

Certain Structures of Q-Fuzzy Soft Ideals of Near-Ring

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Available online at: www.isroset.org

Received: 19/Jul/2019, Accepted: 16/Aug/2019, Online: 31/Aug/2019

Abstract: In this paper, we introduce and study Q-fuzzy soft sub near-ring and Q-fuzzy soft ideals of near-rings by Molodtsov's definition of the soft set. Some related properties are investigated and illustrated by a great ideal of example.

Keywords: Near-ring, soft set, Q-fuzzy set, Q-fuzzy soft set, Q-fuzzy soft sub near-ring, Q-fuzzy soft ideal.

I. INTRODUCTION

The concept of gamma in algebra was introduced and studied first by N.Nobusawa [7] in 1964 and further established Γ -ring. Infact, there have been a few slightly different definition on a Γ -ring. In 1995, M.K.Rao [6] introduced the notion Γ -semi ring as a generalization of Γ -ring as well as semi ring and studied the concepts of Γ -semi ring and its sub Γ -semi ring with a left(right) unity. Later on much has been developed and this concepts by different researchers. Fuzzy sets introduced by L.A.Zadeh[9] and there after several researchers developed algebraic structures and applied it on different branches of pure and applied mathematics. Further on Γ -semi rings, the properties of fuzzy ideals, fuzzy prime ideals, fuzzy semi prime ideal and their generalizations play an important role in their structure theory. However the properties of a fuzzy ideal in semi rings and Γ -semi rings are some what different from the properties of the usual ring ideals. In 1992, Jun and Lee [4] introduced the notion of fuzzy ideal in Γ -ring and studied few properties. In 2005, Dutta and Chanda[2] studied the structures of fuzzy ideals in Γ -ring via operation rings of Γ -ring.

II. BASIC DEFINITIONS AND RESULTS

In this section, some basic definitions and results on soft sets with suitable examples, much of which were introduced in (Molodtsov 1999, Maji et. al 2003, Pie and Miao 2005, Ali et. al 2009).

Definition 2.1. Let U be a common universe, E be a set of

parameters and $A \subseteq E$. Then a pair f_A is called a fuzzy soft set over U , where F is a mapping given by $F : A \rightarrow F(U)$, when $F(U)$ is the family of all fuzzy subsets of U .

Definition 2.2. For two fuzzy soft sets f_A and g_B over a common universe U , we say that f_A is a fuzzy soft subset of g_B if

(i) $A \subseteq B$

(ii) $f_a \leq g_b$ for all $a \in A$, In this case, we write $f_A \subseteq g_B$.

Definition 2.3. The relative complement of a fuzzy soft set f_A is denoted⁴ by $f_A^c : A \rightarrow f(U)$ is a mapping given by $f^c(a) = 1 - f(a)$ for all $a \in A$.

Definition 2.4. (i) A fuzzy soft set f_A is said to be the absolute fuzzy soft set over U , denoted by ∇ if $f(a) = LU$ for all $a \in A$.

(ii) A fuzzy soft set f_A is said to be the null fuzzy soft set over U , denoted by π , if $f(a) = OU$ for all $a \in A$.

Definition 2.5. A Q-fuzzy set v in a near-ring N is called Q-fuzzy sub near ring of N , if

(i) $v(x \rightarrow y, q) \geq \min\{v(x, q), v(y, q)\}$

(ii) $v(xy, q) = \min\{v(x, q), v(y, q)\}$, for all $x, y \in N$ and $q \in Q$.

Definition 2.6. (Molodtsov 1999)

Let U be a initial universe set and E be a set of parameters with respect to U . Let $P(U)$ denote the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$.

