.6749530
M	-20	0.380811193	0.483910361	1156.267784
	-10	0.390104887	0.463511184	1123.102266
	10	0.403033913	0.410777463	1049.112867
	20	0.405813407	0.376638945	1007.206089
p	-20	0.4257939317	0.495377228	1122.714025
	-10	0.410936892	0.467170842	1105.65196
	10	0.385458855	0.411536418	1068.248323
	20	0.374110603	0.383240078	1047.6749530

Table 2: **Partial Sensitivity Analysis Based on Numerical Example – 2**

Authors' Biography

Santosh Rohida is a Research scholar in Statistics department at Gujarat University. his research interests in Formulation of Inventory Systems for Deteriorating Items. He has received M.Phil. M.Sc. and B.Sc. in Statistics from Gujarat University, Ahmedabad, Gujarat, India.

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