Example 2.7. Suppose a universe U is the set of six

houses under construction given by $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, the parameters set $E = \{e_1, e_2, e_3, e_4, e_5\}$ where each parameter $e_i, i = 1, 2, 3, 4, 5$ stands for expensive, beautiful, cheap, modern, wooden respectively, and $A = \{e_1, e_2, e_3\} \subset E$. Now consider the mapping F , where $F : A \rightarrow P(U)$ is given by $F(e_1) = \{h_2, h_4\}$, $F(e_2) = \{h_1, h_3, h_5\}$, $F(e_3) = \{h_1, h_3, h_6\}$. Then the soft set (F, A) is a parameterized family $\{F(e_i), i = 1, 2, 3\}$ of subsets of the universe U given by $(F, A) = \{\{h_2, h_4\}, \{h_1, h_3, h_5\}, \{h_1, h_3, h_6\}\}$.

Table 1

$z U$	1	1	1	Choice
h_1	0	1	1	2
h_2	1	0	0	1
h_3	0	1	1	2
h_4	1	0	0	1
h_5	0	1	0	1
h_6	0	0	1	1

Tabular representation of (F, A)

Definition 2.8. Soft subset defined (Pie and Miao 2005), for two soft sets (F, A) and (G, B) over a universe U , $(F, A) \subseteq (G, B)$ if, (i) $A \subseteq B$, (ii) $\forall e \in A, F(e) \subseteq G(e)$.

Example 2.9. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a universe set and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters. Let $A = \{e_1, e_2, e_3\} \subseteq E$ and $B = \{e_1, e_2, e_3, e_5\} \subset E$.

Suppose (F, A) and (G, B) are two soft sets over U , where $F(e_1) = \{u_2, u_4\}$, $F(e_2) = \{u_1, u_4, u_5\}$, $F(e_3) = \{u_1\}$ and $G(e_1) = \{u_2, u_4\}$, $G(e_2) = \{u_1, u_3, u_4, u_5\}$, $G(e_3) = \{u_3, u_4, u_5\}$, then $(F, A) \subseteq (G, B)$, since $A \subset B$ and $F(e) \subset G(e), \forall e \in A$, but $(G, B) \not\subseteq (F, A)$. Hence $(F, A) \not\subseteq (G, B)$.

Definition 2.10. (Feng, F et.al). The bi-intersection of two soft sets (F, A) and (G, B) over a common universe U is defined to be the soft set (H, C) , where $C = A \cap B$ and $H : C \rightarrow P(U)$ is mapping given by $H(x) = F(x) \cap G(x)$ for all $x \in C$. This is denoted by $(F, A) \cap (G, B) = (H, C)$ and also Cartesian product is defined as $(F, A) \times (G, B) = (H, C)$.

By a near-ring, we shall mean on algebraic system $(N, +, \cdot)$, where

- (i) $(N, +)$ forms a group (not necessary abelian)
- (ii) (N, \cdot) forms a semi group and
- (iii) $(a + b)c = ac + bc \forall a, b, c \in N$ (i.e we study on right near-rings)

Throughout this paper, N will always denoted a right near-ring. A subgroup H of N with $M, M \subseteq M$ is called a sub near-ring of N . A normal subgroup I of N is called a right ideal if $IN \subseteq I$ denoted by $IV_r N$. It is

called a left ideal if $n(s+i) - ns \in I \forall n, s \in N$ and $i \in I$ and denoted by $IV_l N$. If such a normal subgroup I is both left and right ideal in N . A subgroup H of N is called a left N -subgroup of N if $NH \subseteq H$ and H is called a right N -subgroup of N if $HN \subseteq H$.

III. Q-FUZZY SOFT SUB NORMAL RINGS

In the sequel, let N be a near-ring and A be non-empty set. α will refer to an arbitrary binary relation between an element of A and an element of N , that is α is a subset of $A \times N$ without otherwise specified. A set valued function $F : A \rightarrow P(N)$ can be defined as $F(x) = \{y \in N / (x, y) \in \alpha\} \forall x \in A$. Then the pair (F, A) is a soft set over N , which is derived from the relation α .

Definition 3.1. Let M be a sub near-ring of N and let (F, M) be a Q-fuzzy soft set over N . If for all $x, y \in M$

- (QFSNR - 1): $F(x \rightarrow y, q) \geq F(x, q) \cap F(y, q)$ and
- (QFSNR - 2): $F(xy, q) \geq F(x, q) \cap F(y, q)$. Then the Q-fuzzy soft set (F, M) is called a Q-fuzzy soft sub near-ring of N and denoted by $O(F, M) \sim N$ or $F_M \nabla N$.

Example 3.2. Let the additive group $(Z_6, +)$ under a multiplication given in the following, $(Z_6, +, \cdot)$ is a (right) near-ring. Let the Q-fuzzy soft (F, N) over $N = Z_6$, when $F : N \times Q \rightarrow P(Z_6)$ is a set-valued function defined by

$F(x) = \{y \in Z_6 / xay \leftrightarrow xy \in \{0, 3\}\}$ for all $x \in N$. Then $F(0) = F(3) = Z_6$ and

$F(1) = F(2) = F(4) = F(5) = \{0, 3\}$. Hence, it is seen that $F_N \nabla N$.

Let the sub near-ring $M = \{0, 2, 4\}$ of N and left Q-fuzzy soft set (G, M) over N , where $G : M \times Q \rightarrow P(N)$ is defined by $G(x) = \{y \in M / xay \leftrightarrow xy \in \{0, 1, 2\}\} \forall x \in M$. Then $G(0) = \{0, 2, 4\}$, $G(2) = \{0, 4\}$ and $G(4) = \{0, 2\}$. Since $G(0-4) = G(2) = \{0, 4\} \not\supseteq G(0) \cap G(4) = \{0, 2\}$. (G, M) is not a sub near-ring of N . For a near-ring N , the zero-symmetric part of N denoted by N_0 is defined by $N_0 = \{n \in N / n_0 = 0\}$ and the constant part of N denoted by N_c is defined by $N_c = \{n \in N / n_0 = n\}$. It is well known that N_0 and N_c are sub near-rings of N [Pilz]. For a near-ring N , we can obtain at least two Q-fuzzy soft sub near-rings of N using N_0 and N_c .

Example 3.3. Let N be a near-ring and let $F_0 : N_0 \times Q \rightarrow P(N)$ be a set valued function defined by $F_0(x) = \{y \in N_0 / xy \in N_0\} \forall x \in N_0$. Then (F_0, N_0) is a Q-fuzzy soft sub near-ring of N . Infact, for all $x, y \in N_0$ assume that $a \in F_0(x, q) \cap F_0(y, q)$. Then $xa \in N_0$ and $ya \in N_0$. Since N_0 is a sub near-ring of N , then $xa - ya = (x - y)a \in N_0$.

Hence $F(x - y, q) \geq F_0(x, q) \cap F_0(y, q)$, (i.e) the condition (QFSNR - 1) is satisfied.

Since $x \in N_0$ and $ya \in N_0$, then $((xy)a) 0 = x((ya)0) = x_0 = 0$, (i.e) $(xy)a \in N_0$. Hence $a \in F_0(xy,q)$. (i.e) $F_0(xy,q) \geq F_0(x,q) \cap F_0(y,q)$ and this shows us that the condition QFSNR-2 is satisfied. Therefore $(F_0, N_0) \sim N$.

Let $F_c: N_c \times Q \rightarrow P(N)$ be a set valued function defined by $F_c(x) = \{y \in N / (xy, q) \in N\} \forall x \in N_c$. Then (F_c, N_c) is a Q-fuzzy soft sub near- ring of N .

Infact, for all $x, y \in N_c$ assume that $a \in F_c(x, q) \cap F_c(y, q)$. Then $x_a \in N_c$ and $y_a \in N_c$. Since N_c is a sub near-ring of N , then $xa - ya = (x - y)a \in N_c$. Hence $F_c(x - y, q) \geq F_c(x, q) \cap F_c(y, q)$, (i.e) the condition (QFSNR-1) is satisfied. Since $ya \in N_c$, then $((xy)a)0 = (x((ya)0)) = x(ya) = (xy)a$. (i.e) $(xy)a \in N_c$. Hence $a \in F_c(xy, q)$, (i.e) $F_c(xy, q) \geq F_c(x, q) \cap F_c(y, q)$ and this shows us (QFSNR-2) is satisfied. Therefore $(F_c, N_c) \sim N$.

Theorem 3.4. If $F_M \sim N$ and $G_K \sim N$, then $F_M \cap G_K \sim N$.

Proof: By definition 2.10, Let $F_M \cap G_K = (F, M) \cap (G, K) = (H, M \cap K)$,

where $H(x, q) = F(x, q) \cap G(x, q) \forall x \in M$ and $q \in Q$, then for all $x, y \in M \cap K, q \in Q$,

(QFSNR - 1) : $H(x \rightarrow y, q) = F(x - y, q) \cap G(x - y, q) \geq (F(x, q) \cap F(y, q)) \cap (G(x, q) \cap G(y, q)) = (F(x, q) \cap G(x, q)) \cap (F(y, q) \cap G(y, q)) = H(x, q) \cap H(y, q)$ and

(QFSNR - 2) : $H(xy, q) = F(xy, q) \cap G(xy, q) = (F(x, q) \cap F(y, q)) \cap (G(x, q) \cap G(y, q)) = (F(x, q) \cap G(x, q)) \cap (F(y, q) \cap G(y, q)) = H(x, q) \cap H(y, q)$

Therefore $F_M \cap G_K \sim N$ or $H_{M \cap K} \sim N$.

Definition 3.5. Let N_1 and N_2 be near -rings and let $F_M \sim N_1, G_K \sim N_2$. The product of Q-fuzzy soft sub near -rings (F, M) and (G, K) is defined as $(F, M) \times (G, K) = (H, M \times K)$ where $H(x, y)_q = F(x, q) \times G(y, q) \forall (x, y) \in M \times K$ and $q \in Q$.

Theorem 3.6. If $F_M \sim N_1$ and $G_K \sim N_2$, then $F_M \times G_K \sim N_1 \times N_2$.

Proof: Since M and K are sub near-rings of N_1 and N_2 respectively, then $M \times K$ is a sub near-ring of $N_1 \times N_2$. By definition-3.5, Let $F_M \times G_K = (F, M) \times (G, K) = (H, M \times K)$, where $H(x, y)_q = F(x, q) \times G(y, q) \forall (x, y) \in M \times K$ and $q \in Q$. Then $\forall (x_1, y_1), (x_2, y_2) \in M \times K$,

(QFSNR - 1) : $H((x_1, y_1)_q - (x_2, y_2)_q) = H((x_1 - x_2), (y_1 - y_2))_q = F((x_1 - x_2)_q \times (y_1 - y_2)_q) \geq (F(x_1, q) \cap F(x_2, q)) \times (G(y_1, q) \cap G(y_2, q)) = (F(x_1, q) \times G(y_1, q)) \cap (F(x_2, q) \times G(y_2, q)) = H(x_1, y_1)_q \cap H(x_2, y_2)_q$.

(QFSNR - 2) : $H((x_1, y_1)_q (x_2, y_2)_q) = H((x_1 x_2), (y_1 y_2))_q = F(x_1, x_2, q) \times G(y_1, y_2, q) \geq (F(x_1, q) \cap G(x_2, q)) \times (G(y_1, q) \cap G(y_2, q)) = (F(x_1, q) \times G(y_1, q)) \cap (F(x_2, q) \times G(y_2, q))$

$= H(x_1, y_1)_q \cap H(x_2, y_2)_q$

Hence $F_M \times G_K = H_{M \times K} \sim N_1 \times N_2$.

Lemma 3.7. If $F_M \sim N$, then $F(0, q) \geq F(x, q) \forall x \in M$ and $q \in Q$.

Proof: Since (F, M) is a Q-fuzzy soft sub near-ring of N , $F(0, q) = F(x_q - x_q) \geq F(x, q) \cap F(x, q) = F(x, q) \forall x \in M$ and $q \in Q$.

Proposition 3.8. If $F_M \sim N$, then $M_F = \{x \in M / F(x, q) = F(0, q)\}$ is a sub near- ring of N .

Proof: We need to show that $x - y \in M_F$ and $xy \in M_F \forall x, y \in M_F$ and then to show that $F(x - y, q) = F(0, q)$ and $F(xy, q) = F(0, q) \forall x, y \in M_F$ and $q \in Q$.

Since $x, y \in M_F$, then $F(x, q) = F(y, q) = 0$. By lemma - 3.7, $F(0, q) \geq F(x - y, q)$ and $F(0, q) \geq F(xy, q) \forall x, y \in M$. Since (F, M) is a Q-fuzzy soft sub near -ring of N , then $F(x - y, q) \geq F(x, q) \cap F(y, q) = F(0, q)$ and $F(xy, q) \geq F(x, q) \cap F(y, q) = F(0, q) \forall x, y \in M_F$.

Hence $F(x - y, q) = F(0, q)$ and $(xy, q) = F(0, q) \forall x, y \in M_F$.

Therefore M_F is a sub near-ring of N .

4. Q-FUZZY SOFT IDEALS OF NEAR-RINGS

Definition 4.1. Let $I \sim N$ and let (F, I) be a Q-fuzzy soft set over N . If for all $x, y \in I$ and for all $t, s \in N$,

(QFSI₁) : $F(x - y, q) \geq F(x, q) \cap F(y, q)$

(QFSI₂) : $F(t + x - t, q) \geq F(x, q)$

(QFSI₃) : $F(xt, q) \geq F(x, q)$

(QFSI₄) : $F(t(s + x) - ns, q) \geq F(x, q)$,

then (F, I) is called a Q-fuzzy soft ideal of N and denoted by $(F, I) \sim N$ or simply $F_I \sim N$.

If $I \sim_1 N$, (F, I) is a Q-fuzzy soft set over N and if the condition QFSI₁, QFSI₂ and QFSI₄ are satisfied. Then (F, I) is called Q-fuzzy soft left ideal of N and denoted by $(F, I) \sim_l N$ or simply $F_l \sim_l N$ of $I \sim_r N$, (F, I) is called Q-fuzzy soft right ideal of N and denoted by $(F, I) \sim_r N$ or simply $F_r \sim_r N$.

Example 4.2. Let $N = (Z_6, +, \cdot)$ be the near-ring given in Example- 3.2 and let the ideal $I = \{0, 2, 4\}$ of N . Then (F, I) is a Q-fuzzy soft set over N , where $F : I \times Q \rightarrow P(N)$ is a set valued function defined by $F(x, q) = \{y \in I / xy = 0\}, \forall x \in I$, then $F(0, q) = \{0, 2, 4\}$ and $F(2, q) = F(4, q) = \{0\}$. Hence it is such that $F_I \sim N$. Let the ideal $J = \{0, 3\}$ of N and let Q-fuzzy soft set (G, J) over N , where $G : J \rightarrow P(N)$ is a set valued function defined by $G(x, q) = \{y \in J / xy \in \{0, 2, 4\}\} \forall x \in J$. Then we have $G(0, q) = \{0, 3\}$ and $G(3, q) = \emptyset$. It is rarely such that $G_J \sim N$.

Example 4.3. Let $N = \{0, 1, 2, 3, 4, 5\}$ be a near-ring with two binary operations $+$ and \cdot . Let Q-fuzzy soft set (G, N)

) over N , where $G: N \times Q \rightarrow P(N)$ is a set valued function defined by $G(x, q) = \{y \in N/x\alpha y \leftrightarrow xy \in \{0, 1\}\}$, $\forall x \in N$. Then $G(0, q) = G(1, q) = N$ and $G(2, q) = G(3, q) = G(4, q) = G(5, q) = \{0, 1\}$.

Since $G(4+1-4, q) = G(3, q) = \{0, 1\} = G(1, q) = N$, then (G, N) is a Q-fuzzy soft ideal of N .

Theorem 4.4. If $F_1 \sim N$ and $G_j \sim N$, then $F_1 \cap G_j \sim N$.

Proof: We give the proof for Q-fuzzy soft ideals, the same proof can be seen for Q-fuzzy soft left ideals and Q-fuzzy soft right ideals. Since $I, J \sim N$, then $I \cap J \sim N$. By definition - 2.10, $F_1 \cap G_j = (F, I) \cap (G, J) = (H, I \cap J)$, where $H(x, q) = F(x, q) \cap G(x, q) \forall x \in I \cap J$. Then $\forall x, y \in I \cap J$ and $\forall t, s \in N$

$$(QFSI_1) : H(x - y, q) = F(x - y, q) \cap G(x - y, q)$$

$$\geq (F(x, q) \cap F(y, q)) \cap (G(x, q) \cap G(y, q))$$

$$= (F(x, q) \cap G(x, q)) \cap (F(y, q) \cap G(y, q))$$

$$= H(x, q) \cap G(y, q)$$

$$(QFSI_2) : H(t + x - t, q) = F(t + x - t, q) \cap G(t + x - t, q)$$

$$\geq F(x, q) \cap G(x, q)$$

$$= H(x, q)$$

$$(QFSI_3) : H(xt, q) = F(xt, q) \cap G(xt, q)$$

$$\geq F(x, q) \cap G(x, q)$$

$$= H(x, q)$$

$$(QFSI_4) : H(t(s + x) - ts, q) = F(t(s + x) - ts, q) \cap G(t(s + x) - ts, q)$$

$$\geq F(x, q) \cap G(x, q)$$

$$= H(x, q). \text{Therefore } F_1 \cap G_j = H_{I \cap J} \sim N.$$

Definition 4.5. Let N_1 and N_2 be two near-rings and let $F_1 \sim F_j; G_j \sim N_2$. The product of Q-fuzzy soft ideals (F, I) and (G, J) is defined as $(F, I) \times (G, J) = (H, I \times J)$, where $H(x, y)_q = F(x, q) \times G(y, q) \forall (x, y) \in I \times J$.

Theorem 4.6. If $F_1 \sim N_1$ and $G_j \sim N_2$, then $F_1 \times G_j \sim N_1 \times N_2$ (resp., $F_1 \times G_j \sim_r N_1 \times N_2, F_1 \times G_j \sim_r N_1 \times N_2$).

Proof: We give the proof for Q-fuzzy soft ideals, the same proof can be seen for Q-fuzzy soft left ideals and Q-fuzzy soft right ideals. Since $I \sim N_1$ and $J \sim N_2, I \times J \sim N_1 \times N_2$. By definition - 4.5,

$F_1 \times G_j = (F, I) \times (G, J) = (H, I \times J)$, where $H(x, y)_q = F(x, q) \times G(y, q) \forall x, y \in I \times J, q \in Q$, then $\forall (x_1, y_1), (x_2, y_2) \in (I, J)$ and $\forall (t_1, t_2), (s_1, s_2) \in N_1 \times N_2$,

$$(QFSI_1) : H((x_1, x_2) - (y_1, y_2), q)$$

$$= H((x_1 - x_2, y_1 - y_2), q)$$

$$= F(x_1 - x_2)_q \times G(y_1 - y_2)_q$$

$$(\leq (F(x_1, q) \cap F(x_2, q)) \cap (G(y_1, q) \cap G(y_2, q)))$$

$$= F(x_1, q) \times G(y_1, q) \cap F(x_2, q) \times G(y_2, q)$$

$$= H(x_1, y_1)_q \cap H(x_2, y_2)_q$$

$$(QFSI_2) : H((t_1, t_2) + (x_1, y_1) - (t_1, t_2), q)$$

$$= H((t_1 + x_1 - t_1, t_2 + y_1 - t_2), q)$$

$$= F((t_1 + x_1 - t_1), q) \times G((t_2 + y_1 - t_2), q)$$

$$\geq F(x_1, q) \times G(y_1, q)$$

$$= H(x_1, y_1)$$

$$(QFSI_3) : H((x_1, y_1)(t_1, t_2), q)$$

$$= H(x_1 t_1, y_1 t_2)_q$$

$$= F(x_1, q) \times G(y_1, q)$$

$$\geq F(x_1, q) \times G(y_1, q)$$

$$= H(x_1, y_1)_q$$

$$(QFSI_4) : H(((t_1, t_2)((s_1 s_2) + (x_1 y_1)) - (t_1 t_2)(s_1 s_2)), q)$$

$$= H(t_1(s_1 + x_1) - t_1 s_1, t_2(s_2 + y_1) - t_2 s_2, q)$$

$$= F(t_1(s_1 + x_1) - t_1 s_1, q) \times G(t_2(s_2 + y_1) - t_2 s_2, q)$$

$$\geq F(x_1, q) \times G(y_1, q)$$

$$= H(x_1, y_1)_q$$

Therefore $F_1 \times G_j = H_{I \times J} \sim N_1 \times N_2$.

Proposition 4.7. If $F_1 \sim N$, then $I_F = \{x \in I/F(x, q) = F(0, q)\}$ is an ideal of N .

Proof: We need to show that

$$(i) x - y \in I_F$$

$$(ii) t + x - t \in I_F$$

$$(iii) xt \in I_F \text{ and}$$

$$(iv) t(s + x) - ts \in I_F, \forall x, y \in I_F \text{ and } t, s \in N.$$

If $x, y \in I_F$, then $F(x, q) = F(y, q) = 0$. By Lemma - 3.7,

$$F(0, q) \geq F(x - y, q)$$

$$F(0, q) \geq F(t - x - t, q)$$

$$F(0, q) \geq F(xt, q) \text{ and}$$

$$F(0, q) \geq F(t(s + x) - ts) \forall x, y \in I \text{ and } \forall t, s \in N$$

Since (F, I) is Q-fuzzy soft ideal, then $\forall x, y \in I_F$ and $\forall t, s \in N$

$$(i) F(x - y, q) \geq F(x, q) \cap F(y, q) = F(0, q)$$

$$(ii) F(t + x - t, q) \geq F(x, q) = F(0, q)$$

$$(iii) F(xt, q) \geq F(x, q) = F(0, q) \text{ and}$$

$$(iv) F(t(s + x) - ts, q) \geq F(x, q) = F(0, q)$$

Hence $F(x - y, q) = F(0, q), F(t + x - t, q) = F(0, q)$,

$$F(xt, q) = F(0, q) \text{ and } F(t(s + x) - ts, q) = F(0, q),$$

$\forall x, y \in I_F$ and $t, s \in N$. Therefore I_F is an ideal of N .

CONCLUSION

Throughout this paper in a near-ring structure, we study the algebraic properties of Q-fuzzy soft sets which were introduced by Molodtsov as a new mathematical tool for dealing with uncertainty. This work based on Q-fuzzy soft sub near-rings and soft ideals of near-rings. Since every associative ring is near-ring, the results in this study are also true for associative rings.

FUTURE WORK

One could study the Q-fuzzification of soft sub structures of other algebraic structures such as semi rings and fields.

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